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Procurement and Pricing Decision for Trapezoidal Demand Rate and Time Dependent Deterioration

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Abstract: **Deterioration rate may be constant or varies with time. In real life time dependent deterioration is observed in many products like fruits, bakery products, milk products etc. Generally deterioration rate increases as the time passes. In this paper we present an inventory model for the trapezoidal type demand function and time dependent deterioration rate. Demand rate depends on time as well as price. Shortages are allowed with partial backlogging. In order to illustrate solution procedure numerical examples and sensitivity analysis have been demonstrated.**

Keywords: Inventory model, Trapezoidal type demand, deteriorating items, price and time dependent demand

I. INTRODUCTION

In recent years, inventory model for deteriorating items gets focus of many researchers. Deteriorating items having more importance due to high margin which also results high loss associated with them. In real life deterioration is a natural phenomenon which can be defined as a process of becoming impaired or inferior in quality from being used for its original purpose. It is quite essential for the consideration of deterioration rate in the analysis of items like fruits, vegetables, perfumes, pharmaceutical, radioactive substances etc.. With a view to address this research problem and provide pragmatic solution on offer several inventory models have been developed in the past. These models began with classical inventory model [24]-[25] these studies explicitly states that depletion of inventory model is primarily due to constant demand rate. Effects of deterioration on fashion products after their prescribed dates were studied by [26]-[27]. [1] established no shortage inventory model for deteriorating items with constant demand rate and constant deterioration rate. Similarly [2] considered classical Economic Order Quantity (EOQ) model with linearly deterministic demand rate.

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Since then, many researchers have exploited deteriorating model with time varying demand, presuming various assumptions for example[3]-[6] and [3] examined the non-shortage inventory model with a linear dependent on time trend in demand whereas [4] considered EOQ model with constant deterioration and demand rate changes linearly. [6] Developed Lot size inventory models with continuous time varying demand under three replenishment policies for deteriorating items.

However, most of the above cited studies have utilized inventory model with linearly or exponentially, increasing or decreasing demand rate, which limits applicability of these model in real life, as demand for item cannot increase linearly. [7] Was the pioneer researcher to demonstrate inventory model with ramp type demand function, which is much practical in real life as demand linearly increases in the beginning and become constant in second stage when market gets stable. Afterwards [8] considered deterministic and probabilistic demand situation with ramp type demand function of time for the deteriorating items. Study by [9] introduced two possible shortage models (i.e. model starting with no shortage and model starting with shortage) with ramp type demand function of time for the deteriorating items. [10] presented an inventory model with ramp type demand function which depends on time and deterioration rate follows Weibull distribution with partial backlogging. Recently [23] adopted ramp type demand to upstream and downstream trade credit finance with consideration of minimum cost at per unit of time. [22] Presented deterministic inventory model with seasonal demand is price and time dependent and shortage allowed with partial backlogging.

In this paper we have developed an inventory model with time dependent deterioration rate, shortage allowed with partial backlogging presuming trapezoidal type demand function with infinite replenishment. Three phases of trapezoidal demand function has been adopted with three different price and time dependent demand functions. During shortage, number of customer decrease as waiting time increases. Graphical analysis approach used to show concavity of profit function with respect to decision variables. Trapezoidal type demand function consists with three stages, in first stage demand increases then it reached to saturation in second stage in last demand rate decreases. This demand pattern is commonly observed in fruits, vegetables, sea foods items etc. Such model was firstly studied by [17] and [18] who proposed an Inventory model where deterioration rate is time ependent, with trapezoidal type demand and partial backlogging is allowed in the model.

[19] Extended [17] work and presented two models with shortage and no shortage to find out optimal ordered quantity and the optimal pricing policies. [20] Introduced inventory model for Weibull-distributed deteriorating items with trapezoidal type demand where shortage are allowed. [21] Consider inventory model for perishable items with Quadratic trapezoidal demand rate under constant deterioration and partial backlogging.

To this end, the paper is presented in the following manner, notations and assumptions of the model are presented. In section 3 analysis of the model is presented. Section 4 contains mathematical solution procedure and numerical analysis. Section 5 contains the sensitivity analysis at different segments and lastly article end with vivid concluding remarks.

