

# Dualities between Laplace-Carson Transform and Some Useful Integral Transforms

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**Abstract:** Integral transforms have wide applications in the various disciplines of engineering and science to solve the problems of heat transfer, springs, mixing problems, electrical networks, bending of beams, carbon dating problems, Newton's second law of motion, signal processing, exponential growth and decay problems. In this paper, we will discuss the dualities between Laplace-Carson transform and some useful integral transforms namely Laplace transform, Kamal transform, Aboodh transform, Sumudu transform, Elzaki transform, Mohand transform and Sawi transform. To visualize the importance of dualities between Laplace-Carson transform and mention integral transforms, we give tabular presentation of the integral transforms (Laplace transform, Kamal transform, Aboodh transform, Sumudu transform, Elzaki transform, Mohand transform and Sawi transform) of mostly used basic functions by using mention dualities relations. Results show that the mention integral transforms are strongly related with Laplace-Carson transform.

**Keywords:** Laplace; Kamal; Aboodh; Sumudu; Elzaki; Mohand; Sawi; Laplace-Carson transforms.

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## I. INTRODUCTION

Many process and phenomenon of science, engineering and real life can be expressed mathematically and solved by using integral transforms. The problems arise in the field of signal processing, statistics, thermal science, medicine, fractional calculus, aerodynamics, civil engineering, control theory, cardiology, quantum mechanics, space science, marine science, biology, gravitation, nuclear magnetic resonance, heat conduction, economics, telecommunications, nuclear reactors, detection of diabetes, chemistry, stress analysis, electricity, physics, potential theory, mathematics, deflection of beams, vibration of plates, defense, Brownian motion and many other fields can be easily handle with the help of integral transforms by converting them into mathematical form. In the advanced time, researchers are interested in solving the advance problems of research, science, space, engineering and real life by introducing new integral transforms. Aggarwal and Chaudhary [1] discussed Mohand and Laplace transforms comparatively by solving system of differential equations using both integral transforms.

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Recently many scholars [2-6, 9] used different integral transforms namely Kamal transform, Aboodh transform, Sumudu transform, Elzaki transform, Mohand transform and Mahgoub (Laplace-Carson) transform for evaluating improper integrals which contains error function in the integrand. Mahgoub [7] gave Sawi transform which is a new integral transform. Singh and Aggarwal [8] solved the problems of growth and decay by using Sawi transform.

The aim of this study is to establish duality relations between Laplace-Carson transform and some useful integral transforms namely Laplace transform, Kamal transform, Aboodh transform, Sumudu transform, Elzaki transform, Mohand transform and Sawi transform.

## II. LAPLACE TRANSFORM

The Laplace transform of the function  $Z(\gamma)$ ,  $\gamma \geq 0$  is given by [1]

$$L\{Z(\gamma)\} = \int_0^{\infty} Z(\gamma)e^{-\epsilon\gamma} d\gamma = B(\epsilon) \quad (1)$$

## III. KAMAL TRANSFORM

Kamal transform of the function  $Z(\gamma)$ ,  $\gamma \geq 0$  is given by [2]

$$K\{Z(\gamma)\} = \int_0^{\infty} Z(\gamma)e^{-\frac{\gamma}{\epsilon}} d\gamma = C(\epsilon), \quad (2)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

## III. ABOODH TRANSFORM

Aboodh transform of the function  $Z(\gamma)$ ,  $\gamma \geq 0$  is given by [3]

$$A\{Z(\gamma)\} = \frac{1}{\epsilon} \int_0^{\infty} Z(\gamma)e^{-\epsilon\gamma} d\gamma = D(\epsilon), \quad (3)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

## IV. SUMUDU TRANSFORM

Sumudu transform of the function  $Z(\gamma)$ ,  $\gamma \geq 0$  is given by [4]

$$S\{Z(\gamma)\} = \int_0^{\infty} Z(\epsilon\gamma)e^{-\gamma} d\gamma = F(\epsilon), \quad (4)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

## V. ELZAKI TRANSFORM

Elzaki transform of the function  $Z(\gamma)$ ,  $\gamma \geq 0$  is given by [5]

$$E\{Z(\gamma)\} = \epsilon \int_0^{\infty} Z(\gamma)e^{-\frac{\gamma}{\epsilon}} d\gamma = G(\epsilon), \quad (5)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

