Procurement and Pricing Decision for Trapezoidal Demand Rate and Time Dependent Deterioration

Jitendra Kaushik, Ashish Sharma

Abstract: Deterioration rate may be constant or varies with time. In real life time dependent deterioration is observed in many products like fruits, bakery products, milk products etc. Generally deterioration rate increases as the time passes. In this paper we present an inventory model for the trapezoidal type demand function and time dependent deterioration rate. Demand rate depends on time as well as price. Shortages are allowed with partial backlogging. In order to illustrate solution procedure numerical examples and sensitivity analysis have been demonstrated.

Keywords: Inventory model, Trapezoidal type demand, deteriorating items, price and time dependent demand

1. INTRODUCTION

In recent years, inventory model for deteriorating items gets focus of many researchers. Deteriorating items having more importance due to high margin which also results high loss associated with them. In real life deterioration is a natural phenomenon which can be defined as a process of becoming impaired or inferior in quality from being used for its original purpose. It is quite essential for the consideration of deterioration rate in the analysis of items like fruits, vegetables, perfumes, pharmaceutical, radioactive substances etc.. With a view to address this research problem and provide pragmatic solution on offer several inventory models have been developed in the past. These models began with classical inventory model [24]-[25] these studies explicitly states that depletion of inventory model is primarily due to constant demand rate. Effects of deterioration on fashion products after their prescribed dates were studied by [26]-[27]. [1] established no shortage inventory model for deteriorating items with constant demand rate and constant deterioration rate. Similarly [2] considered classical Economic Order Quantity (EOQ) model with linearly deterministic demand rate. Since then, many researchers have exploited deteriorating model with time varying demand, presuming various assumptions for example[3]-[6] and [3] examined the non-shortage inventory model with a linear dependent on time trend in demand whereas [4] considered EOQ model with constant deterioration and demand rate changes linearly. [6] Developed Lot size inventory models with continuous time varying demand under three replenishment policies for deteriorating items. However, most of the above cited studies have utilized inventory model with linearly or exponentially, increasing or decreasing demand rate, which limits applicability of these model in real life, as demand for item cannot increase linearly. [7] Was the pioneer researcher to demonstrate inventory model with ramp type demand function, which is much practical in real life as demand linearly increases in the beginning and become constant in second stage when market gets stable. Afterwards [8] considered deterministic and probabilistic demand situation with ramp type demand function of time for the deteriorating items. Study by [9] introduced two possible shortage models (i.e. model starting with no shortage and model starting with shortage) with ramp type demand function of time for the deteriorating items. [10] presented an inventory model with ramp type demand function which depends on time and deterioration rate follows Weibull distribution with partial backlogging. Recently [23] adopted ramp type demand to upstream and downstream trade credit finance with consideration of minimum cost at per unit of time. [22] Presented deterministic inventory model with seasonal demand is price and time dependent and shortage allowed with partial backlogging.

In this paper we have developed an inventory model with time dependent deterioration rate, shortage allowed with partial backlogging presuming trapezoidal type demand function with infinite replenishment. Three phases of trapezoidal demand function has been adopted with three different price and time dependent demand functions. During shortage, number of customer decrease as waiting time increases. Graphical analysis approach used to show concavity of profit function with respect to decision variables. Trapezoidal type demand function consists with three stages, in first stage demand increases then it reached to saturation in second stage in last demand rate decreases. This demand pattern is commonly observed in fruits, vegetables, sea foods items etc. Such model was firstly studied by [17] and [18] who proposed an Inventory model where deterioration rate is time dependent, with trapezoidal type demand and partial backlogging is allowed in the model.

[19] Extended [17] work and presented two models with shortage and no shortage to find out optimal ordered quantity and the optimal pricing policies. [20] Introduced inventory

Revised Manuscript Received on October 05, 2019.

Jitendra Kaushik, Department of Master of Business Administration and PGDM, Sunstone Eduniversity, Lower Ground, 006 Spring House, Grand Mall, MG Road Gurugram, Haryana 122002, INDIA., AND, Department of Mathematics, Institute of Applied sciences and Humanities, GLA University, NH-2, Mathura-281406, INDIA. Email: jitendrakaushik1986@gmail.com

Ashish Sharma, Department of Mathematics, Institute of Applied sciences and Humanities, GLA University, NH-2, Mathura-281406, INDIA. Email: ashishsharma@glau.ac.in
model for Weibull-distributed deteriorating items with trapezoidal type demand where shortage are allowed. [21]
Consider inventory model for perishable items with Quadratic trapezoidal demand rate under constant deterioration and partial backlogging.
To this end, the paper is presented in the following manner, notations and assumptions of the model are presented. In section 3 analysis of the model is presented. Section 4 contains mathematical solution procedure and numerical analysis. Section 5 contains the sensitivity analysis at different segments and lastly article end with vivid concluding remarks.

