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Fault-Tolerant Resolvability of Oxide Interconnections

M.Somasundari, F.Simon Raj

Abstract: ***A collection of well-defined set W=$\{w_1, w_2, \dots, w_k\}$ of nodes of a graph G is named as a resolving set, if all the nodes of G are distinctively identified by the ordered set of distances to the nodes in W. The metric index of G is the smallest cardinality of a resolving set of G. A resolving set W for G is called fault-tolerant if $W\backslash\{w_i\}$ is also a resolving set, for each w_i in W. The smallest cardinality of such a set is called fault-tolerant metric index of G. In this paper fault-tolerant metric index of oxide interconnection is found.***

Keywords: ***Metric Dimension, Fault-Tolerant metric dimension, oxide interconnection.***

I. INTRODUCTION

Graph Theory has a wide range of applications in the field of Computer & Mathematical Sciences. In particular, Chemical Graph Theory is well-known among researchers because of more applications providing in mathematical chemistry. In 1976, Haray *et al* introduced the concept of resolvability in graphs. They have characterized the metric dimension for trees. In 2011, Bharathi *et al* studied conditional resolvability of Honeycomb and Hexagonal networks. In 2012, Bharathi *et al* investigated conditional resolving parameters on Enhanced Hypercube networks. They found one factor and one size resolving sets for enhanced Hypercube networks. In 2014, Carmen Hernando discussed fault-tolerant metric dimension of graphs. They showed that the values of the fault tolerant metric dimension are bounded. In 2014, M.A. Chaudry *et al* found fault-tolerant metric and partition dimension of complete and complete bipartite graphs. In 2016, Sathish K. and B. Rajan found the fault-tolerant resolving number of Certain Crystal Structures. In 2017, C. Monica *et al* discovered Star resolving number of Honeycomb network and Honeycomb Rectangular Mesh. Silicate and Oxide interconnections are studied extensively in[11,12,13]. Silicates are found by combining metal oxides or metal carbonates with sand. Basically all the silicates contain SiO_4 tetrahedra. In chemistry, the corner nodes of SiO_4 tetrahedran denotes oxygen ions and the middle node signifies the silicon ion. In graph theory, we call the corner nodes as oxygen nodes and the middle mode as silicon node.

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***** Correspondence Author
M.Somasundari*, Department of Mathematics, Hindustan Institute of Technology and Science, Padur,Chennai
F.Simon Raj, Department of Mathematics, Hindustan Institute of Technology and Science, Padur,Chennai

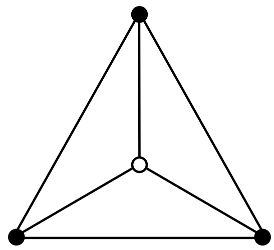

Figure 1: SiO_4 tetrahedra

Oxide interconnection can be obtained from the silicate network by removing silicon nodes. In general, oxide interconnection is denoted by $OX(n)$.

Theorem [12] Let $G = OX(n)$, then G has $9n^2 + 3n$ nodes and $18n^2$ links.