## VI. MOHAND TRANSFORM

Mohand transform of the function  $Z(\gamma)$ ,  $\gamma \geq 0$  is given by [1, 6]

$$M\{Z(\gamma)\} = \epsilon^2 \int_0^{\infty} Z(\gamma)e^{-\epsilon\gamma} d\gamma = H(\epsilon), \quad (6)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

**VII. SAWI TRANSFORM**

Sawi transform of the function  $Z(\gamma), \gamma \geq 0$  is given by [7, 8]

$$S^*\{Z(\gamma)\} = \frac{1}{\epsilon^2} \int_0^\infty Z(\gamma) e^{-\frac{\gamma}{\epsilon}} d\gamma = I(\epsilon), \quad (7)$$

$$0 < k_1 \leq \epsilon \leq k_2$$

**VIII. LAPLACE-CARSON TRANSFORM**

Laplace-Carson transform of the function  $Z(\gamma), \gamma \geq 0$  is given by [9]

$$L_*\{Z(\gamma)\} = \epsilon \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma = J(\epsilon), \quad (8)$$

$$0 < k_1 \leq \epsilon \leq k_2$$

**IX. DUALITIES OF LAPLACE-CARSON TRANSFORM WITH SOME USEFUL INTEGRAL TRANSFORMS**

In this section, we define the dualities between Laplace-Carson transform and some useful integral transforms namely Laplace transform, Kamal transform, Aboodh transform, Sumudu transform, Elzaki transform, Mohand transform and Sawi transform.

**A. Duality between Laplace-Carson and Laplace Transforms**

If Laplace-Carson and Laplace transforms of  $Z(\gamma)$  are  $J(\epsilon)$  and  $B(\epsilon)$  respectively then

$$J(\epsilon) = \epsilon B(\epsilon) \quad (9)$$

$$\text{and } B(\epsilon) = \frac{1}{\epsilon} J(\epsilon) \quad (10)$$

**Proof:** From (8),

$$J(\epsilon) = \epsilon \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma$$

Now, using (1) in above Equation, we obtain

$$J(\epsilon) = \epsilon B(\epsilon).$$

To drive (10), we use (1)

$$B(\epsilon) = \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma$$

$$\Rightarrow B(\epsilon) = \frac{1}{\epsilon} \left[ \epsilon \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma \right] \quad (11)$$

It is immediately concluded using (8) in (11),

$$B(\epsilon) = \frac{1}{\epsilon} J(\epsilon).$$

**B. Duality between Laplace-Carson and Kamal Transforms**

If Laplace-Carson and Kamal transforms of  $Z(\gamma)$  are  $J(\epsilon)$  and  $C(\epsilon)$  respectively then

$$J(\epsilon) = \epsilon C\left(\frac{1}{\epsilon}\right) \quad (12)$$

$$\text{and } C(\epsilon) = \epsilon J\left(\frac{1}{\epsilon}\right) \quad (13)$$

**Proof:** Using (8) follows

$$J(\epsilon) = \epsilon \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma \quad (14)$$

Now, using (2) in above equation, we obtain

$$J(\epsilon) = \epsilon C\left(\frac{1}{\epsilon}\right).$$

To drive (13), we use (2)

$$C(\epsilon) = \int_0^\infty Z(\gamma) e^{-\frac{\gamma}{\epsilon}} d\gamma$$

$$\Rightarrow C(\epsilon) = \epsilon \left[ \frac{1}{\epsilon} \int_0^\infty Z(\gamma) e^{-\frac{\gamma}{\epsilon}} d\gamma \right]$$

It is immediately concluded using (8) in above equation,

$$C(\epsilon) = \epsilon J\left(\frac{1}{\epsilon}\right).$$

**C. Duality between Laplace-Carson and Aboodh Transforms**

If Laplace-Carson and Aboodh transforms of  $Z(\gamma)$  are  $J(\epsilon)$  and  $D(\epsilon)$  respectively then

$$J(\epsilon) = \epsilon^2 D(\epsilon) \quad (15)$$

$$\text{and } D(\epsilon) = \frac{1}{\epsilon^2} J(\epsilon) \quad (16)$$

**Proof:** It is immediately concluded from (8)

$$J(\epsilon) = \epsilon \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma$$

$$\Rightarrow J(\epsilon) = \epsilon^2 \left[ \frac{1}{\epsilon} \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma \right]$$