II. NOTATIONS AND ASSUMPTIONS

In this model we considered price and time depended on demand function under the presence of constant deterioration. The demand function is linear in time and power function in price. Inventory once restore will use in one complete demand season. There are three possibilities that inventory will deplete either in phase 1, 2 or 3. All three situations are discussed in three cases. After getting the optimal profit for all the three cases we will select the maximum. This will give the overall optimum value of the profit function. Shortages are allowed and partially backlogged since we assume that the probability of purchasing the item by a customer will decrease with the increase of the waiting time. We provide a discounted price to loyal customers for those who are waiting up to the next replenishment. In this inventory model, we are using the following notations

1.1. Notations

T: Interval length
T1: Time up to which demand increases.
T2: Time up to which demand remains constant and then decreases.
T1f: Time epoch at which inventory depletes and shortages start. F = 1, 2 and 3 (Decision variable)
I: Initial Inventory level
Ib: backlogged shortage
C: ordering cost
C1: holding cost, per unit per unit time
P: Per unit selling price of item. (Decision variable)
λP: backlogged price ; (0 < λ < 1)
θt: time dependent deterioration rate
P: Per unit selling price of item. (Decision variable)
λP: backlogged price ; (0 < λ < 1)

1.2. Demand function

We consider a price and time-dependent iso-elatic demand function with constant deterioration rate. Demand is a linear function of time and a power function of price. For three different phases, three demand functions are defined as follows:

For phase I: \( f_1 (P, t) = \frac{a + bt}{P^j} \); \( 0 \leq t \leq T_1 \)

For phase II: \( f_2 (P, t) = \frac{a + bT_1}{P^j} \); \( T_1 \leq t \leq T_2 \)

For phase III: \( f_3 (P, t) = \frac{a - bt}{P^j} + r \); \( T_2 \leq t \leq T \)

Here a, b and j are the constant demand parameters. Parameter ‘r’ provides the flexibility that the demand in the third phase may start with a jump.

I.3. Assumption

Following are the main assumptions of the presented model

1) The probability that the customer waits during the shortage depends up on the waiting time, as waiting time increases the probability that the customer will wait up to the next replenishment decreases \( \gamma(\eta) = 1 - (\eta / T) \); \( 0 \leq \eta \leq T \)

2) Backorder price is \( \lambda P \) such that \( C < \lambda P < P \) for this \( C < \lambda P < 1 \)

3) Demand function is positive; \( f_1 (P, T) > 0 \)

4) Selling price per unit should be greater than purchasing price and the total holding cost \( P > C + TC_1 \). The RHS of the expression is known as price floor.

5) Replenishment rate is infinite.

6) Demand decrease as price increase

7) Backlogs may clear at arrival of next replenishment and shortage time cannot be exceed the length T

III. ANALYSIS

In this model, we have three different cases according to the depletion of inventory in the growth, constant and decline phase

Case 1: Inventory depletes in growth phase

\( 0 \leq T_{1f} \leq T_1 \)

Holding cost is given by

\[ H_1 = C [ \int_0^{T_{1f}} (T_{1f} - 0) f_1(P, t) + \theta t) dt ] \]

The revenue earned

\[ R_1 = \int_0^{T_{1f}} P f_1(P, t) dt + \lambda PI_{bl} \]

Here \( I_{bl} \) is the backlogged amount is given by

\[ I_{bl} = \int_{T_{1f}}^{T_1} f_1(P, t) \gamma(T_{1f} - t) dt + \int_{T_{1f}}^{T_2} f_1(P, t) \gamma(T_{2} - t) dt \]

\[ + \int_{T_{1f}}^{T} f_2(P, t) \gamma(T - t) dt \]

The initial inventory level is

\[ I_1 = \int_0^{T_{1f}} \{ f_1(P, t) + \theta t \} dt \]

Profit per unit time for the first stage

\[ \text{Net}_1 = (R_1 - H_1 - C_0 - C (I_1 + I_{bl})) / T \]

Case 2: Inventory depletes in constant phase

\( T_{1f} \leq T_{12} \leq T_2 \)

Holding cost is given by

\[ \int_0^{T_{12}} f_2(P, t) \gamma(T_{12} - t) dt \]

\[ + \int_{T_{12}}^{T} f_3(P, t) \gamma(T - t) dt \]