II. NOTATIONS AND ASSUMPTIONS

In this model we considered price and time depended on demand function under the presence of constant deterioration. The demand function is linear in time and power function in price. Inventory once restore will use in one complete demand season. There are three possibilities that inventory will deplete either in phase 1, 2 or 3. All three situations are discussed in three cases. After getting the optimal profit for all the three cases we will select the maximum. This will give the overall optimum value of the profit function. Shortages are allowed and partially backlogged since we assume that the probability of purchasing the item by a customer will decrease with the increase of the waiting time. We provide a discounted price to loyal customers for those who are waiting up to the next replenishment. In this inventory model, we are using the following notations

1.1. Notations

T: Interval length

T_1: Time up to which demand increases.

T_2: Time up to which demand remains constant and then decreases.

T_{LF}: Time epoch at which inventory depletes and shortages start. F =1, 2 and 3 (Decision variable)

I: Initial Inventory level

I_b: backlogged shortage

C_0: ordering cost

C_1: holding cost, per unit per unit time

C: purchase price per unit

θt: time dependent deterioration rate

P: Per unit selling price of item. (Decision variable)

λP : backlogged price ; $(0 < \lambda < 1)$

1.2. Demand function

We consider a price and time-dependent iso-elastic demand function with constant deterioration rate. Demand is a linear function of time and a power function of price. For three different phases, three demand functions are defined as follows:

For phase I: $f_1(P,t) = \dfrac{a+bt}{P^j}$; $0 \le t \le T_1$

For phase II: $f_2(P,t) = \dfrac{a+bT_1}{P^j}$; $T_1 \le t \le T_2$

For phase III: $f_3(P,t) = \dfrac{a-bt}{P^j} + r$; $T_2 \le t \le T$

Here a, b and j are the constant demand parameters. Parameter 'r' provides the flexibility that the demand in the third phase may start with a jump.

1.3. Assumption

Following are the main assumptions of the presented model

(1) The probability that the customer waits during the shortage depends up on the waiting time, as waiting time increases the probability that the customer will wait up to the next replenishment decreases

$\gamma(\eta) = 1-(\eta/T)$; $0 \le \eta \le T$

(2) Backorder price is λP such that $C < \lambda P < P$ for this $\dfrac{C}{P} < \lambda < 1$

(3) Demand function is positive; $f_F(P,T) > 0$

(4) Selling price per unit should be greater than purchasing price and the total holding cost $P > C + TC_1$. The RHS of the expression is known as price floor.

(5) Replenishment rate is infinite.

(6) Demand decrease as price increase

(7) Backlogs may clear at arrival of next replenishment and shortage time cannot be exceed the length T

III. ANALYSIS

In this model, we have three different cases according to the depletion of inventory in the growth, constant and decline phase

Case 1: Inventory depletes in growth phase

$(0 \le T_{L1} \le T_1)$

Holding cost is given by

$$H_1 = C_1[\int_0^{T_{L1}} \{(T_{L1}-0) f_1(P,t) + \theta t\} dt]$$

The revenue earned

$$R_1 = \int_0^{T_{L1}} Pf_1(P,t)dt + \lambda P I_{b1}$$

Here I_{b1} is the backlogged amount is given by

$$I_{b1} = \int_{T_{L1}}^{T_1} f_1(P,t)\gamma(T_1-t)\,dt + \int_{T_1}^{T_2} f_2(P,t)\gamma(T_2-t)\,dt$$
$$+ \int_{T_2}^{T} (f_3(P,t)+r)\gamma(T-t)\,dt$$

The initial inventory level is

$$I_1 = \int_0^{T_{L1}} \{f_1(p,t) + \theta t\} dt$$

Profit per unit time for the first stage

$Net_1 = (R_1 - H_1 - C_0 - C(I_1 + I_{b1}))/T$ (1)

Case 2: Inventory depletes in constant phase

($T_1 \le T_{L2} \le T_2$)

Holding cost is given by

$$H_2 = C_1[\int_0^{T_1} \{(T_1 - 0) f_1(P,t) + \theta t\} dt + \int_{T_1}^{T_{L2}} \{(T_{L2} - 0) f_2(P,t) + \theta t\} dt\ ]$$

R_2 the revenue earned

$$R_2 = \int_0^{T_1} P f_1(P,t) dt + \int_{T_1}^{T_{L2}} P f_2(P,t) dt + \lambda PI_{b2}$$

