Enhanced Similarity for Spectral Clustering using Local Steering Features

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Abstract:In the field of clustering, spectral clustering (SC) has become an effective tool to analyze complex non-convex data using only pairwise affinity between the data points. Many novel affinity metrics have been proposed in the literature which use local features such as color, spatial coordinates, and texture. Some of these methods used SC for image segmentation [1, 2]. In this work, we have used the covariance matrix of the pixels in a patch and proposed an orientation based feature of a pixel called steering feature. This feature is robust and data-driven. The steering feature is used to enhance the construction of affinity metric for spectral clustering proposed by Shi and Malik [1]. Using the Nystrom framework [2] on images from BSD500 benchmark dataset, we have shown that the proposed affinity metric gives better result than Shi and Malik [1].

Keywords:Spectral clustering, Affinity matrix, Steering Kernel Regression, Nystrom method.

I. INTRODUCTION

Spectral Clustering(SC) is a prominent clustering technique which uses local similarity to produce optimal clusters. It can also cluster complex data such as non-convex data. With the relative ease of implementation of SC, it has become a popular method for clustering. This ease of implementation led to its applications in fields such as clustering, image segmentation, handwritten character recognition, and web data clustering. The critical step in SC is the construction of the affinity matrix, which encapsulates the similarity between the data points. It has been shown in the literature [3, 4, 5] that local features such as color, density, and texture play an important role in enhancing the pairwise affinity.

SC has been used to provide a solution to the image segmentation problem in the works proposed by Shi et al.[1], Chang and Yeung[6], and Fowlkes et al. [2]. Apart from spatial and color features of the images, additional features such as local gradients and texture could help in constructing strong similarity metric. To enhance the pairwise similarity, we have used the local dominant orientation of the gradients estimated from the local covariance matrices. The dominant orientation of the gradients at a point is related to eigenvectors of the covariance matrix. We note that the same covariance matrix is used in the works of Harris et al.[7] for

detecting corners and edges. In their work, the authors make use of the structure tensor, which is an unbiased estimate of the covariance matrix of the gradients for the pixels in the local window. A naive estimation of the covariance matrix is given in Eq (1).

$$C_i =$$
$$\begin{bmatrix} \sum_{x_j \in w_i} z_x(x_j)z_x(x_j) & \sum_{x_j \in w_i} z_x(x_j)z_y(x_j) \\ \sum_{x_j \in w_i} z_x(x_j)z_y(x_j) & \sum_{x_j \in w_i} z_y(x_j)z_y(x_j) \end{bmatrix} (1)$$

where z_x and z_y are first gradients along x and y directions and w_i is a local analysis window around the pixel of consideration x_i. The naive covariance matrix might be rank deficient or unstable. To this end, we have considered the works of Takeda et al.[8, 9] in which a data-driven approach for the construction of the covariance matrix has been proposed. The authors use the construction of a covariance matrix in steering kernel regression, which is further used for performing image processing operations such as denoising and upscaling. The covariance matrices thus constructed are robust to noise and data-driven.

To arrive at the proximity between two pixels, we make use of the corresponding covariance matrices as proposed by Takeda et al[8]. In our work, a steering feature vector is proposed as an additional feature for an image pixel which is based on covariance matrices obtained from local neighborhoods. The steering feature proposed by us is used to enhance the similarity definition proposed by Shi and Malik[1].

The outline of this paper is as follows: In section II, different types of affinity metrics proposed in the literature are discussed. In section III, we discuss the traditional spectral clustering algorithm given by Shi and Malik[1] and the theoretical background of the covariance matrix. The proposed similarity measure is presented in section IV. Section V discusses the results obtained. We conclude our work in section VI.