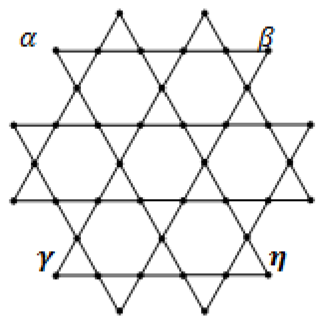

Figure 2: Oxide Interconnection of dimension 2

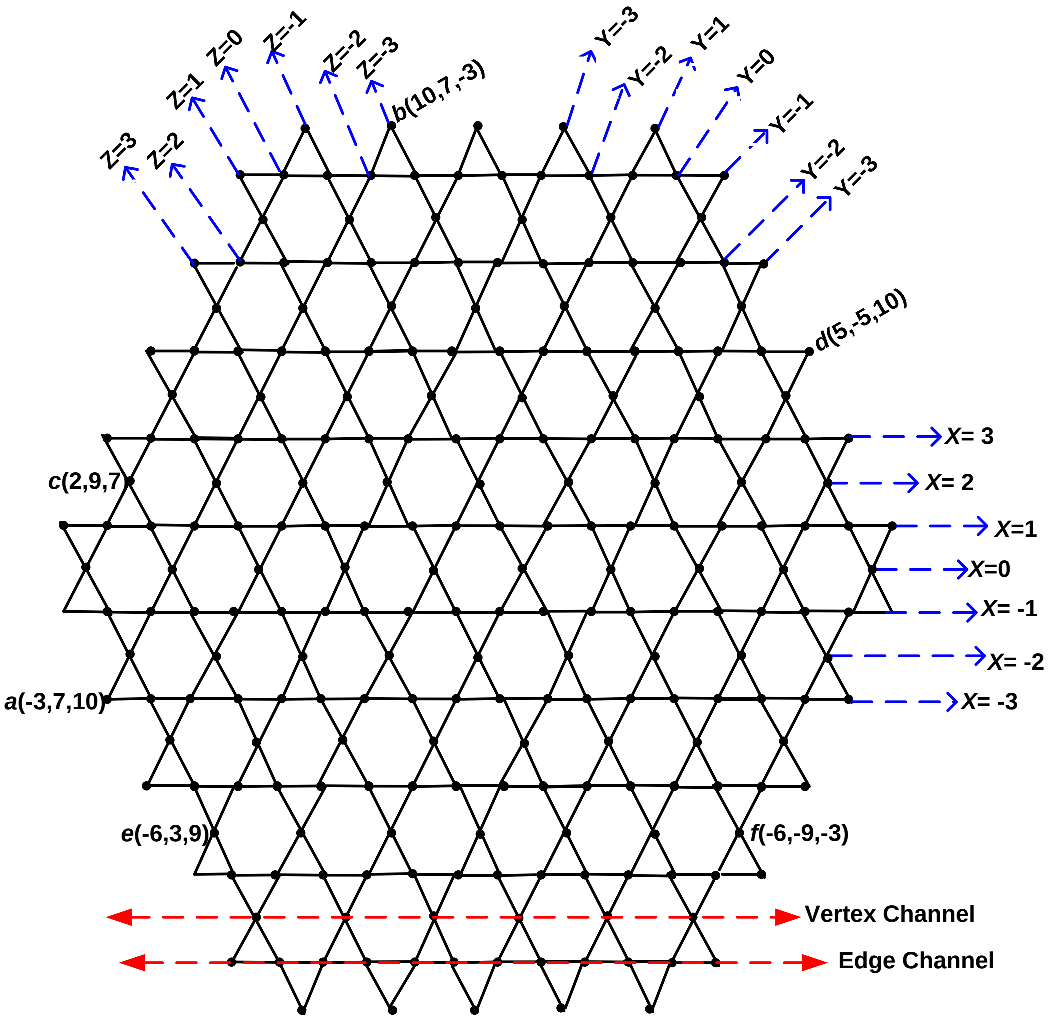

Figure 3: Oxide Interconnection of dimension 2

Proposed methodology.

In this paper we use graph coordinate system to prove the main result.

The distance between any two nodes $u = (x_1, y_1, z_1)$ and $v = (x_2, y_2, z_2)$ in the oxide intereconnection is equal to

$$\begin{cases} \frac{1}{2}\{|x_2 - x_1| + |y_2 - y_1| + |z_2 - z_1| + 2\} & \text{if } u \text{ and } v \text{ lies on the same vertex channel} \\ \frac{1}{2}\{|x_2 - x_1| + |y_2 - y_1| + |z_2 - z_1|\} & \text{otherwise} \end{cases}$$

In Figure 3, the distance between a(-3,7,10) and b(10,7,-3) is 13, the distance between c(2,9,7) and d(5,-5,-10) is 17, the distance between e(-6,3,9) and f(-6,-9,-3) is 13.

Main Result

Theorem [14]: Let $G = OX(n)$, then $dim(G) = \begin{cases} 2 & \omega = 1 \\ 3 & \omega \geq 2 \end{cases}$

Proof:

Any pair of vertices in the oxide interconnection will come under one of the category discussed below lemmata from (1-6).