Now, using (3) in above Equation, we have

$$J(\epsilon) = \epsilon^2 D(\epsilon).$$

To drive (16), we use (3)

$$D(\epsilon) = \frac{1}{\epsilon} \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma$$

$$\Rightarrow D(\epsilon) = \frac{1}{\epsilon^2} \left[ \epsilon \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma \right]$$

It is immediately concluded using (8) in above equation,

$$D(\epsilon) = \frac{1}{\epsilon^2} J(\epsilon).$$

**D. Duality between Laplace-Carson and Sumudu Transforms**

If Laplace-Carson and Sumudu transforms of  $Z(\gamma)$  are  $J(\epsilon)$  and  $F(\epsilon)$  respectively then

$$J(\epsilon) = F\left(\frac{1}{\epsilon}\right) \quad (17)$$

$$\text{and } F(\epsilon) = J\left(\frac{1}{\epsilon}\right) \quad (18)$$

**Proof:** From (8), we have

$$J(\epsilon) = \epsilon \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma$$

Put  $\epsilon\gamma = u \Rightarrow d\gamma = \frac{du}{\epsilon}$  in above equation, we have

$$J(\epsilon) = \epsilon \int_0^\infty Z\left(\frac{u}{\epsilon}\right) e^{-u} \frac{du}{\epsilon}$$

$$\Rightarrow J(\epsilon) = \int_0^\infty Z\left(\frac{u}{\epsilon}\right) e^{-u} du$$

Now, using (4) in above equation, we have

$$J(\epsilon) = F\left(\frac{1}{\epsilon}\right).$$

To drive (18), we use (4)

$$F(\epsilon) = \int_0^\infty Z(\epsilon\gamma) e^{-\gamma} d\gamma$$

Put  $\epsilon\gamma = u \Rightarrow d\gamma = \frac{du}{\epsilon}$  in above equation, we have

$$F(\epsilon) = \int_0^\infty Z(u) e^{-\frac{u}{\epsilon}} \frac{du}{\epsilon}$$

$$\Rightarrow F(\epsilon) = \frac{1}{\epsilon} \int_0^\infty Z(u) e^{-\frac{u}{\epsilon}} du$$

It is immediately concluded using (8) in above equation,

$$F(\epsilon) = J\left(\frac{1}{\epsilon}\right).$$

**E. Duality between Laplace-Carson and Elzaki Transforms**

If Laplace-Carson and Elzaki transforms of  $Z(\gamma)$  are  $J(\epsilon)$  and  $G(\epsilon)$  respectively then

$$J(\epsilon) = \epsilon^2 G\left(\frac{1}{\epsilon}\right) \quad (19)$$

$$\text{and } G(\epsilon) = \epsilon^2 J\left(\frac{1}{\epsilon}\right) \quad (20)$$

**Proof:** From (8), we have



$$J(\epsilon) = \epsilon \int_0^\infty Z(\gamma)e^{-\epsilon\gamma} d\gamma$$

$$\Rightarrow J(\epsilon) = \epsilon^2 \left[ \frac{1}{\epsilon} \int_0^\infty Z(\gamma)e^{-\epsilon\gamma} d\gamma \right]$$

Now, using (5) in above equation, we have

$$J(\epsilon) = \epsilon^2 G \left( \frac{1}{\epsilon} \right).$$

To drive (20), we use (5)

$$G(\epsilon) = \epsilon \int_0^\infty Z(\gamma)e^{-\frac{\gamma}{\epsilon}} d\gamma$$

$$G(\epsilon) = \epsilon^2 \left[ \frac{1}{\epsilon} \int_0^\infty Z(\gamma)e^{-\frac{\gamma}{\epsilon}} d\gamma \right]$$