The initial inventory level is

\[ I_1 = \int_0^{T_{12}} \{ f_2(P, t) + \theta t \} dt \]

Profit per unit time for the first stage

\[ \text{Net}_1 = (R_1 - H_1 - C_0 - C (I_2 + I_{bl})) / T \]

Published By: Blue Eyes Intelligence Engineering & Sciences Publication

Retrieval Number: L30291081219/2019©BEIESP
DOI: 10.35940/ijitee.L3029.1081219

2827
\[ H_2 = C_i \left[ \int_0^{T_1} (T_1 - 0) f_1(P, t) + \theta t \, dt \right] + \int_{T_1}^{T_2} (T_2 - 0) f_2(P, t) + \theta t \, dt \]

R_2, the revenue earned

\[ R_2 = \int_0^{T_1} Pf_1(P, t) \, dt + \int_{T_1}^{T_2} Pf_2(P, t) + \lambda \pi b_3 \]

Here backlogged amount is

\[ I_{b3} = \int_0^{T_1} f_2(P, t) \gamma (T_1 - t) \, dt + \int_{T_1}^{T_2} f_3(P, t) + \theta t \, dt \]

And the initial inventory level is

\[ I_2 = \left\{ f_1(P, t) + \theta t \right\} dt + \int_{T_1}^{T_2} f_3(P, t) + \theta t \, dt \]

Profit per unit time for the second stage

\[ \text{Net}_2 = \left( R_2 - H_2 - C_0 - C (I_2 + I_{b3}) \right) / T \] (2)

Case 3: Inventory depletes in decreasing phase

\( T_2 \leq T_{L3} \leq T \)

Holding cost is given by

\[ H_3 = C_i \left[ \int_0^{T_1} (T_1 - 0) f_1(P, t) + \theta t \, dt \right] + \int_{T_1}^{T_2} (T_2 - 0) f_2(P, t) + \theta t \, dt \]

\[ + \int_{T_2}^{T_3} (T_3 - 0) f_3(P, t) + \theta t \, dt \]

The revenue earned is

\[ R_3 = \int_0^{T_1} Pf_1(P, t) \, dt + \int_{T_1}^{T_2} Pf_2(P, t) + \theta t \, dt + \int_{T_2}^{T_3} Pf_3(P, t) + \gamma (T_3 - t) \, dt \]

Here backlogged amount is

\[ I_{b3} = \int_{T_1}^{T_2} f_3(P, t) + \theta t \, dt \]

Profit per unit time for the third stage

\[ \text{Net}_3 = \left( R_3 - H_3 - C_0 - C (I_2 + I_{b3}) \right) / T \] (3)

Now we discuss the concavity of the profit function Net_3 with respect to the decision variables T_{L1}.

**Theorem 1:**

(a) Net_1 is concave function in T_{L1}.

(b) Net_2 is concave function in T_{L2}.

(c) Net_3 is concave function in T_{L3} if

\[ C(a - 4T_1) + P \lambda (r + 2T_1 - a) + T_3 < \frac{P^j - 1 - (r - C - T_3 \beta)}{C_i (r + T_1 \beta - a) - T_2 C_i (\theta - b) + b(C - P + 2T_1)} \]

**Proof:**

(a) On partially differentiating Net_1 from expression (1) with respect to T_{L1}, we get

\[ P^j (a(-CT_1 + T_1(C - P + C I_1) + PT_1 \lambda) - T_1(T_1 P^j (C + C I_1) \theta + b(-CT_1 + T_1(C - P + C I_1) + PT_1 \lambda))) \]

\[ \frac{1}{T_1^3} \] (5)

Now the rhs of the above expression is

\[ \frac{[T_1(P^j \theta - b(C - P) - T_1^2 (2C_1(P^j \theta - b) - P \lambda(2 + aC_1))]}{T_1^3} ] < P^j - (C(a + 2) + P \lambda) \]

Thus on simplification

\[ \frac{[T_1(P^j \theta - b(C - P) - T_1^2 (2C_1(P^j \theta - b) - P \lambda(2 + aC_1))]}{T_1^3} ] < 0 \]

Since C < P\lambda, therefore the above expression is negative. Thus Net_1 is a concave function with respect to T_{L1}.

