Determination of Johnson Cook Parameters in Turning of Micro and Nano Reinforced Aluminum Composites using Trust Region Reflective Algorithm

Ravi Sekhar, T. P. Singh

Abstract: ***Accuracy of shear stress estimations plays a vital role in correct prediction of cutting forces in machining. In this study, efforts were directed to obtain better flow stress estimates through determination of novel Johnson Cook (JC) parameters for a number of micro and nano reinforced composite materials. Trust region reflective algorithm (non linear least squares) was used to determine optimum JC constants for each developed material subjected to varying machining parameters under high strain rate conditions. The newer JC constants yielded substantially better shear stress estimates as compared to base alloy JC constants; thus in turn improving cutting force predictability in machining of developed composite materials.***

Keywords: ***metal matrix composites, cutting forces, shear stress, Johnson Cook, non linear least squares***

I. INTRODUCTION

Matrix composites are being preferred in a number of aerospace applications as alternatives to conventional alloys primarily due to superior strength to weight ratio. However, they generate higher forces during machining. Prediction of cutting forces during machining of composites is therefore, an important objective of contemporary research. Force prediction relies upon, among other factors, material flow strength. Flow strength in turn depends upon machining parameters and mechanisms such as strain, strain rate and thermal softening. Johnson Cook's (JC) constitutive equation predicts material shear stress taking into account strain hardening, thermal softening and strain rate effects [1]. This equation has five constants associated with certain mechanisms that are unique for every material. Therefore, successful force predictions in machining composites depend upon the determination of these constants unique to every composite composition. This constitutive equation (JC) is widely used to predict flow stresses subjected to high strain rate sensitivity of metals and alloys. It is quite popular for material modelling of metals due to its inherent simplicity and ease of parameter determination [2, 3, 4, 5]. It has been used to predict failure and evolution of damage in many materials as well [6, 7].

A. Methods to determine JC parameters and strain rate effects

Material shear strength k is obtained from Johnson-Cook's constitutive model as given below [8]

$$k = \frac{1}{\sqrt{3}}[A + B_1\, \varepsilon_1{}^{n_1}][1 + C_1 \ln \dot{\varepsilon}_1]\left[1 - \left(\frac{T - T_r}{T_m - T_r}\right)^m\right] \qquad (1)$$

Wherein, T is cutting temperature, Tr reference temperature, Tm melting point, ε1 equivalent shear strain and $\dot{\varepsilon}_1$ equivalent shear strain rate. A is yield stress, B is strain hardening coefficient, n strain hardening exponent [6], C strain rate strengthening coefficient and m thermal softening function [3]. These five parameters (A, B, n, C and m) are unique to every material. The remaining functions are dependent on machining conditions.

The five JC constants can be estimated experimentally for material modelling. A, B and n can be determined directly from tension, compression and torsion tests. B and n can be established from the power law curve of the true stress strain graph from the tensile test, A can also be obtained from this graph [6]. Parameter m is obtained through quasi static tensile tests under varying adiabatic heating conditions. Temperature rise due to plastic deformation should also be considered for parameter m [3]. C is the slope of shear stress versus log of shear strain curves [6].

However, it is important to note that JC constants vary according to strain rate changes. Banerjee et al [6] emphasise that JC model parameters should be evaluated based on experiments in dynamic conditions as compared to quasi static conditions. This is because engineering materials behave differently under these two conditions due to inertia effects, strain rate sensitivity and stress reflections [9]. Inertia effects get neglected in low strain rate tests. As per Huang et al [3], universal testing machine and Split Hopkinson Pressure Bar (SHPB) tests can be used to obtain flow stress data under different temperature and strain rate conditions. SHPB is specifically meant for compression experiments, whereas Kolsky bar is its more generalized form, to be used for compression, tension or torsion [10]. Huang et al [3] advise to obtain the JC strain hardening parameters firstly through quasi static tests like uniaxial tensile and torsional tests at strain rates of 10^-1 per second; which later can be modified at the practical strain rate values using SHPB, from 103 to 104 per second. Similarly, compression tests using hydraulic testing machines