II. RELATED METHODS

In this section, we present some of the relevant works in SC. Shi and Malik[1] modeled the image segmentation as graph partitioning problem and proposed a normalized cut (Ncut) for segmenting the graph effectively. The Ncut is used to increase the within cluster similarity and decrease the without cluster similarity. The similarity between data points p_k and p_l is given as:

$$A_{kl} = \begin{cases} \exp(\frac{-||X_k - X_l||^2}{2 \times \sigma^2}) & ||X_k - X_l|| < \epsilon \\ 0 & otherwise \end{cases}$$
$$. \exp(\frac{-||F_k - F_l||^2}{2 \times \sigma^2}).$$

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where X_k and X_l are the spatial features and F_k and F_l are any other features of points p_k and p_l respectively. The feature could be anything from color to texture information. If we consider the spatial and color coordinates of the pixels as features of consideration. The following would be the similarity between two data points p_i and p_j

$$A_{ij} = e^{\frac{-||x_i - x_j||^2}{2\sigma_x^2}} . e^{\frac{-||c_i - c_j||^2}{2\sigma_c^2}} \quad (4)$$

where x_i, x_j are the spatial coordinates, c_i and c_j are the color coordinates of p_i and p_j respectively. The parameter σ_x and σ_c are sigma values for space and color coordinates respectively.

Chang and Yeung[6] proposed a robust path-based spectral clustering method, in which the paths between data points in the graph are used to define pairwise similarity. The intuition behind their work is that if the paths between the corresponding nodes go through high-density regions, then the similarity between them is higher. Chang and Yeung applied their method on image data set BSD500 [10] and obtained encouraging results.

Zhang et al.[11] have used Common Nearest Neighbors(CNN) based affinity measure. They proposed that, if two points have a high number of CNN, then they have high similarity and vice versa. The similarity proposed by them is:

$$A_{kl} = \begin{cases} \exp(\frac{-d_{kl}^2}{2 \times \sigma^2 \times (CNN_{kl}+1)}) & k \neq l \\ 0 & k = l \end{cases} (5)$$

where p_k, $p_l \in$ S, the data set. σ is the scale parameter and the number of common nearest neighbors between p_k, p_l is given by CNN_{kl}.

Yang et al.[12] have proposed a density-based similarity metric in which two points are similar if there are paths are connecting them which are lying in high-density regions. The following is the similarity measure defined by them. Adjustable line segment length in their work is defined as :

$$ALS(p_k, p_l) = (e^{\rho \ dist(p_k, p_l)} - 1)^{1/\rho}$$

where $dist(p_k, p_l)$ is given by Euclidean distance between p_k and p_l, $\rho > 1$ is density factor. The distance between two points p_k and p_l is defined as the minimum sum of adjustable line segment length between points in the path from p_k and p_l. The metric is sensitive to local density and is data-dependent. However, this method is computationally expensive.

Li et al.[13] have shown a way to improve the affinity matrix based on the neighborhood of points. If two points p_i, p_j are within ϵ distance of each other they are termed as neighbors. Then neighborhood matrix N is constructed such that, if points p_i, p_j are neighbors then $n_{ij} = 1$. Using N the neighborhood property is propagated based on following conditions: if $n_{ij} = 1$, $n_{jk} = 1$ and $n_{ik} = 0$ then set $n_{ik} = 1$ and $n_{ki} = 1$. Also set the similarity between p_i, p_k, s_{ik} as average of s_{ij} and s_{jk}. In this way, both similarity and neighborhood matrices are updated till convergence.

Diao et al.[14] proposed a similarity using local projection distance-based measure for SC. They define Local Projection Neighborhood(LPN) of p_i and p_j, where the points in the overlap of neighborhoods of both the points are considered as LPN. The two points in consideration are connected using a line. The points in the LPN are projected onto the line segment connecting the points p_i and p_j. A novel distance between the projected points is defined, and the summation of these projection distances is used to create a modified similarity metric. From the literature review we can observe that the local features play an important role in estimating pairwise simlarity effectively.

III. SC ALGORITHM AND THEORETICAL BACKGROUND

In this section, we discuss the SC algorithm as given by Shi and Malik[1] (NCUTS) and the theoretical background of the steering kernel regression, proposed by Takeda et al.[8]. In a typical SC algorithm, the data is modeled as a graph, and it is partitioned optimally to reveal the inherent clusters. To handle large datasets, KNN graph is constructed in our experiments. This method of graph construction keeps the graph sparse and reduces the overall computation.