(b) On partially differentiating Net_2 from expression (2) with respect to T_{L2}, we get

\[ P^j (-T_2^2 P^j T_1 (C + C_1(-T_1 + T_1)) \theta + a(T_2(C + T_1 C_1 + P - C_1 T_1) + T_1(C - P \lambda) + bT_1(T_2 - C + T_1 C_1 + P - C_1 T_1) + T_1(C - P \lambda)) \]

\[ \frac{\partial \text{Net}_2}{\partial T_{L2}} = \frac{-T_2^2}{T_2 T_3} \] (7)

\[ \frac{\partial^2 \text{Net}_2}{\partial T_{L2}^2} = \frac{a(C - T_2 C_1 - P \lambda) + bT_1(C - T_2 C_1 - P \lambda))}{T_2^2 T_3} \] (8)

The R.H.S. of the above expression is

\[ [T_2(P^j \theta(-C - T_1 C_1 - 2C_1 T_1) + C_1(-a - bT_1)) + bT_1(C - T_1 P \lambda)] \]

\[ < P^j - a(C - P \lambda) \]
\[ T_{1}(P^j \theta - C - T_{2}C_{3} - 2C_{2}T_{1}) + C_{1}(-a - b\theta T_{1}) + bT_{1}(C - T_{1}\mu) < 0 \]

Since \( C < \lambda P \) therefore the above expression is negative.

(e) On partially differentiating \( N_{t} \) from expression (3) with respect to \( T_{1,3} \), we get

\[
P^{-j}(-T_{3}(-P^{j+1} + \alpha \mu T_{1} + C_{1}P^{j}r T_{1} - bC_{1}T_{1}^{2} + P^{j}(r + T_{1}\theta) - T_{2}C_{1}(-bT_{1} + P^{j}(r + T_{1}\theta) + T_{1}(P^{j} - bT_{1})(C - P\mu) + a(T_{3}(-C + T_{2}C_{1} + P - C_{1}T_{1}) + T_{1}(C - P\mu)))
\]

\[ T_{2}^{3} \tag{9} \]

The RHS of the expression (9) is negative iff

\[
P^{-j}[(aC - aC_{1}C_{a}P\mu) - P^{-j}(C\theta + T_{3}\theta)] + T_{1}C_{1}(r + T_{1}\theta + 2T_{2}\theta) + Pr\mu + b(T_{1}C_{1} - T_{2}C_{1} - T_{3}P + 2T_{1}C_{T1} - 2T_{1}C_{T1} + 2T_{1}P < P^{j} \]

That is

\[
aC_{a}C_{1}C_{a}P\mu + P\theta(C_{1} + T_{3}\theta) + T_{1}C_{1}(r + T_{1}\theta + 2T_{2}\theta) + Pr\mu + b(T_{1}C_{1} - T_{2}C_{1} - T_{3}P + 2T_{1}C_{T1} - 2T_{1}C_{T1} + 2T_{1}P < P^{j} \]

On simplification

\[
(aC_{a}C_{1}C_{a}P\mu + P\theta(C_{1} + T_{3}\theta) + T_{1}C_{1}(r + T_{1}\theta + 2T_{2}\theta) + Pr\mu + b(T_{1}C_{1} - T_{2}C_{1} - T_{3}P + 2T_{1}C_{T1} - 2T_{1}C_{T1} + 2T_{1}P < P^{j} \]

Thus implies that

\[
C(a - 4T_{1}) + P\theta(r + T_{1}\theta - a) + T_{3}C_{1}(r + T_{1}\theta - a) + T_{1}C_{1}(\theta - b) + b(C - P + 2T_{1}) < P^{j} \]

Thus

\[
C(a - 4T_{1}) + P\theta(r + T_{1}\theta - a) + T_{3}
\]

\[
< \frac{P^{j}[1 - r - C - T_{1}\theta]}{C_{1}(r + T_{1}\theta - a) - T_{2}C_{1}(\theta - b) + b(C - P + 2T_{1})}
\]

Thus if the above condition satisfied profit function is concave with respect to \( T_{1,3} \). Now we check the concavity of the profit function with respect to \( P \).

\[
\frac{\partial^{2} Net_{t1}}{\partial P^{2}} = \frac{1}{6T_{1}T_{2}T_{3}} p^{-1} - j(3T_{1}T_{1}T_{3} - T_{2}^{2})(p^{j} - 3) \mu 
\]

\[
-3\lambda T_{3}T_{3}(C_{j} - (-1 + j)\mu P\mu) - T_{1}T_{2}T_{3}(C_{j} - (-1 + j)\mu P\mu) + T_{2}T_{3}T_{1}(C_{j} - (-1 + j)\mu P\mu) + T_{2}T_{3}(C_{j} - (-1 + j)\mu P\mu) + T_{2}T_{3}(1C_{j} + 2P - 2p + C_{1}T_{1})
\]