Here backlogged amount is

$$I_{b2} = \int_{T_{L2}}^{T_2} f_2(P,t)\gamma(T_2 - t) dt + \int_{T_2}^{T} (f_3(P,t) + r)\gamma(T - t) dt$$

And the initial inventory level is

$$I_2 = \int_0^{T_1} \{ f_1(p,t) + \theta t \} dt + \int_{T_1}^{T_{L2}} \{ f_2(p,t) + \theta t \} dt$$

Profit per unit time for the Second stage

$Net_2 = (R_2 - H_2 - C_0 - C(I_2 + I_{b2}))/T$ (2)

Case 3: Inventory depletes in decreasing phase

($T_2 \le T_{L3} \le T$)

Holding cost is given by

$$H_3 = C_1[\int_0^{T_1} \{(T_1 - 0) f_1(P,t) + \theta t\} dt + \int_{T_1}^{T_2} \{(T_2 - 0) f_2(P,t) + \theta t\} dt + \int_{T_2}^{T_{L3}} \{(T_{L3} - 0) f_3(P,t) + \theta t\} dt]$$

The revenue earned is

$$R_3 = \int_0^{T_1} P f_1(P,t) dt + \int_{T_1}^{T_2} P f_2(P,t) dt + \int_{T_2}^{T_{L3}} P(f_3(P,t) + r) dt + \lambda PI_{b3}$$

Here backlogged amount is

$$I_{b3} = \int_{T_{L3}}^{T} (f_3(P,t) + r) * \gamma(T - t) dt$$

Profit per unit time for the third stage

$Net_3 = (R_3 - H_3 - C_0 - C(I + I_{b3}))/T$ (3)

Now we discuss the concavity of the profit function Net_F with respect to the decision variables T_{LF}.

Theorem 1: (a) Net_1 is concave function in T_{L1}.

(b) Net_2 is concave function in T_{L2}.

(c) Net_3 is concave function in T_{L3} if

$$C(a - 4T_1) + P\lambda(r + 2T_1 - a) + T_3 < \frac{P^j[1 - r(C - T_3\theta)]}{C_1(r + T_1\theta - a) - T_2C_1(\theta - b) + b(C - P + 2T_1)} \quad (4)$$

Proof: (a) On partially differentiating Net_1 from expression (1) with respect to T_{L1}, we get

$$\frac{P^{-j}(-a(-CT_1 + T_1(C - P + C_1T_1) + PT_1\lambda) - T_1(T_1P^j(C + C_1T_1)\theta + b(-CT_1 + T_1(C - P + C_1T_1) + PT_1\lambda)))}{T_1T_3} \quad (5)$$

$$\frac{P^{-j}\begin{pmatrix} -T_1P^j(C + 2C_1T_1)\theta + a(C - T_1C_1 - P\lambda) \\ +b(T_1(-C + P - 2C_1T_1) + 2T_1(C - P\lambda)) \end{pmatrix}}{T_1T_3} \quad (6)$$

Now the rhs of the above expression is

$$\{T_1(P^{-j}\theta - b(C - P) - T_1^2(2C_1(P^j\theta - b) - P\lambda(2 + aC_1)\} < P^j - C(a + 2) + P\lambda$$

Thus on simplification

$$\frac{\{T_1(P^{-j}\theta - b(C - P) - T_1^2(2C_1(P^j\theta - b) - P\lambda(2 + aC_1)\}}{P^j - C(a + 2) + P\lambda} < 0$$

Since $C < P\lambda$ therefore the above expression is negative. Thus Net_1 is a concave function with respect to T_{L1}

(b) On partially differentiating Net_2 from expression (2) with respect to T_{L2}, we get

$$\frac{\partial Net_2}{\partial T_{L2}} = \frac{\begin{gathered} P^{-j}(-T_2P^jT_1(C + C_1(-T_1 + T_1))\theta + \\ a(T_2(-C + T_1C_1 + P - C_1T_1) + T_1(C - P\lambda) + \\ bT_1(T_2(-C + T_1C_1 + P - C_1T_1) + T_1(C - P\lambda) \\ +bT_1(T_2(-C + T_1C_1 + P - C_1T_1) + T_1(C - P\lambda))) \end{gathered}}{T_2T_3} \quad (7)$$

$$\frac{\partial^2 Net_2}{\partial T_{L2}^2} = \frac{P^{-j}(-T_2P^{-j}(C - T_1C_1 + 2C_1T_1)\theta + a(C - T_2C_1 - P\lambda) + bT_1(C - T_2C_1 - P\lambda))}{T_2T_3}$$