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have been conducted for lower strain rates (up to 1 per second). Investigators have also implemented numerical modelling to simulate these results [6]. Tension, compression and torsion tests have been performed at varying temperatures to get thermal softening data [11]. Duncan and Pearce [12] state that low and intermediate strain rate conditions upto 10^-2 per second can be simulated using electromechanical test machines, while servo hydraulic test machines can provide upto 10^2 per second. As per Ramesh [10], strain rates above 10^2 are high strain rates, above 10^4 are very high strain rates and above 10 ^6 are called ultra high. Rates below 10^-3 are quasistatic and below 10^-6 are in creep. Drop weight tests develop strain rates upto 10^2 [6, 10]. To measure shear stresses at strain rates upto 10^6, high strain rate pressure shear (HSPRS) plate impact test was devised. Strain rates upto 10^8 can be produced for short time durations using shock wave propagation experiments [10]. Researchers claim that the high strain rate behaviour of many materials is yet to be understood [13].

B. Determination of JC parameters in machining

During machining, strain rates of the order of 10^6 per second are encountered. This high strain rate cannot be simulated in most of the experimental setups discussed above. Also, costs are prohibitive in many cases of testing in dynamic range as compared to standard tests in quasi static range [14]. Huang et al [3] further state that these experimental methods of determining parameters are inadequate as they do not cover all points in experimental data; plus there is lack of continuity in response curves. Plus, there are different measurement techniques for different ranges of strain rates; hence they provide data pertaining to that specific range only. Data for one range cannot be implemented in the other. It is clear that the high strain rates affect changes in the material structure and properties, thus increasing the value of yield point A and others [6]. Some researchers indicate that the tensile strength of metal can increase twofold from quasi static condition to a high strain rate condition of 10^4 per second or more [15]. Sierakowsky [15] states that there have been very limited efforts to develop constitutive equations for composites as compared to metals. Presently, the research effort appears to be inclined towards obtaining models for metal matrix composites as derivatives of the parent alloy performance equations. So, researchers [3, 5 and 16] advocate the use of optimization algorithms within the domain of selected parameter ranges and go over iterations to arrive at the optimum parameter settings that minimize the digression between predicted and experimental results.

Thus, in the present work, optimization algorithms were utilized to arrive at novel Johnson Cook constants for machining of micro and nano reinforced metal matrix composites under study.

II. METHODOLOGY

A number of metal matrix composites (MMC) were prepared using stir casting method with Al6063 as the matrix alloy. Two types of micro reinforcements (SiC and B4C) and one kind of nano particulates (carbon nano tubes, CNT) were employed to compose nine varieties of MMC materials (Table 1). Volume fractions were so chosen such that the substantial difference in shear stress predictions among base alloy and MMC JC constants gets evident at even low reinforcements. Nano reinforcement fractions were kept low to avoid particle agglomeration/coagulation issues.

Turning experiments were performed on prepared materials varying one machining parameter at a time and keeping the others constant (Table 2). This kind of experimental design was necessary to test the validity of optimised JC constants over machining parameters not explored often in such cases, like the depth of cut. Each experimental run was repeated thrice and average chip thicknesses, cutting force components were recorded using micrometer and lathe tool dynamometer, respectively. Dry cutting conditions were maintained for all experimental runs, with turning length kept constant at 10mm. Cutting tool (Seco CNMG-120408 CVD coated tool inserts) was kept common for the three runs comprising variation of each machining parameter. This enabled inclusion of worn tool effects on shear stresses, forces and eventually, JC model parameters with regards rising feed rate and depth of cut in particular.

Table 1 . Metal matrix compositions

S. No.	Matrix Alloy	Particulate Material	Volume Fraction (%)
1.	Al 6063	SiC	3
2.	Al 6063	SiC	5
3.	Al 6063	SiC	7
4.	Al 6063	B4C	3
5.	Al 6063	B4C	5
6.	Al 6063	B4C	7
7.	Al 6063	CNT	0.3
8.	Al 6063	CNT	0.5
9.	Al 6063	CNT	0.7

Table 2 Experimental Design

Run No.	Speed, m/min	Feed, mm/rev	Depth of Cut, mm
1.	75	0.15	0.75
2.	125	0.15	0.75
3.	175	0.15	0.75
4.	125	0.1	0.75
5.	125	0.15	0.75
6.	125	0.2	0.75
7.	125	0.15	0.25
8.	125	0.15	0.75
9.	125	0.15	1.25

The nine runs shown in Table 2 were conducted for all MMC compositions, taking the total number to 81 experiments. Strain, strain rate and material flow stresses required in the JC equation were determined experimentally based on chip thickness ratios [17] and cutting forces [8] for each experimental run.