It is immediately concluded using (8) in above equation,

$$G(\epsilon) = \epsilon^2 J \left( \frac{1}{\epsilon} \right).$$

#### F. Duality between Laplace-Carson and Mohand Transforms

If Laplace-Carson and Mohand transforms of  $Z(\gamma)$  are  $J(\epsilon)$  and  $H(\epsilon)$  respectively then

$$J(\epsilon) = \frac{1}{\epsilon} H(\epsilon) \tag{21}$$

$$\text{and } H(\epsilon) = \epsilon J(\epsilon) \tag{22}$$

**Proof:** From (8), we have

$$J(\epsilon) = \epsilon \int_0^\infty Z(\gamma)e^{-\epsilon\gamma} d\gamma$$

$$\Rightarrow J(\epsilon) = \frac{1}{\epsilon} \left[ \epsilon^2 \int_0^\infty Z(\gamma)e^{-\epsilon\gamma} d\gamma \right]$$

Now, using (6) in above equation, we have

$$J(\epsilon) = \frac{1}{\epsilon} H(\epsilon).$$

To drive (22), we use (6)

$$H(\epsilon) = \epsilon^2 \int_0^\infty Z(\gamma)e^{-\epsilon\gamma} d\gamma$$

$$\Rightarrow H(\epsilon) = \epsilon \left[ \epsilon \int_0^\infty Z(\gamma)e^{-\epsilon\gamma} d\gamma \right]$$

It is immediately concluded using (8) in above equation,

$$H(\epsilon) = \epsilon J(\epsilon).$$

#### G. Duality between Laplace-Carson and Sawi Transforms

If Laplace-Carson and Sawi transforms of  $Z(\gamma)$  are  $J(\epsilon)$  and  $I(\epsilon)$  respectively then

$$J(\epsilon) = \frac{1}{\epsilon} I \left( \frac{1}{\epsilon} \right) \tag{23}$$

$$\text{and } I(\epsilon) = \frac{1}{\epsilon} J \left( \frac{1}{\epsilon} \right) \tag{24}$$

**Proof:** Using (8) follows

$$J(\epsilon) = \epsilon \int_0^\infty Z(\gamma)e^{-\epsilon\gamma} d\gamma$$

$$\Rightarrow J(\epsilon) = \frac{1}{\epsilon} \left[ \epsilon^2 \int_0^\infty Z(\gamma)e^{-\epsilon\gamma} d\gamma \right]$$

Now, using (7) in above equation, we obtain

$$J(\epsilon) = \frac{1}{\epsilon} I \left( \frac{1}{\epsilon} \right).$$

To drive (24), we use (7)

$$I(\epsilon) = \frac{1}{\epsilon^2} \int_0^\infty Z(\gamma)e^{-\frac{\gamma}{\epsilon}} d\gamma$$

$$\Rightarrow I(\epsilon) = \frac{1}{\epsilon} \left[ \frac{1}{\epsilon} \int_0^\infty Z(\gamma)e^{-\frac{\gamma}{\epsilon}} d\gamma \right]$$

Now, using (8) in above equation, we obtain

$$I(\epsilon) = \frac{1}{\epsilon} J \left( \frac{1}{\epsilon} \right).$$

### X. APPLICATIONS OF MENTION DUALITY RELATIONS FOR FINDING INTEGRAL TRANSFORMS (LAPLACE TRANSFORM, KAMAL TRANSFORM, ABOODH TRANSFORM, SUMUDU TRANSFORM, ELZAKI TRANSFORM, MOHAND TRANSFORM AND SAWI TRANSFORM) OF USEFUL BASIC FUNCTIONS

We are giving tabular presentation of the integral transforms of mostly used basic functions by using mention dualities relations to visualize the usefulness of dualities between Laplace-Carson transform and mention integral transforms in the application field.

**Table-I: Laplace transform of useful basic functions with the help of relation of duality between Laplace – Carson and Laplace transforms**