(8)

The RHS of the above expression is

$$[T_2\{P^j\theta(-C-T_1C_1-2C_1T_1)+C_1(-a-bT_1)\}+bT_1(C-T_1P\lambda)] < P^j - a(C-P\lambda)$$

$$\frac{[T_2\{P^j\theta(-C-T_1C_1-2C_1T_1)+C_1(-a-bT_1)\}+bT_1(C-T_1P\lambda)]}{P^j-a(C-P\lambda)} < 0$$

Since $C < \lambda P$ therefore the above expression is negative

(c) On partially differentiating Net_3 from expression (3) with respect to T_{L3}, we get

$$\frac{\begin{aligned}&P^{-j}(-T_3(-P^{1+j}r+bPT_1+C_1P^jrT_1-bC_1T_1^2\\&+C_1P^jT_1^2\theta+C(-bT_1+P^j(r+T_1\theta)\\&-T_2C_1(-bT_1+P^j(r+T_1\theta)+T_1(P^jr-bT_1)(C-P\lambda)\\&+a(T_3(-C+T_2C_1+P-C_1T_1)+T_1(C-P\lambda)))\end{aligned}}{T^2{}_3} \tag{9}$$

$$\frac{\partial^2 Net_3}{\partial T^2{}_{L3}} = \frac{\begin{aligned}&P^{-j}(a(C-T_3C_1-P\lambda)-P^{-j}(C(-r+T_3\theta)\\&+T_3C_1(r-T_2\theta+2T_1\theta)+\Pr\lambda)+\\&b(T_3(C-T_2C_1-P+2C_1T_1-2T_1(C-P\lambda)))\end{aligned}}{T^2{}_3} \tag{10}$$

The RHS of the expression (9) is negative iff

$$\begin{aligned}&P^{-j}[(aC\text{-}aT_3C_1\text{-}aP\lambda)-P^j(\text{-}Cr\text{+}T_3r\theta)\\&+T_3C_1r\text{-}T_2T_3C_1\theta\text{+}T_3C_1T_1\theta\text{+}Pr\lambda)\\&+b(T_3C\text{-}T_2T_3C_1\text{-}T_3P\text{+}2C_1T_1T_3\text{-}2T_1C\text{+}2T_1P\lambda)< P^j\end{aligned}$$

That is

$$\begin{aligned}&aC\text{-}aT_3C_1\text{-}aP\lambda\text{+}P^jCr\text{-}P^{-j}T_3r\theta\text{+}T_3C_1r\text{-}T_2T_3C_1\theta\\&+T_3C_1T_1\theta\text{+}Pr\lambda\text{+}bT_3C\text{-}bT_2T_3C_1\text{-}bT_3P\\&+2T_1bT_3\text{-}2T_1C\text{+}2T_1P\lambda< P^j\end{aligned}$$

On simplification

$$\begin{aligned}&(aC\text{-}4T_1C)+P\lambda(r\text{+}2T_1\text{-}a)\text{+}P^jr(C\text{-}T_3\theta)\text{+}T_3C_1(r\text{+}T_1\theta\text{-}a)\\&\text{-}T_2T_3C_1(\theta\text{-}b)\text{+}bT_3(C\text{-}P\text{+}2T_1) < P^j\end{aligned}$$

Implies that

$$\begin{aligned}&C(a-4T_1)+P\lambda(r+2T_1-a)+T_3[C_1(r+T_1\theta-a)-T_2C_1(\theta-b)+b(C-P+2T_1]\\&< \quad P^j[1-r(C-T_3\theta)]\end{aligned}$$

Thus

$$\begin{aligned}&C(a-4T_1)+P\lambda(r+2T_1-a)+T_3\\&< \quad \frac{P^j[1-r(C-T_3\theta)]}{C_1(r+T_1\theta-a)-T_2C_1(\theta-b)+b(C-P+2T_1)}\end{aligned}$$

Thus if the above condition satisfied profit function is concave with respect to T_{L3}. Now we check the concavity of the profit function with respect to P.