\[
+T_{2}(C_{j} + P\mu - j\mu P\mu)) + b(-3T_{1}T_{3}(C_{j} - (-1 + j)\mu P\mu) + 2T_{1}T_{3}(C_{j} - (-1 + j)\mu P\mu) + 3T_{1}T_{2}T_{3}(C_{j} - (-1 + j)\mu P\mu) - 2T_{2}T_{3}(C_{j} - (-1 + j)\mu P\mu) + T_{2}T_{3}(2C_{j} + 3P - 3p + 2C_{2}T_{1})
\]

\[
+T_{3}(T_{1}^{3}(3C_{j} + 3P - 3p + 2C_{2}T_{1}) + 3T_{2}(C_{j} + P\mu - j\mu P\mu))) \tag{11}
\]

Similarly, expressions for phase 2 and phase 3 are also complicated; hence analytically it is difficult to check the concavity of the profit function with respect to \( P \). Thus numerically we will check the concavity of the profit function with respect to \( P \) as well as joint concavity of function \( Net_{t1} \) with respect to \( T_{1,3} \) and \( P \).

Let

\[
U = \frac{\partial^{2} Net_{t1}}{\partial T_{1,3}^{2}} \quad , \quad V = \frac{\partial^{2} Net_{t1}}{\partial T_{1,3}^{2}} \quad \text{and} \quad W = \frac{\partial^{2} Net_{t1}}{\partial T_{1,3}^{2}} - \left( \frac{\partial^{2} Net_{t1}}{\partial T_{1,3}^{2}} \right)^{2}
\]

IV. SOLUTION PROCEDURE

For \( F=1 \)

Step 1: Solve \( \frac{\partial Net_{t1}}{\partial T_{1,3}} = 0 \) from expression (4) and \( \frac{\partial Net_{t1}}{\partial P} = 0 \) from expression (11) to obtain the value of \( T_{1,3}^{*} \) and \( P^{*} \).

Step 2. Check \( 0 < T_{1,3}^{*} < T_{1} \) and \( P^{*} > \) Price floor. If yes goto next step otherwise check the initial values of the parameters.

Step 3. For this set of \( (T_{1,3}^{*}, P^{*}) \) find the value of \( \frac{\partial^{2} Net_{t1}}{\partial T_{1,3}^{2}} \)

from expression (12). If this value is negative go to next step.

Step 4. For this set of \( (T_{1,3}^{*}, P^{*}) \) find the value of \( Net_{t1} \) from expression (1).
Step 5. Repeat step 1 to 4 for F = 2 and 3. From Net₁, Net₂ and Net₃ select the maximum one.

Now we apply the solution procedure in the Numerical example 1 and 2.

**Example 1**: Now for the T₁ = 40, T₂ = 75, T = 100, r = 1, a = 40, b = 3.3, j = 1.5, C₀ = 100, λ = 0.99, C₁ = 0.001, C = 0.5 by applying solution procedure results are given below

**Result**: The optimal value of T₁⁺, P⁺, Net⁺, for the phase I and II are given in Table 1. Since the concavity of Net₁ and Net₂ with respect to T₁ is already proved in Theorem 1 (a) and (b) thus we numerically check the concavity of Net with respect to P in the column ‘v’ and joint concavity in column ‘W’. In Table 2 numerical results are given for case 3. For Net₁ condition for concavity of Net₁ with respect to T₁⁺ are presented under column ‘lhs’ and ‘rhs’. Joint concavity of Net are presented under column ‘W’. From the table 1 and 2, the overall optimum value of the profit function is 24.6016 accomplish in Phase II at θ = .001 where T₁⁺ = 73.8048, P⁺ = 1.63919, T₂⁺ = 2299.37, I⁺ = -2494.22, V = -6.68519 and W = 0.087112

<table>
<thead>
<tr>
<th>Phase I, 0 &lt; T₁⁺ &lt; T₂⁺</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>.001</td>
</tr>
<tr>
<td>.002</td>
</tr>
<tr>
<td>.003</td>
</tr>
<tr>
<td>.004</td>
</tr>
<tr>
<td>.005</td>
</tr>
<tr>
<td>.006</td>
</tr>
<tr>
<td>.007</td>
</tr>
<tr>
<td>.008</td>
</tr>
<tr>
<td>.009</td>
</tr>
<tr>
<td>.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Phase II, T₁⁺ &lt; T₂⁺ &lt; T₂⁺</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>.001</td>
</tr>
<tr>
<td>.002</td>
</tr>
<tr>
<td>.003</td>
</tr>
<tr>
<td>.004</td>
</tr>
<tr>
<td>.005</td>
</tr>
<tr>
<td>.006</td>
</tr>
<tr>
<td>.007</td>
</tr>
<tr>
<td>.008</td>
</tr>
<tr>
<td>.009</td>
</tr>
<tr>
<td>.01</td>
</tr>
</tbody>
</table>
Table 2: Results of numerical example for Phase III