S. N.	$Z(\gamma)$	$L_*\{Z(\gamma)\} = J(\epsilon)$	$L\{Z(\gamma)\} = B(\epsilon)$
1.	1	1	$\frac{1}{\epsilon}$
2.	$\gamma$	$\frac{1}{\epsilon}$	$\frac{1}{\epsilon^2}$
3.	$\gamma^2$	$\frac{2!}{\epsilon^2}$	$\frac{2!}{\epsilon^3}$
4.	$\gamma^n,$ $n \in N$	$\frac{n!}{\epsilon^n}$	$\frac{n!}{\epsilon^{n+1}}$
5.	$\gamma^n,$ $n > -1$	$\frac{\Gamma(n+1)}{\epsilon^n}$	$\frac{\Gamma(n+1)}{\epsilon^{n+1}}$
6.	$e^{a\gamma}$	$\frac{1}{(\epsilon - a)}$	$\frac{1}{(\epsilon - a)}$
7.	$\sin a\gamma$	$\frac{a\epsilon}{(\epsilon^2 + a^2)}$	$\frac{a}{(\epsilon^2 + a^2)}$
8.	$\cos a\gamma$	$\frac{\epsilon^2}{(\epsilon^2 + a^2)}$	$\frac{\epsilon}{(\epsilon^2 + a^2)}$
9.	$\sinh a\gamma$	$\frac{a\epsilon}{(\epsilon^2 - a^2)}$	$\frac{a}{(\epsilon^2 - a^2)}$
10.	$\cosh a\gamma$	$\frac{\epsilon^2}{(\epsilon^2 - a^2)}$	$\frac{\epsilon}{(\epsilon^2 - a^2)}$

**Table-II: Kamal transform of useful basic functions with the help of relation of duality between Laplace – Carson and Kamal transforms**

S.N.	$Z(\gamma)$	$L_*\{Z(\gamma)\} = J(\epsilon)$	$K\{Z(\gamma)\} = C(\epsilon)$
1.	1	1	$\epsilon$
2.	$\gamma$	$\frac{1}{\epsilon}$	$\epsilon^2$
3.	$\gamma^2$	$\frac{2!}{\epsilon^2}$	$2! \epsilon^3$
4.	$\gamma^n,$ $n \in N$	$\frac{n!}{\epsilon^n}$	$n! \epsilon^{n+1}$
5.	$\gamma^n,$ $n > -1$	$\frac{\Gamma(n+1)}{\epsilon^n}$	$\Gamma(n+1) \epsilon^{n+1}$
6.	$e^{a\gamma}$	$\frac{\epsilon}{(\epsilon - a)}$	$\frac{\epsilon}{(1 - a\epsilon)}$
7.	$\sin a\gamma$	$\frac{a\epsilon}{(\epsilon^2 + a^2)}$	$\frac{a\epsilon^2}{(1 + a^2\epsilon^2)}$
8.	$\cos a\gamma$	$\frac{\epsilon^2}{(\epsilon^2 + a^2)}$	$\frac{\epsilon}{(1 + a^2\epsilon^2)}$



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9.	<i>sinhay</i>	$\frac{a\epsilon}{(\epsilon^2 - a^2)}$	$\frac{a\epsilon^2}{(1 - a^2\epsilon^2)}$	10.	<i>coshay</i>	$\frac{\epsilon^2}{(\epsilon^2 - a^2)}$	$\frac{\epsilon}{(1 - a^2\epsilon^2)}$
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**Table-III: Aboodh transform of useful basic functions with the help of relation of duality between Laplace – Carson and Aboodh transforms**

S.N.	Z( $\gamma$ )	L <sub>*</sub> {Z( $\gamma$ )} = J( $\epsilon$ )	A{Z( $\gamma$ )} = D( $\epsilon$ )
1.	1	1	$\frac{1}{\epsilon^2}$
2.	$\gamma$	$\frac{1}{\epsilon}$	$\frac{1}{\epsilon^3}$
3.	$\gamma^2$	$\frac{2!}{\epsilon^2}$	$\frac{2!}{\epsilon^4}$
4.	$\gamma^n, n \in N$	$\frac{n!}{\epsilon^n}$	$\frac{n!}{\epsilon^{n+2}}$
5.	$\gamma^n, n > -1$	$\frac{\Gamma(n+1)}{\epsilon^n}$	$\frac{\Gamma(n+1)}{\epsilon^{n+2}}$
6.	$e^{a\gamma}$	$\frac{\epsilon}{(\epsilon - a)}$	$\frac{1}{\epsilon(\epsilon - a)}$
7.	<i>sinay</i>	$\frac{a\epsilon}{(\epsilon^2 + a^2)}$	$\frac{a}{\epsilon(\epsilon^2 + a^2)}$
8.	<i>cosay</i>	$\frac{\epsilon^2}{(\epsilon^2 + a^2)}$	$\frac{1}{(\epsilon^2 + a^2)}$
9.	<i>sinhay</i>	$\frac{a\epsilon}{(\epsilon^2 - a^2)}$	$\frac{a}{\epsilon(\epsilon^2 - a^2)}$
10.	<i>coshay</i>	$\frac{\epsilon^2}{(\epsilon^2 - a^2)}$	$\frac{1}{(\epsilon^2 - a^2)}$