$$\begin{aligned}\frac{\partial Net_1}{\partial P} &= \frac{1}{6T_1T_2T_3^2}P^{-1-j}(-3T_1T_3(T_2^2-T_3^2)P^{1-j}r\lambda\\&-3a(T_1^3T_3(C_j-(-1+j)P\lambda)-T_1^2T_2T_3(C_j-(-1+j)P\lambda)\\&+T_2T_3\mathrm{T}_1^2(C_j-(-1+j)P\lambda)+T_1T_2(T_2^2(C_j-(-1+j)P\lambda)\\&-T_2T_3\mathrm{T}_1^2(C_j-(-1+j)P\lambda)+T_1T_2(T_2^2(C_j-(-1+j)P\lambda)\\&-T_2T_3(C_j-(-1+j)P\lambda)-T_3(T_1(2C_j+2P-2jP+C_1jT_1)\\&+T_3(C_j+P\lambda-jP\lambda))))+\mathrm{b}(-3\mathrm{T}_1^4T_3(C_j-(-1+j)P\lambda)\\&+2T_1^3T_2T_3(C_j-(-1+j)P\lambda)+3T_1^2T_2^2T_3(C_j-(-1+j)P\lambda)\\&-2\mathrm{T}_2T_3T_1^3(C_j-(-1+j)P\lambda)+T_1T_2(2T_2^3(C_j-(-1+j)P\lambda)\\&+T_3(T_1^2(3C_j+3P-3jP+2C_1jT_1)-2T_3^2(C_j+P\lambda-jP\lambda)))))\end{aligned} \tag{11}$$

and

$$\begin{aligned}\frac{\partial^2 Net_1}{\partial P^2} &= \frac{1}{6T_1T_2T_3^2}jP^{-2-j}(3aT_1^3T_3(\mathrm{C}+C_j+P\lambda-jP\lambda)\\&-T_1^2T_2T_3(\mathrm{C}+\mathrm{C}_j+P\lambda-jP\lambda)+T_1T_2(T_2^2(\mathrm{C}+C_j+P\lambda-jP\lambda)\\&-T_2T_3(C+C\,\mathrm{j}+P\lambda-jP\lambda)-T_3(T_1(2\,\mathrm{C}(1+\mathrm{j})-2(-1+\mathrm{j})\,\mathrm{P}\\&+C_1(1+j)T_1)+T_3(C+C\,\mathrm{j}+P\lambda-jP\lambda)))\\&b(3T_1^4T_3(C+C\,\mathrm{j}+P\lambda-jP\lambda)-2T_1^3T_2T_3(C+C\,\mathrm{j}+P\lambda-jP\lambda)\\&-3T_1^2T_2^2T_3(C+C\,\mathrm{j}+P\lambda-jP\lambda)+2T_2T_3T_1^3(C+C\,\mathrm{j}+P\lambda-jP\lambda)\\&-T_1T_2(2T_2^3(C+C\,\mathrm{j}+P\lambda-jP\lambda)+T_3(T_1^2(3C(1+j)-3(-1+j)P\\&+2C_1(1+j)T_1)-2T_3^2(C+C\,\mathrm{j}+P\lambda-jP\lambda)))\end{aligned}$$

Similarly, expressions for phase 2 and phase 3 are also complicated; hence analytically it is difficult to check the concavity of the profit function with respect to P. Thus numerically we will check the concavity of the profit function with respect to P as well as joint concavity of function Net_F with respect to T_{LF} and P.

Let $U = \frac{\partial^2 Net_F}{\partial T_{LF}^2}$, $V = \frac{\partial^2 Net_F}{\partial P_{LF}^2}$ and

$$W = \left(\frac{\partial^2 Net_F}{\partial T_{LF}^2} * \frac{\partial^2 Net_F}{\partial P_{LF}^2}\right) - \left(\frac{\partial^2 Net_F}{\partial T \partial P}\right)^2$$

IV. SOLUTION PROCEDURE

For F =1

Step 1: Solve $\frac{\partial Net_1}{\partial T_{L1}} = 0$ from expression (4) and $\frac{\partial Net_1}{\partial P} = 0$ from expression (11) to obtain the value of T_{L1}^* and P^*.

Step 2. Check $0 < T_{L1}^* < T_1$ and P^* >Price floor. If yes goto next step otherwise check the initial values of the parameters.