Thereafter, a total of 81 objective functions (for a total of 81 experiments, 9 runs x 9 compositions) were constructed of the following form –

$$F = k - \frac{1}{\sqrt{3}}[A + B_1\, \varepsilon_1{}^{n_1}][1 + C_1 \ln\dot{\varepsilon}_1]\left[1 - \left(\frac{T - T_r}{T_m - T_r}\right)^m\right]$$

Following is an example of nine objective functions corresponding to the nine experimental runs grouped under one Matlab function for 7pc boron carbide MMC material.

```
•  function F=rjc_7b4c(x)
•  %X(1)=A;
•  %X(2)=B;
•  %X(3)=n;
•  %X(4)=c;
•  %X(5)=m;
•  % Upper range [200 400 0.6 0.01 1.5]
•  % Lower range, [90 180 0.2 0.001 0.2]
•  % F = sh.str. - root3(A+B*sh.strn.^n)(1+c*ln(eq.sh.strn.rt.)(1-T
     fraction^m)
•  F(1)=abs(290-0.577*(x(1)+x(2)*(2.79)^x(3))*(1+x(4)*6.99)*(1-
     0.02^x(5)));
•  F(2)=abs(294-0.577*(x(1)+x(2)*(3.27)^x(3))*(1+x(4)*7.32)*(1-
     0.02^x(5)));
•  F(3)=abs(265-0.577*(x(1)+x(2)*(3.35)^x(3))*(1+x(4)*7.63)*(1-
     0.03^x(5)));
•  F(4)=abs(384-0.577*(x(1)+x(2)*(3.48)^x(3))*(1+x(4)*7.27)*(1-
     0.02^x(5)));
•  F(5)=abs(249-0.577*(x(1)+x(2)*(4.25)^x(3))*(1+x(4)*7.04)*(1-
     0.02^x(5)));
•  F(6)=abs(229-0.577*(x(1)+x(2)*(3.93)^x(3))*(1+x(4)*7.1)*(1-0
    .03^x(5)));
•  F(7)=abs(500-0.577*(x(1)+x(2)*(2.09)^x(3))*(1+x(4)*8.43)*(1-
     0.02^x(5)));
•  F(8)=abs(362-0.577*(x(1)+x(2)*(3.71)^x(3))*(1+x(4)*7.18)*(1-
     0.02^x(5)));
•  F(9)=abs(176-0.577*(x(1)+x(2)*(4.67)^x(3))*(1+x(4)*6.71)*(1-
     0.02^x(5)));
•  F=sum(F);
•  end
```

This objective function (F) stands for the residue of experimental (k) versus estimated shear stress (JC equation) with strain, strain rate etc. specific to each experiment. Since there will be nine such equations for each MMC composition, this non-linear function has to be minimised using suitable algorithm(s) and in the process, optimum JC parameter settings are to be determined for each composition. Thus, this function can be solved as a least squares problem (NLSE), since least square problems minimize the divergence between a data set and model estimations. The Levenberg-Marquardt method is commonly used to minimize least square problems [18]. Shrot and Baker [16] used the Levenberg-Marquardt algorithm to optimise JC constitutive constants in their machining simulations. The same approach was applied in the present study as well, using Matlab software (R2016b). However, the trust region reflective algorithm proved to be more useful for minimizing the present case of nonlinear function with bounds. The upper and lower bounds of JC constants were decided on the basis of literature [5, 12], wherein expected increments in yield stress and other parameters at higher shear strains have been investigated and guidelines for extrapolation provided to fix bounds for optimization. For instance, the tensile modulus (Young's) defined as slope of stress by strain in tensile test in the strain range of 0.0005 to 0.0025 (0.05% to 0.25%), as per ISO 527-2; varies almost linearly with log of strain rate. Researchers' [6] advice for estimating modulus at high strain rates is that 5% increase in modulus can be considered for every decade of rate. Since modulus can be accurately determined only at low strain rates [19, 20], so for higher strain rates it has to be extrapolated. Table 3 shows the fixation of JC parameter bounds for optimization.

Table 3 Johnson Cook parameter bounds and base alloy matrix JC values

Constants	Lower Bound	Upper Bound	Parent Alloy Values [8]
A	90	200	102
B	180	400	201
n	0.2	0.6	0.415
C	0.001	0.01	0.002
M	0.2	1.5	1.05

To confirm the obtained results, the systems of JC equations (9 equations for each composition; one for each run) were similarly solved as optimization problems using branch and bound method (Matlab R2016b).