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( T^*_{L1} )</th>
<th>( P^* )</th>
<th>Net(^*)</th>
<th>( I^* )</th>
<th>( I^*_b )</th>
<th>lhs</th>
<th>rhs</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>.001</td>
<td>99.3265</td>
<td>1.63859</td>
<td>20.5895</td>
<td>1956.93</td>
<td>-4853.28</td>
<td>128.954</td>
<td>-352305</td>
<td>.243925</td>
</tr>
<tr>
<td>.003</td>
<td>99.3887</td>
<td>1.63833</td>
<td>20.5384</td>
<td>1967.22</td>
<td>-4853.3</td>
<td>128.939</td>
<td>-351508</td>
<td>.244069</td>
</tr>
<tr>
<td>.004</td>
<td>99.4198</td>
<td>1.63819</td>
<td>20.5128</td>
<td>1972.38</td>
<td>-4853.31</td>
<td>128.93</td>
<td>-351107</td>
<td>.244182</td>
</tr>
<tr>
<td>.005</td>
<td>99.4509</td>
<td>1.63809</td>
<td>20.4872</td>
<td>1977.55</td>
<td>-4853.32</td>
<td>128.924</td>
<td>-350718</td>
<td>.244353</td>
</tr>
<tr>
<td>.006</td>
<td>99.482</td>
<td>1.63793</td>
<td>20.4616</td>
<td>1982.72</td>
<td>-4853.33</td>
<td>128.915</td>
<td>-350311</td>
<td>.244353</td>
</tr>
<tr>
<td>.007</td>
<td>99.531</td>
<td>1.6378</td>
<td>20.436</td>
<td>1987.91</td>
<td>-4853.34</td>
<td>128.907</td>
<td>-349913</td>
<td>.244439</td>
</tr>
<tr>
<td>.009</td>
<td>99.5753</td>
<td>1.63753</td>
<td>20.3847</td>
<td>1998.3</td>
<td>-4853.36</td>
<td>128.89</td>
<td>-349114</td>
<td>.24461</td>
</tr>
<tr>
<td>.01</td>
<td>99.6064</td>
<td>1.63739</td>
<td>20.359</td>
<td>2003.51</td>
<td>-4853.37</td>
<td>128.882</td>
<td>-348713</td>
<td>.244696</td>
</tr>
</tbody>
</table>

lhs and rhs are the left hand side and right hand sight of expression (4)

Figure 2(a), 2(b) and 2(c) shows the joint concavity of the profit function Net\(_1\), Net\(_2\) and Net\(_3\) respectively with respect to \( T_{L1} \) and \( P \).

**Figure 2(a): Joint concavity of Net\(_1\) with respect to \( T_{L1} \) and \( P \).**

**Figure 2(b): Joint concavity of Net\(_2\) with respect to \( T_{L2} \) and \( P \).**

**Figure 2(c): Joint concavity of Net\(_3\) with respect to \( T_{L3} \) and \( P \).**

Now we solve Example 1 for the case when \( j = 0 \), that is the demand function is independent of price and linear in 't' to observe the change in optimal profit and the ordered inventory.

**Example 2:** All parameters of example 1 are same except \( j = 0 \)

**Result:** From table 3 we observe that the maximum profit 34.7905 accomplish in Phase II at \( \theta = .001 \) where \( T^*_{L1} = 64.5149, P^* = 1.63919, I^* = 4692.25, I^*_b = -3785.12, V = -0.000813361. \)
Table 3: Results of numerical example 2.