The key steps in NCUTS method as proposed by Shi and Malik[1] are summarized as:

1. Given an image or image sequence, set up a weighted graph $G = (V, E)$ and set the weight on edge connecting two nodes to be a measure of the similarity between the two nodes. Let W be the matrix in which the similarities between the points are incorporated, and D be the degree matrix. The similarity is obtained using Eq. (2).
2. Solve $(D - W)x = \lambda x$ for eigenvectors with the smallest eigenvalues.
3. Use the eigenvector with the second smallest eigenvalue to bipartition the graph.
4. Decide if the current partition should be subdivided and recursively repartition the segmented parts if necessary.

After discussing the key steps in the NCUTS algorithm, the following section explores the theoretical background of steering matrices.

A. *Theoretical background*

In this subsection, the theoretical background of the steering matrix as given by Takeda et al. [9] is discussed. The main contribution of this work is to estimate local orientation using the local gradients. The covariance matrix incorporates the local gradients as given in Eq. (1). As mentioned in the introduction, the covariance matrix proposed by Eq. (1) could be rank deficient or unstable. To overcome these issues, we have used the construction of the covariance matrix as proposed in Takeda et al.[8]. The following is the theoretical background of the steering matrix used in their work.

1) *Steering matrix:*

Takeda et al.[8] proposed data adapted steering kernel regression method, which incorporates radiometric properties into regression in addition to sample location and density. The idea of this method is to align the kernel along edges so that more relevant information is obtained using the kernel. They proposed the following steps in this process:

1. Using any standard gradient estimator, an initial estimate of the image gradients is obtained.
2. The dominant orientation of local gradients is evaluated using the initial estimate.
3. Using the local orientation, the kernel is steered in shape and size so that it aligns along edges.

Takeda et al.[8] define H_is as data-dependent full matrices called steering matrices. At a data point X_i, it is defined as:

$$H_i^{steer} = h\mu_i C_i^{-\frac{1}{2}} \quad (6)$$

where h is a global smoothing parameter, μ_i is the local density parameter and C_i is the symmetric covariance matrix based on differences in local gray values. In order to incorporate the local gradient features in estimating the covariance matrix, they define the covariance matrix as follows:

$$C_i = \gamma_i U_{\theta_i} \Lambda_i U_{\theta_i}^T,$$
$$U_{\theta_i} = \begin{bmatrix} cos\theta_i & sin\theta_i \\ -sin\theta_i & cos\theta_i \end{bmatrix}$$
$$\Lambda_i = \begin{bmatrix} \sigma_i & 0 \\ 0 & \sigma_i^{-1} \end{bmatrix}$$

where U_{θ_i} is the rotation matrix and Λ_i is the elongation matrix. The three parameters θ_i, γ_i and σ_i are used to define covariance matrix. These parameters correspond to rotation, scaling and elongation parameters respectively and are obtained from the local gradients[8].

The matrix C_i that gives the dominant local orientation information at a pixel (p_i) is used as an additional feature in our work. In the following section, we describe the construction of steering feature vector from C_i.

IV. PROPOSED SIMILARITY METRIC BASED ON STEERING FEATURE(SFS)

In this section, we present the construction of the steering feature (SF) at a pixel and propose the similarity metric using the SFs. As mentioned in the theoretical background of the steering matrix, at each pixel (p_i), we obtain a covariance matrix (C_i) which contains local gradients.
The following are the steps in constructing SF at all the pixels.

1. Calculate the covariance matrix C_i at all the pixels (p_is) using Eq. (7).
2. Convert all C_is into vectors of size 1x4.
3. At each pixel (p_i) take the vectors of points in its 8-neighborhood and concatenate them into a Steering Feature (SF) of size 1x36.
4. The constructed SF incorporates orientation information at p_i.