Following are examples of NLSE and Branch & Bound codes invoking the 7pc boron carbide Matlab function detailed above.

```
% Non linear Least square method
clc,clear all;
lb=[90 180 0.2 0.001 0.2];
ub=[200 400 0.6 0.01 1.5];
 opts                                                              =
optimoptions(@lsqnonlin,'Display','off','Algorithm','levenberg-mar
quardt');
%opts                                                              =
optimoptions(@lsqnonlin,'Display','off','Algorithm','trust-region-ref
lective');
x = lsqnonlin(@rjc_7b4c,zeros(1,5),lb,ub,opts)

% branch and bound method
clc,clear all;
lb=[90 180 0.2 0.001 0.2];
ub=[200 400 0.6 0.01 1.5];
% x = fmincon(@rjc_sir,zeros(1,5),[],[],[],[],lb,ub)
[errmsg,Z,x,t,c,fail] = BNB20('rjc_7b4c',lb',zeros(1,5)',lb',ub')
```

Thereafter, the novel JC constant values (A, B, n, C and m) were utilised to compute theoretical shear stress (k) estimates for each individual experimental run for each composition using the Johnson Cook's equation (1). For comparison, theoretical shear stresses were obtained using base alloy JC constant values as well, using the same equation. The other functions of equation (1) like equivalent shear strain etc. were determined experimentally through the formulations given by Pramanik et al [21]. The formulation for determining experimental shear stress [21] based on cutting forces measured by dynamometer, chip thickness ratio, feed rate and depth of cut is given as follows -

$$\tau_s = \frac{\lfloor (F_{cs} cos\varnothing) - (F_{ts} sin\varnothing) \rfloor sin\varnothing}{A_c}$$

Where, τ_s is material flow stress, F_{cs} and F_{ts} are cutting and thrust force components, A_c is the area of cut and $\varnothing$ is the shear plane angle obtained from chip thickness ratio. The theoretical flow stress values were compared with experimental flow stresses for the same runs; and percent errors were obtained.

III. RESULTS AND DISCUSSION

Tables 4 and 5 show the optimized values of JC constants for all the developed materials, as per the NLSE and branch and bound techniques.

The tables 6 and 7 show the percentage errors in shear stress estimations for each composition using the JC constants from either of the optimization techniques. This percentage error is the average over nine runs for each composition. It can be observed that the NLSE JC results resemble those from branch and bound method results in most cases. The percentage errors (JC opt.) of both methods match closely for each composition in every case. The error percentages vary from five percent (0.3%CNT MMC) to twenty one percent (5% B4C MMC). Laakso et al [5] report that JC parameters generally show an error of up to 20%. The authors [5] achieved an average error of less than 6%; however, they didn't vary the feed rate and depth of cut in their study. Although it can be argued that in the present study JC constants are applicable only to the range of experimental parameters in scope, it can also be appreciated that these have been obtained by considering variations in all parameters, viz. speed, feed and depth of cut. So, they should be able to cater to wider range of parameters (especially cutting speed) with better accuracy than that obtained in this study. Another significant result is the drastic reductions in avg. % errors in shear stress estimations using optimized JC constants as opposed to the JC constants of the base alloy, as used by other researchers [21] (Fig. 1). This proves beyond doubt that JC constants should be obtained for every MMC composition for better shear stress and cutting force predictions as compared to simply applying base alloy JC constants for the same. Among compositions, the applied optimization techniques have not been able to achieve as accurate results for boron carbide particulate composites as obtained for silicon carbide MMCs. Results are very positive for the low reinforcement carbon nano tube MMCs, showing significant improvement in shear stress prediction accuracy over usage of base alloy JC constants for these materials.