<table>
<thead>
<tr>
<th>Phase I, $0 &lt; T_L &lt; T_1$</th>
<th>$T_{L1}$</th>
<th>$P^*$</th>
<th>Net $I^*$</th>
<th>$I_b^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.001</td>
<td>33.3671</td>
<td>1.6259</td>
<td>30.5976</td>
<td>3611.62</td>
</tr>
<tr>
<td>.002</td>
<td>33.3626</td>
<td>1.6292</td>
<td>30.5953</td>
<td>3612.06</td>
</tr>
<tr>
<td>.003</td>
<td>33.3583</td>
<td>1.6288</td>
<td>30.5916</td>
<td>3612.51</td>
</tr>
<tr>
<td>.004</td>
<td>33.3541</td>
<td>1.6253</td>
<td>30.5876</td>
<td>3612.96</td>
</tr>
<tr>
<td>.005</td>
<td>33.3498</td>
<td>1.6278</td>
<td>30.5837</td>
<td>3613.41</td>
</tr>
<tr>
<td>.006</td>
<td>33.3456</td>
<td>1.6257</td>
<td>30.5798</td>
<td>3613.86</td>
</tr>
<tr>
<td>.007</td>
<td>33.3413</td>
<td>1.6256</td>
<td>30.5759</td>
<td>3614.31</td>
</tr>
<tr>
<td>.008</td>
<td>33.337</td>
<td>1.6254</td>
<td>30.5722</td>
<td>3614.75</td>
</tr>
<tr>
<td>.009</td>
<td>33.3328</td>
<td>1.6259</td>
<td>30.5683</td>
<td>3615.2</td>
</tr>
<tr>
<td>.01</td>
<td>33.3285</td>
<td>1.6254</td>
<td>30.5443</td>
<td>3615.65</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Phase II, $T_1 &lt; T_L &lt; T_2$</th>
<th>$T_{L2}$</th>
<th>$P^*$</th>
<th>Net $I^*$</th>
<th>$I_b^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.001</td>
<td>64.5149</td>
<td>1.6391</td>
<td>34.7905</td>
<td>4692.25</td>
</tr>
<tr>
<td>.002</td>
<td>64.5031</td>
<td>1.6390</td>
<td>34.7776</td>
<td>4675.32</td>
</tr>
<tr>
<td>.003</td>
<td>64.4913</td>
<td>1.639</td>
<td>34.7649</td>
<td>4677.11</td>
</tr>
<tr>
<td>.004</td>
<td>64.4794</td>
<td>1.6391</td>
<td>34.7522</td>
<td>4678.9</td>
</tr>
<tr>
<td>.005</td>
<td>64.4677</td>
<td>1.6388</td>
<td>34.7393</td>
<td>4680.69</td>
</tr>
<tr>
<td>.006</td>
<td>64.4558</td>
<td>1.6387</td>
<td>34.7266</td>
<td>4682.48</td>
</tr>
<tr>
<td>.007</td>
<td>64.444</td>
<td>1.6386</td>
<td>34.7139</td>
<td>4684.26</td>
</tr>
<tr>
<td>.008</td>
<td>64.4322</td>
<td>1.6385</td>
<td>34.7012</td>
<td>4686.05</td>
</tr>
<tr>
<td>.009</td>
<td>64.4184</td>
<td>1.6384</td>
<td>34.6968</td>
<td>4687.78</td>
</tr>
<tr>
<td>.01</td>
<td>64.4086</td>
<td>1.6385</td>
<td>34.6757</td>
<td>4689.61</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Phase III, $T_2 &lt; T_L &lt; T$</th>
<th>$T_{L3}$</th>
<th>$P^*$</th>
<th>Net $I^*$</th>
<th>$I_b^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.001</td>
<td>1.63859</td>
<td>99.7752</td>
<td>43.6012</td>
<td>4071.3</td>
</tr>
<tr>
<td>.002</td>
<td>1.63846</td>
<td>99.7898</td>
<td>43.5701</td>
<td>4076.27</td>
</tr>
<tr>
<td>.003</td>
<td>1.63833</td>
<td>99.8044</td>
<td>43.5391</td>
<td>4081.25</td>
</tr>
<tr>
<td>.004</td>
<td>1.63819</td>
<td>99.819</td>
<td>43.5077</td>
<td>4086.22</td>
</tr>
<tr>
<td>.005</td>
<td>1.63809</td>
<td>99.8337</td>
<td>43.4778</td>
<td>4091.21</td>
</tr>
<tr>
<td>.006</td>
<td>1.63793</td>
<td>99.8483</td>
<td>43.4455</td>
<td>4096.19</td>
</tr>
<tr>
<td>.007</td>
<td>1.6378</td>
<td>99.8629</td>
<td>43.4145</td>
<td>4101.18</td>
</tr>
<tr>
<td>.008</td>
<td>1.63766</td>
<td>99.8775</td>
<td>43.383</td>
<td>4106.17</td>
</tr>
<tr>
<td>.009</td>
<td>1.63753</td>
<td>99.8922</td>
<td>43.3519</td>
<td>4111.17</td>
</tr>
<tr>
<td>.01</td>
<td>1.63739</td>
<td>99.9068</td>
<td>43.3204</td>
<td>4116.17</td>
</tr>
</tbody>
</table>
Table 5 Sensitivity analysis profit function