Step 3. For this set of (T_{L1}^*, P^*) find the value of $\frac{\partial^2 Net_1}{\partial P^2}$ from expression (12). If this value is negative go to next step.
Step 4. For this set of (T_{L1}^*, P^*) find the value of Net_1 from expression (1).
Step 5. Repeat step 1 to 4 for F = 2 and 3. From Net_1 , Net_2 and Net_3 select the maximum one.

Now we apply the solution procedure in the Numerical example 1 and 2.

Example 1: Now for the $T_1 = 40$, $T_2 = 75$, T = 100, r = 1, a = 40, b = 3.3, j = 1.5, $C_0 = 100$, $\lambda = 0.99$, $C_1 = 0.001$, C = 0.5 by applying solution procedure results are given below

Result: The optimal value of T_L^*, P^*, Net*, for the phase I and II are given in Table 1. Since the concavity of Net_1 and Net_2 with respect to T_L is already proved in Theorem 1 (a) and (b)thus we numerically check the concavity of Net with respect to P in the column 'v' and joint concavity in column 'W'. In Table 2 numerical results are given for case 3. For Net_3 condition for concavity of Net_3 with respect to T_{L3} are presented under column 'lhs' and 'rhs'. Joint concavity of Net are presented under column 'W'. From the table 1 and 2, the overall optimum value of the profit function is 24.6016 accomplish in Phase II at $\theta = .001$ where $T_L^* = 73.8048$, P^* = 1.63919, $I^* = 2299.37$, $I_b^* = -2494.22$, V = -6.68519 and W = .087112

Table 1: Results of numerical example for Phase I and II

Phase I , $0< T_{L1}< T_1$							
θ	T_{L1}^*	P^*	Net*	I^*	I_b^*	V	W
.001	**39.165**	**1.6259**	**17.1278**	**1647.79**	**-329.28**	**-4.67529**	**.109085**
.002	39.156	1.62592	17.1238	1648.61	-328.573	-4.67558	.109081
.003	39.1469	1.62588	17.1198	1649.43	-327.158	-4.67587	.109078
.004	39.1379	1.62583	17.1157	1650.26	-327.865	-4.67616	.109078
.005	39.1289	1.62578	17.1117	1651.08	-326.452	-4.67645	.10907
.006	39.1198	1.62573	17.1077	1651.89	-325.745	-4.67673	.109066
.007	39.1108	1.62568	17.1036	1652.71	-325.039	-4.67702	.109063
.008	39.1017	1.62564	17.0996	1653.53	-324.334	-4.67731	.109059
.009	39.0927	1.62559	17.0956	1654.35	-323.629	-4.67759	.109055
.01	39.0836	1.62554	17.0916	1655.16	-322.924	-4.67788	.109051
Phase II , $T_1< T_{L2}< T_2$							
θ	T_{L2}^*	P^*	Net*	I^*	I_b^*	V	W
.001	**73.8048**	**1.63919**	**24.6016**	**2299.37**	**-2494.22**	**-6.68519**	**.087112**
.002	73.7745	1.63909	24.5874	2302.25	-2492	-6.68598	.0871649
.003	73.7443	1.639	24.5732	2305.12	-2489.78	-6.6876	.0872179
.004	73.7142	1.63891	24.5591	2307.98	-2487.56	-6.68754	.0872707
.005	73.684	1.63881	24.5449	2310.84	-2485.35	-6.68831	.0873234
.006	73.6539	1.63872	24.5308	2313.69	-2483.14	-6.68908	.0873761
.007	73.6239	1.63863	24.5167	2316.53	-2480.93	-6.68984	.0874287
.008	73.5938	1.63854	24.5026	2319.37	-2478.72	-6.6906	.0874813
.009	73.5638	1.63884	24.4885	2322.21	-2476.52	-6.69135	.0875338
.01	73.5338	1.63835	24.4744	2325.03	-2474.31	-6.6921	.0875862