**Table-IV: Sumudu transform of useful basic functions with the help of relation of duality between Laplace – Carson and Sumudu transforms**

S.N.	Z( $\gamma$ )	L <sub>*</sub> {Z( $\gamma$ )} = J( $\epsilon$ )	S{Z( $\gamma$ )} = F( $\epsilon$ )
1.	1	1	1
2.	$\gamma$	$\frac{1}{\epsilon}$	$\epsilon$
3.	$\gamma^2$	$\frac{2!}{\epsilon^2}$	$2! \epsilon^2$
4.	$\gamma^n, n \in N$	$\frac{n!}{\epsilon^n}$	$n! \epsilon^n$
5.	$\gamma^n, n > -1$	$\frac{\Gamma(n+1)}{\epsilon^n}$	$\Gamma(n+1)\epsilon^n$
6.	$e^{a\gamma}$	$\frac{\epsilon}{(\epsilon - a)}$	$\frac{1}{(1 - a\epsilon)}$
7.	<i>sinay</i>	$\frac{a\epsilon}{(\epsilon^2 + a^2)}$	$\frac{a\epsilon}{(1 + a^2\epsilon^2)}$

8.	<i>cosay</i>	$\frac{\epsilon^2}{(\epsilon^2 + a^2)}$	$\frac{1}{(1 + a^2\epsilon^2)}$
9.	<i>sinhay</i>	$\frac{a\epsilon}{(\epsilon^2 - a^2)}$	$\frac{a\epsilon}{(1 - a^2\epsilon^2)}$
10.	<i>coshay</i>	$\frac{\epsilon^2}{(\epsilon^2 - a^2)}$	$\frac{1}{(1 - a^2\epsilon^2)}$

**Table-V: Elzaki transform of useful basic functions with the help of relation of duality between Laplace – Carson and Elzaki transforms**

S.N.	$Z(\gamma)$	$L_*\{Z(\gamma)\} = J(\epsilon)$	$E\{Z(\gamma)\} = G(\epsilon)$
1.	1	1	$\epsilon^2$
2.	$\gamma$	$\frac{1}{\epsilon}$	$\epsilon^3$
3.	$\gamma^2$	$\frac{2!}{\epsilon^2}$	$2! \epsilon^4$
4.	$\gamma^n, n \in N$	$\frac{n!}{\epsilon^n}$	$n! \epsilon^{n+2}$
5.	$\gamma^n, n > -1$	$\frac{\Gamma(n + 1)}{\epsilon^n}$	$\Gamma(n + 1)\epsilon^{n+2}$
6.	$e^{a\gamma}$	$\frac{\epsilon}{(\epsilon - a)}$	$\frac{\epsilon^2}{(1 - a\epsilon)}$
7.	<i>sinay</i>	$\frac{a\epsilon}{(\epsilon^2 + a^2)}$	$\frac{a\epsilon^3}{(1 + a^2\epsilon^2)}$
8.	<i>cosay</i>	$\frac{\epsilon^2}{(\epsilon^2 + a^2)}$	$\frac{\epsilon^2}{(1 + a^2\epsilon^2)}$
9.	<i>sinhay</i>	$\frac{a\epsilon}{(\epsilon^2 - a^2)}$	$\frac{a\epsilon^3}{(1 - a^2\epsilon^2)}$
10.	<i>coshay</i>	$\frac{\epsilon^2}{(\epsilon^2 - a^2)}$	$\frac{\epsilon^2}{(1 - a^2\epsilon^2)}$

**Table-VI: Mohand transform of useful basic functions with the help of relation of duality between Laplace – Carson and Mohand transforms**