<table>
<thead>
<tr>
<th></th>
<th>-100%</th>
<th>-75%</th>
<th>-50%</th>
<th>-25%</th>
<th>0%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.84135</td>
<td>0.631015</td>
<td>0.420677</td>
<td>0.210339</td>
<td>0</td>
<td>-0.210337</td>
<td>-0.420675</td>
<td>-0.631013</td>
<td>-0.84135</td>
</tr>
<tr>
<td>b</td>
<td>0.18988</td>
<td>0.142417</td>
<td>0.094944</td>
<td>0.047472</td>
<td>0</td>
<td>-0.0474714</td>
<td>-0.094944</td>
<td>-0.142416</td>
<td>-0.18989</td>
</tr>
<tr>
<td>j</td>
<td>-1.13299</td>
<td>-0.76687</td>
<td>-0.462695</td>
<td>-0.209971</td>
<td>0</td>
<td>0.174452</td>
<td>0.319392</td>
<td>0.439813</td>
<td>0.53986</td>
</tr>
<tr>
<td>T</td>
<td>1.44966*</td>
<td>0.815444</td>
<td>0.362423</td>
<td>0.090606</td>
<td>0</td>
<td>0.0906095</td>
<td>0.362438</td>
<td>0.815493</td>
<td>1.44978*</td>
</tr>
<tr>
<td>P</td>
<td>Not valid</td>
<td>1.98671</td>
<td>0.288849</td>
<td>0.036982</td>
<td>0</td>
<td>0.0156089</td>
<td>0.045615</td>
<td>0.078504</td>
<td>0.11041</td>
</tr>
</tbody>
</table>

V. SENSITIVITY ANALYSIS

In this section sensitivity analysis of the profit function with respect to decision variable and the demand parameters are presented. The sensitivity analysis of the optimal profit function which lies in phase 2 for θ = 0.001. In Table 3 contains the percent loss with respect to change in the demand parameters (a, b and j) and the decision variables (T, P).

From Table 5 inference can be drawn that there is a linear change with respect to a and b but in opposite directions. For instance if we decreases ‘a’ by 25 % then the percent loss will be 0.210339 while if we increase the ‘a’ by 25 % then the percent loss will be -0.210337. It means that we can increase the profit by increasing the value of ‘a’. It is due to by increasing ‘a’ the demand rate increases. Similar type of change is observed with respect to ‘b’ but with a different rate. If we decreases ‘b’ by 25% the percent loss will be 0.0474727 while by increasing ‘b’ by 25% the percent loss will be -0.0474714

In case of ‘j’ the profit function increases with the increment in ‘j’ and decreases with the decrement in ‘j’. The rate of change in profit function is more in case of negative change in ‘j’ as compare to positive change in ‘j’.

VI. CONCLUSION

We develop a mathematical model for trapezoidal type demand function which is price and time dependent, deterioration rate is time dependent, shortage and partial backlogging allowed. Fruit vegetables and sea food ruin very fast so we adopted trapezoidal type demand function which may grow fast in first stage and reached on saturation point in second stage then get decline in last stage. We use numerical examples to analyzing profit function and applied sensitivity analysis to get change in profit as change in different parameters. We accomplished maximum profit in second stage.

REFERENCES


22. Pratibha Sharma, Ashish Sharma and Sanjay Jain “Inventory model for deteriorating items with price and time-dependent seasonal demand” Int. J. Procurement Management, Vol. 12, No. 4, 2019

23. Hui-Ling Yang “An Inventory Model for Ramp-Type Demand with Two-Level Trade Credit Financing Linked to Order Quantity” Open Journal of Business and Management, 2019, 7, 427-446.

24. F.W. Harris, How many parts to make at once, (1913).


AUTHORS PROFILE

Jitendra Kaushik, Assistant Professor of Mathematics in the Department of MBA and PGDM at Sunstone Eduversity and Ph.D. Research scholar in Department of Mathematics, GLA University Mathura. He received degree of M.Phil in Mathematics from Bundelkhand University as merit holder. His research interest in Inventory modelling.

Dr. Ashish Sharma, is Associate Professor in the Department of Mathematics at GLA University, Mathura, India. He received the Ph.D. degree in Statistics from Devi Ahilya University, Indore, India. His research interests are primarily in inventory modeling, pricing and facility allocation. His articles have appeared in journals such as International journal of production economics, Computer and Industrial Engineering, Mathematical and Computer Modelling.