Table 2: Results of numerical example for Phase III

θ	T_L^*	P^*	Net^*	I^*	I_b^*	lhs	rhs	W
.001	**99.3265**	**1.63859**	**20.5895**	**1956.93**	**-4853.28**	**128.954**	**-.352305**	**.243925**
.002	99.3576	1.63846	20.5639	1962.07	-4853.29	128.947	-.351906	.24401
.003	99.3887	1.63833	20.5384	1967.22	-4853.3	128.939	-.351508	.244096
.004	99.4198	1.63819	20.5128	1972.38	-4853.31	128.93	-.351107	.244182
.005	99.4509	1.63809	20.4872	1977.55	-4853.32	128.924	-.350718	.244353
.006	99.482	1.63793	20.4616	1982.72	-4853.33	128.915	-.350311	.244353
.007	99.531	1.6378	20.436	1987.91	-4853.34	128.907	-.349913	.244439
.008	99.5442	1.63766	20.4103	1993.1	-4853.35	128.898	-.349512	.244525
.009	99.5753	1.63753	20.3847	1998.3	-4853.36	128.89	-.349114	.24461
.01	99.6064	1.63739	20.359	2003.51	-4853.37	128.882	-.348713	.244696

lhs and rhs are the left hand side and right hand sight of expression (4)

Figure 2(a), 2(b) and 2(c) shows the joint concavity of the profit function Net_1, Net_2 and Net_3 respectively with respect to $T_{L1}$and P.

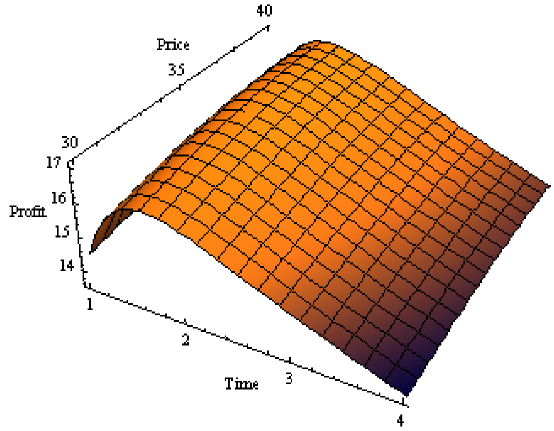

Figure 2(a):Joint concavity of Net_1 with respect to $T_{L1}$and P.

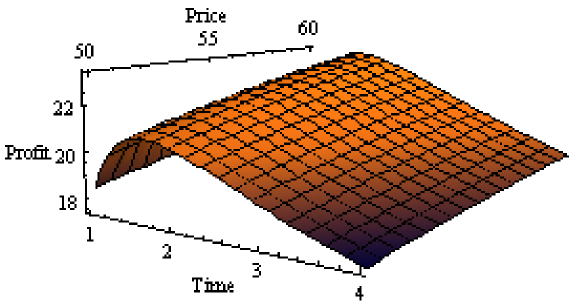

Figure 2(b): Joint concavity of Net_2 with respect to $T_{L2}$and P

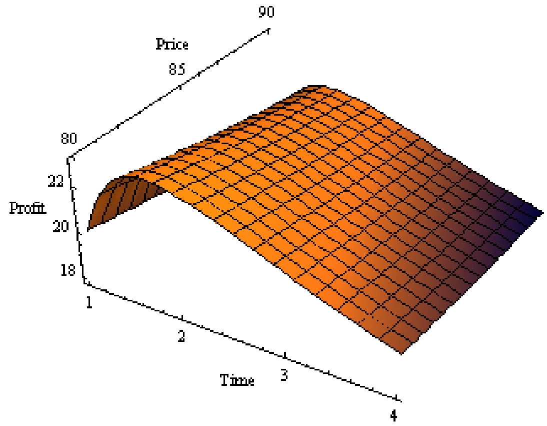

Figure 2(c): Joint concavity of Net_3 with respect to $T_{L3}$and P.

Now we solve Example 1 for the case when j = 0, that is the demand function is independent of price and linear in ‘t’ to observe the change in optimal profit and the ordered inventory.

Example 2: All parameters of example 1 are same except j = 0

Result: From table 3 we observe that the maximum profit 34.7905 accomplish in Phase II at θ = .001 where $T_{L_2}^*$ = 64.5149, P^* = 1.63919, I^* = 4692.25, I_b^* = -3785.12, V = -0.000813361.

Table 3: Results of numerical example 2.