S. N.	$Z(\gamma)$	$L_*\{Z(\gamma)\} = J(\epsilon)$	$M\{Z(\gamma)\} = H(\epsilon)$
1.	1	1	$\epsilon$
2.	$\gamma$	$\frac{1}{\epsilon}$	1
3.	$\gamma^2$	$\frac{2!}{\epsilon^2}$	$\frac{2!}{\epsilon}$
4.	$\gamma^n, n \in N$	$\frac{n!}{\epsilon^n}$	$\frac{n!}{\epsilon^{n-1}}$

5.	$\gamma^n, n > -1$	$\frac{\Gamma(n + 1)}{\epsilon^n}$	$\frac{\Gamma(n + 1)}{\epsilon^{n-1}}$
6.	$e^{a\gamma}$	$\frac{\epsilon}{(\epsilon - a)}$	$\frac{\epsilon^2}{(\epsilon - a)}$
7.	<i>sinay</i>	$\frac{a\epsilon}{(\epsilon^2 + a^2)}$	$\frac{a\epsilon^2}{(\epsilon^2 + a^2)}$
8.	<i>cosay</i>	$\frac{\epsilon^2}{(\epsilon^2 + a^2)}$	$\frac{\epsilon^3}{(\epsilon^2 + a^2)}$
9.	<i>sinhay</i>	$\frac{a\epsilon}{(\epsilon^2 - a^2)}$	$\frac{a\epsilon^2}{(\epsilon^2 - a^2)}$
10.	<i>coshay</i>	$\frac{\epsilon^2}{(\epsilon^2 - a^2)}$	$\frac{\epsilon^3}{(\epsilon^2 - a^2)}$

**Table-VII: Sawi transform of useful basic functions with the help of relation of duality between Laplace – Carson and Sawi transforms**

S.N.	$Z(\gamma)$	$L_*\{Z(\gamma)\} = J(\epsilon)$	$S^*\{Z(\gamma)\} = I(\epsilon)$
1.	1	1	$\frac{1}{\epsilon}$
2.	$\gamma$	$\frac{1}{\epsilon}$	1
3.	$\gamma^2$	$\frac{2!}{\epsilon^2}$	$2! \epsilon$
4.	$\gamma^n, n \in N$	$\frac{n!}{\epsilon^n}$	$n! \epsilon^{n-1}$
5.	$\gamma^n, n > -1$	$\frac{\Gamma(n+1)}{\epsilon^n}$	$\Gamma(n+1) \epsilon^{n-1}$
6.	$e^{a\gamma}$	$\frac{\epsilon}{(\epsilon - a)}$	$\frac{1}{\epsilon(1 - a\epsilon)}$
7.	$\sin a\gamma$	$\frac{a\epsilon}{(\epsilon^2 + a^2)}$	$\frac{a}{(1 + a^2\epsilon^2)}$
8.	$\cos a\gamma$	$\frac{\epsilon^2}{(\epsilon^2 + a^2)}$	$\frac{1}{\epsilon(1 + a^2\epsilon^2)}$
9.	$\sinh a\gamma$	$\frac{a\epsilon}{(\epsilon^2 - a^2)}$	$\frac{a}{(1 - a^2\epsilon^2)}$
10.	$\cosh a\gamma$	$\frac{\epsilon^2}{(\epsilon^2 - a^2)}$	$\frac{1}{\epsilon(1 - a^2\epsilon^2)}$

## XI. CONCLUSIONS

In the present paper, duality relations between Laplace-Carson transform and some useful integral transforms namely Laplace transform, Kamal transform, Aboodh transform, Sumudu transform, Elzaki transform, Mohand transform and Sawi transform are established successfully. Tabular presentation of the integral transforms (Laplace transform, Kamal transform, Aboodh transform, Sumudu transform, Elzaki transform, Mohand transform and Sawi transform) of mostly used basic functions are given with the help of mention dualities relations to visualize the importance of dualities between Laplace-Carson transform and mention integral transforms. Results show that the Laplace-Carson transform and mention integral transforms in this paper are strongly related to each others. In future using these duality relations, we can easily solved many advanced problems of modern era such as motion of coupled harmonic oscillators, drug distribution in the body, arms race models, Brownian motion and the common health problem such as detection of diabetes.

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