Lemma 1:

Let $A = \{u_r(-r-1, -1, r) \,/\, 2 \leq r \leq 2\omega - 1\}$ and $B = \{v_r(-r-1, -r, 1) \,/\, 2 \leq r \leq 2\omega - 1\}$, $u_i \in A$ and $v_i \in B$ then $\{\alpha, \beta\}$ is not a resolving set for (u_i, v_i).

Proof:

Let $\alpha = (2\omega - 1, 2\omega, 1)$and $\beta = (2\omega - 1, -1, -2\omega)$ where ω is the dimension of oxide interconnection

Let $u = (x_1, y_1, z_1)$ & $v = (x_2, y_2, z_2)$ be any two points in $OX(\omega)$.

$$d(u, v) = \frac{1}{2}\{|x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2|\}$$

$$d(u_i, \alpha) = \tfrac{1}{2}\{|-r - 1 - 2\omega + 1| + |-1 - 2\omega| + |r - 1|\}$$

$$= \tfrac{1}{2}\{|-(r + 1) - 2\omega + 1| + |1 + 2\omega| + |r - 1|\}$$

$= \frac{1}{2}\{|-(r + 1) - 2\omega + 1| + |2\omega + 1 + r - 1$(since $r-1 \geq 1$ implies r is positive)

$$= \tfrac{1}{2}\{|-(r + 1) - 2\omega + 1| + |2\omega + r|\}$$

$d(v_i,\alpha)=\frac{1}{2}\{|-(r+1)-2\omega+1|+|-r-2\omega|+|1-1|\}$

$=\frac{1}{2}\{|-(r+1)-2\omega+1|+|r+2\omega|\}=d(u_i,\alpha)$

$$d(u_i,\beta)=\frac{1}{2}\{|-(r+1)-2\omega+1|+|-1+1|+|r+2\omega|$$
$$=\frac{1}{2}\{|r+2\omega|+|r+2\omega|$$
$$=|r+2\omega|$$

$$d(v_i,\beta)=\frac{1}{2}\{|-(r+1)-2\omega+1|+|-r+1|+|1+2\omega|$$
$$=\frac{1}{2}\{|r+2\omega|+|r-1|+|1+2\omega|\}$$
$$=\frac{1}{2}\{|r+2\omega|+|r-1+1+2\omega\}$$
$$=\frac{1}{2}\{|r+2\omega|+|r+2\omega|=|r+2\omega|=d(u_i,\beta)$$

Hence the lemma 1.

Lemma 2:

Let $C=\{u_l(2r,2\omega-1,2\omega-1-2r)/0\le r\le\omega-2\ \&\ \omega\ge 2\}$

$D=\{v_l(2r+1,2\omega,2\omega-2r-1)/0\le r\le\omega-2\ \&\ \omega\ge 2\}$

$E=\{u_r(2r,-(2\omega-1-2r),-(2\omega-1)/0\le r\le\omega-2\ \&\ \omega\ge 2\}$

$F=\{v_r(2r+1,-(2\omega-1-2r),-2\omega)/0\le r\le\omega-2\ \&\ \omega\ge 2\}$

$u_l\in C$, $v_l\in D$, $u_r\in E\&v_r\in F$, then $\{\alpha,\beta\}$ is not a resolving set for $(u_l,v_l)\&(u_r,v_r)$.

Proof

Let $\alpha=(2\omega-1,2\omega,1)\ \&\ \beta=(2\omega-1,-1,-2\omega)$

$d(u_i,\alpha)=\frac{1}{2}\{|x_1-x_2|+|y_1-y_2|+|z_1-z_2|\}$ $\rightarrow 1$

Here $u_l=(2r,2\omega-1,2\omega-1-2r)$, $\alpha=(2\omega-1,2\omega,1)$

$$d(u_i,\alpha)=\frac{1}{2}\{|2r-2\omega+1|+|2\omega-1-2\omega|+|2\omega-1-2r-1|\}$$
$$=\frac{1}{2}\{|2r-2\omega+1|+|-1|+|2\omega-2r-2|\}$$
$$=\frac{1}{2}\{|2\omega-2r-1|+|1|+|2\omega-2r-2|\}$$
$$=\frac{1}{2}\{|2\omega-2r-1+1|+|2\omega-2r-2|\}$$
$$=\frac{1}{2}\{|2\omega-2r|+|2\omega-2r-2|\}$$