Phase I, 0< T_{L_1} < T_1					
θ	T^*_{L1}	P^*	Net*	I*	I^*_b
.001	**33.3671**	**1.6259**	**30.5976**	**3611.62**	**439.326**
.002	33.3626	1.62592	30.5953	3612.06	439.89
.003	33.3583	1.62588	30.5916	3612.51	440.428
.004	33.3541	1.62583	30.5876	3612.96	440.954
.005	33.3498	1.62578	30.5837	3613.41	441.492
.006	33.3456	1.62573	30.5798	3613.86	442.017
.007	33.3413	1.62568	30.5759	3614.31	442.555
.008	33.337	1.62564	30.5722	3614.75	443.093
.009	33.3328	1.62559	30.5683	3615.2	443.618
.01	33.3285	1.62554	30.5443	3615.65	444.155
Phase II, T_1< T_{L2} < T_2					
.001	**64.5149**	**1.63919**	**34.7905**	**4692.25**	**-3785.12**
.002	64.5031	1.63909	34.7776	4675.32	-3783.38
.003	64.4913	1.639	34.7649	4677.11	-3781.63
.004	64.4794	1.63891	34.7522	4678.9	-3779.87
.005	64.4677	1.63881	34.7393	4680.69	-3778.14
.006	64.4558	1.63872	34.7266	4682.48	-3776.38
.007	64.444	1.63863	34.7139	4684.26	-3774.64
.008	64.4322	1.63854	34.7012	4686.05	-3772.9
.009	64.4184	1.63884	34.6968	4687.78	-3770.86
.01	64.4086	1.63835	34.6757	4689.61	-3769.41
Phase III, T_2< T_{L3} < T					
.001	**1.63859**	99.7752	43.6012	4071.3	-64.8109
.002	1.63846	99.7898	43.5701	4076.27	-60.6112
.003	1.63833	99.8044	43.5391	4081.25	-56.4101
.004	1.63819	99.819	43.5077	4086.22	-52.2077
.005	1.63809	99.8337	43.4778	4091.21	-47.9752
.006	1.63793	99.8483	43.4455	4096.19	-43.7701
.007	1.6378	99.8629	43.4145	4101.18	-39.5638
.008	1.63766	99.8775	43.383	4106.17	-35.3561
.009	1.63753	99.8922	43.3519	4111.17	-31.1182
.01	1.63739	99.9068	43.3204	4116.17	-26.9079

Table 5 Sensitivity analysis profit function

	-100%	-75%	-50%	-25%	0%	25%	50%	75%	100%
a	0.841353	0.631015	0.420677	0.210339	0	-0.210337	-0.420675	-0.631013	-0.84135
b	0.189889	0.142417	0.0949448	0.0474727	0	-0.0474714	-0.094944	-0.142416	-0.18989
j	-1.13299	-0.766876	-0.462695	-0.209971	0	0.174452	0.319392	0.439813	0.539863
T_1	1.44966*	0.815444	0.362423*	0.0906069	0	0.0906095*	0.362438*	0.815493*	1.44978*
P	Not valid	1.98671	0.288849	0.0369821	0	0.0156089	0.0456158	0.0785047	0.110418

V. SENSITIVITY ANALYSIS

In this section sensitivity analysis of the profit function with respect to decision variable and the demand parameters are presented. The sensitivity analysis of the optimal profit function which lies in phase 2 for θ = 0.001. In Table 3 contains the percent loss with respect to change in the demand parameters (a, b and j) and the decision variables (T_1, P).

From Table 5 inference can be drawn that there is a linear change with respect to a and b but in opposite directions. For instance if we decreases 'a' by 25 % then the percent loss will be 0.210339 while if we increase the 'a' by 25 % then the percent loss will be -0.210337. It means that we can increase the profit by increasing the value of 'a' it is due to by increasing 'a' the demand rate increases. Similar type of change is observed with respect to 'b' but with a different rate. If we decreases 'b' by 25% the percent loss will be 0.0474727 while by increasing 'b' by 25% the percent loss will be -0.0474714

In case of 'j' the profit function increases with the increment in 'j' and decreases with the decrement in 'j'. The rate of change in profit function is more in case of negative change in 'j' as compare to positive change in 'j'.

VI. CONCLUSION

We develop a mathematical model for trapezoidal type demand function which is price and time dependent, deterioration rate is time depended, shortage and partial backlogging allowed. Fruit vegetables and sea food ruin very fast so we adopted trapezoidal type demand function which may grow fast in first stage and reached on saturation point in second stage then get decline in last stage. We use numerical examples to analyzing profit function and applied sensitivity analysis to get change in profit as change in different parameters. We accomplished maximum profit in second stage.

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