SF enables us to capture the orientation information in the neighborhood of a pixel. The orientation is an important feature, especially when the pixel is located on edge or region with gradient.The proposed Steering Feature based Similarity (SFS) is defined as :

$$A_{ij} = e^{\frac{-||x_i - x_j||^2}{2\sigma_x^2}} . e^{\frac{-||c_i - c_j||^2}{2\sigma_c^2}} . e^{\frac{-||s_i - s_j||^2}{2\sigma_s^2}} \quad (10)$$

where x_i, x_j are the spatial coordinates, c_i and c_j are the color coordinates, s_i and s_j are the steering features of pixels p_i, p_j. The three parameters σ_x, σ_c and σ_s are the three sigma values of space, color and steering coordinates respectively.The proposed similarity metric is an

enhancement over the similarity metric proposed in Eq. (4) by Shi and Malik[1].
The advantages of the proposed similarity can be stated as follows:

1. The proposed similarity metric incorporates an additional feature of local orientation by using the covariance matrix over a patch as an additional feature.
2. Additionally, since the steering feature is constructed over a patch, it becomes robust to noise.

We used the proposed similarity SFS in the SC algorithm proposed by Shi and Malik and refer to it as Steering Features based Spectral Clustering (SFSC). The results of SFSC are presented in the following section.

$$(8)$$

V. RESULTS AND ANALYSIS

We present and discuss the results of the SFSC method in this section. For our experiments, we have considered challenging images from Berkeley Segmentation Dataset (BSD500)[10]. We used the Nystrom framework for running our experiments. We have compared the results of NCUTS and the proposed SFSC methods. The three-parameter inputs to finding the similarity in SFSC method are sigma values corresponding to spatial, color, and steering features, respectively. We have estimated the value of sigma using empirical studies. Benchmark code given in BSD500 was used to evaluate the OIS F-measure with respect to the ground truth. Table 1 displays the OIS F-measure of the proposed SFSC method and NCUTS method proposed by Shi and Malik [1].

Table 1: Comparison of methods using F score on imagesfrom BSD500 dataset

Images	NCUTS	Best	SFSC	Best
3063	0.71 $\pm$ 0	0.71	0.52 $\pm$ 0.38	**0.79**
3096	0.65 $\pm$ 0	0.65	0.64 $\pm$ 0.43	**0.79**
8049	0.82 $\pm$ 0	0.82	0.8 $\pm$ 0.06	**0.86**
8068	0.74 $\pm$ 0	0.74	0.76 $\pm$ 0.06	**0.82**
12003	0.71 $\pm$ 0.01	0.71	0.7 $\pm$ 0.04	**0.73**
22090	0.75 $\pm$ 0.01	0.75	0.72 $\pm$ 0.1	**0.79**

Since there is non-determinism in Kmeans step of spectral clustering algorithms, the methods were run 20 times, and the average and best values are presented in Table 1.
The first column of Table 1 presents the test images considered from the test dataset of BSD500 benchmark dataset for our experiments.

The second column displays the average and standard deviation of the results of OIS-F-score using the NCUTS algorithm on the test images. The third column gives the best OIS-F-score obtained by using NCUTS algorithm. The fourth column displays the average and standard deviation of the results obtained using the proposed SFSC method. The fifth column displays the best OIS-F-score obtained using the proposed SFSC method.

We can see from the table that, in the average case, the results of NCUTS method are slightly better than SFSC method. In the case of the best case results, the results of SFSC are consistently better than NCUTS results. This shows that the proposed SFSC method which uses steering features improves over NCUTS by Shi and Malik[1](Eq. 2). The image segmentation result plots of these images are displayed in Fig. 1.

VI. CONCLUSION

In this work, the similarity metric proposed by Shi and Malik [1] has been improved by incorporating in it the local orientation information as an additional feature. The local orientation is obtained by construction of the steering feature at the pixel. The steering feature is derived from the covariance matrices of pixels over a patch around the pixel. We have shown through experiments on challenging natural images from BSD500 benchmark that incorporating local orientation enhances the similarity metric and makes the image segmentation more accurate. In our future work, we would like to explore how this enhanced similarity could be used in several SC applications.