Table 4 Johnson Cook (JC) constants for the developed composites using non linear least squares (NLSE) optimization (trust region reflective algorithm)

NLSE	A	B	n	C	m
7 B4C	200	319	0.2	0.01	0.43
5 B4C	199	289	0.2	0.01	0.52
3 B4C	199	250	0.2	0.01	0.78
0.7CNT	199	209	0.2	0.001	1.5
0.5 CNT	200	400	0.2	0.01	0.36
0.3 CNT	200	400	0.2	0.01	0.32
7 SiC	200	400	0.2	0.01	0.37
5 SiC	200	400	0.2	0.01	0.4
3 SiC	200	400	0.2	0.01	0.35

Table 5 Johnson Cook (JC) constants and % shear stress estimate errors for the developed composites using branch and bound (BnB) method of optimization

BnB	A	B	n	C	m
7 B4C	199.2	397.1	0.2	0.009	0.29
5 B4C	199.9	270.6	0.2	0.009	0.54
3 B4C	199.9	207.57	0.2	0.01	1.49
0.7 CNT	199.1	219.8	0.2	0.001	1.49
0.5 CNT	199.8	282.1	0.2	0.01	1.05
0.3 CNT	199.3	395.1	0.2	0.009	0.33
7 SiC	199.9	399.9	0.2	0.01	0.33
5 SiC	199.7	399.4	0.2	0.01	0.39
3 SiC	101.66	196	0.53	0.009	1.49

Table 6. % shear stress estimate errors for the developed composites using non linear least squares (NLSE) optimization (trust region reflective algorithm)

NLSE	Avg. % error, τ	Avg. % error, τ
	JC (opt.)	JC (base alloy)
7 B4C	17.4	38.01
5 B4C	20.6	37.64
3 B4C	21.1	41.84
0.7 CNT	14.1	49.22
0.5 CNT	8.5	46.37
0.3 CNT	5.3	43.93
7 SiC	14.0	38.89
5 SiC	9.8	42.58
3 SiC	9.4	39.62

Table 7 % shear stress estimate errors for the developed composites using branch and bound (BnB) method of optimization

BnB	Avg. % error, τ	Avg. % error, τ
	JC (opt.)	JC (base alloy)
7 B4C	16.1	38.01
5 B4C	20.5	37.64
3 B4C	19.4	41.84
0.7 CNT	12.9	49.22
0.5 CNT	6.81	46.37
0.3 CNT	5.2	43.93
7 SiC	12.7	38.89
5 SiC	9.6	42.58
3 SiC	11.92	39.62

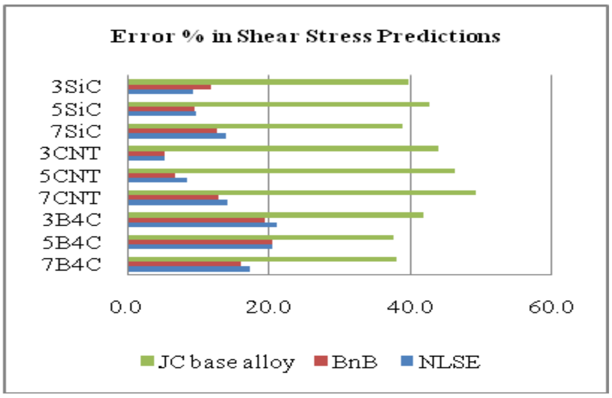

Figure 1 Average percentage errors in shear stress predictions using JC constants of base alloy and those obtained through optimizations – branch and bound (BnB) and non linear least squares (NLSE) in the present study System prototype architecture of FPV

IV. CONCLUSION

In this study, novel Johnson Cook constitutive equation constants were obtained for micro and nano particulate metal matrix composites having Al 6063 as base alloy. Micro particulates include silicon carbide and boron carbide; whereas carbon nano tubes comprise of the nano particulates in the developed MMCs. Three compositions were developed for each particulate material by varying volume fractions, thus creating a total of nine compositions. Each composition was subjected to nine experimental runs, varying speed, feed and depth of cut one at a time, keeping the other two parameters constant at their mid-range values. In this fashion, eighty one turning experiments were performed to obtain machining measures like equivalent shear strain rate to be utilised in the Johnson Cook's equation and to obtain experimental shear stress values. For each MMC composition, nine objective functions were formulated and JC constants were optimized to minimise the residual function (difference between actual and theoretical shear stresses). The trust region reflective algorithm for non linear least squares method was applied and results were confirmed through branch and bound method. Lower and upper bounds were decided by taking guidance from published literature. Results clearly indicate better shear stress prediction accuracy using optimized JC constant values over base alloy values. Better shear stress predictions imply improvement in cutting force estimations for metal matrix composites, which is essential to improve machinability of these new age materials. It is to be noted that the reported novel JC constants cater to high strain rate conditions experienced during machining operations only.

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