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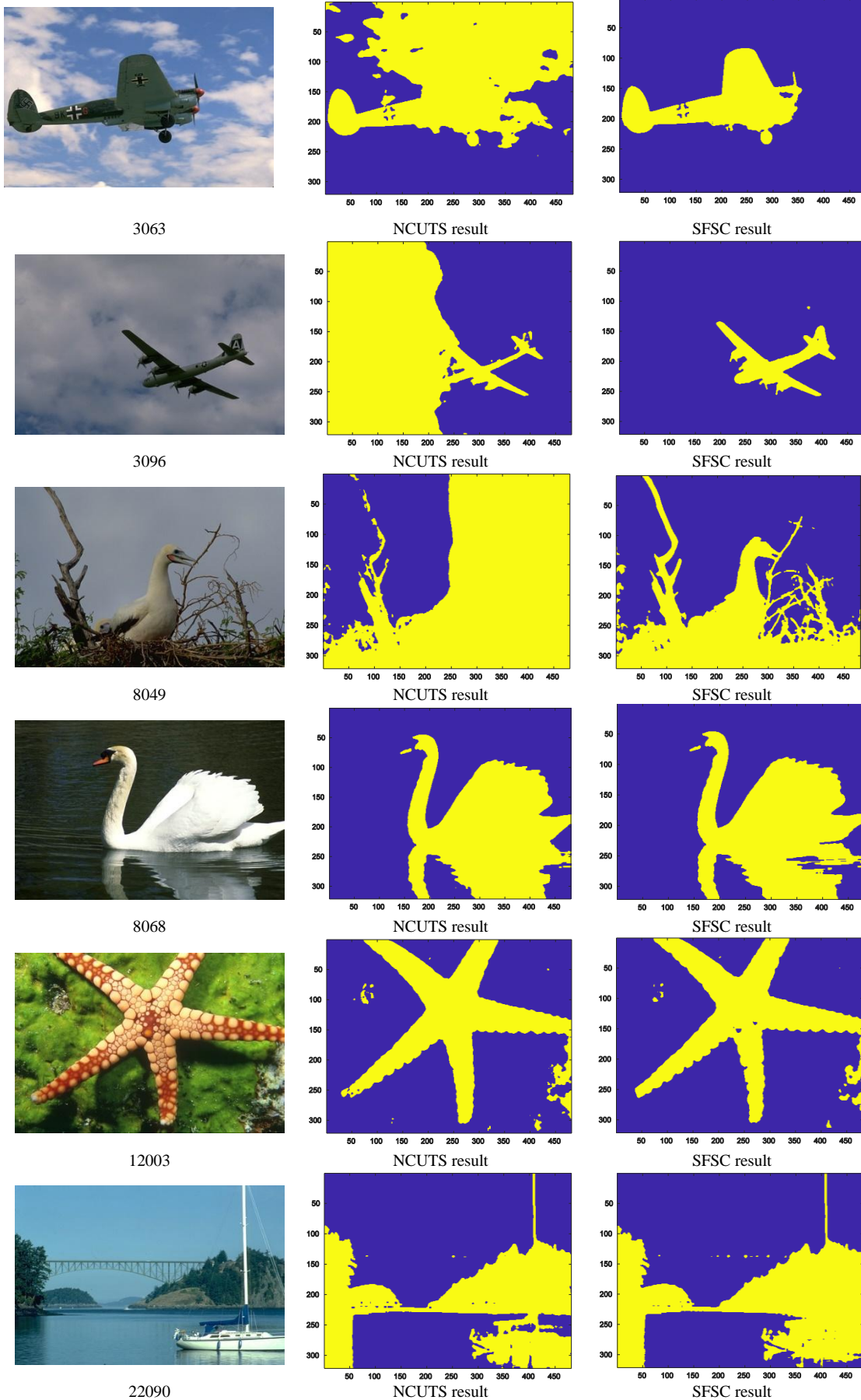
Figure 1: First column represents the original images. The second column presents the result of NCUTSmethod and third columns represent the results of SFSC