Distance between any two points $v_l\&\alpha$ lie on same vertex channel is

$d(v_i,\alpha)=\frac{1}{2}\{|x_1-x_2|+|y_1-y_2|+|z_1-z_2|+|2|\}$ $\rightarrow 2$

Here $v_l=(2r+1,2\omega,2\omega-2r-1)$, $\alpha=(2\omega-1,2\omega,1)$

$$d(v_i,\alpha)=\frac{1}{2}\{|2r+1-2\omega+1|+|2\omega-2\omega|+|2\omega-2r-1-1|+|2|\}$$
$$=\frac{1}{2}\{|2r-2\omega+2|+|2\omega-2r-2|+|2|\}$$
$$=\frac{1}{2}\{|2\omega-2r-2|+|2\omega-2r-2|+|2|\}$$
$$=\frac{1}{2}\{|2\omega-2r-2+2|+|2\omega-2r-2\}$$
$$=\frac{1}{2}\{|2\omega-2r|+|2\omega-2r-2|\}$$
$$=d(u_i,\alpha)$$

Using equation 1, we can find $d(u_l,\beta)\&\ d(v_l,\beta)$.

Here $u_l=(2r,2\omega-1,2\omega-1-2r)$, $\beta=(2\omega-1,-1,-2\omega$

$$d(u_i,\beta)=\frac{1}{2}\{|2r-2\omega+1|+|2\omega-1+1|+|2\omega-1-2r+2\omega|\}$$
$$=\frac{1}{2}\{|2r-2\omega+1|+|2\omega|+|4\omega-2r-1|\}$$
$$=\frac{1}{2}\{|2\omega-2r-1|+|2\omega|+|4\omega-2r-1|\}$$
$$=\frac{1}{2}\{|4\omega-2r-1|+|4\omega-2r-1|\}$$
$$=|4\omega-2r-1|\}$$

To find $d(v_l,\beta)$, here $v_l=(2r+1,2\omega,2\omega-2r-1)$, $\beta=(2\omega-1,-1,-2\omega)$

$$d(v_i,\beta)=\frac{1}{2}\{|2r+1-2\omega+1|+|2\omega+1|+|2\omega-2r-1+2\omega|\}$$
$$=\frac{1}{2}\{|2r-2\omega+2|+|2\omega+1|+|4\omega-2r-1|\}$$
$$=\frac{1}{2}\{|2\omega-2r-2|+|2\omega+1|+|4\omega-2r-1|\}$$
$$=\frac{1}{2}\{|2\omega-2r-2+2\omega+1|+|4\omega-2r-1|\}$$
$$=\frac{1}{2}\{|4\omega-2r-1|+|4\omega-2r-1\}$$
$$=|4\omega-2r-1|$$
$$=d(u_i,\beta)$$

Using equation 1, we can find $d(u_r,\alpha)\&\ d(v_r,\alpha)$ where $u_r=\big(2r,-(2\omega-1-2r),-(2\omega-1)\big)$, $\alpha=(2\omega-1,2\omega,1)$

$$d(u_r,\alpha)=\frac{1}{2}\{|2r-2\omega+1|+|-2\omega+1+2r-2\omega|+|-2\omega+1-1|\}$$
$$=\frac{1}{2}\{|2r-2\omega+1|+|-4\omega+2r+1+-2\omega\}$$
$$=\frac{1}{2}\{|2\omega-2r-1|+|4\omega-2r-1|+|2\omega|\}$$
$$=\frac{1}{2}\{|2\omega-2r-1+2\omega|+|4\omega-2r-1|\}$$
$$=\frac{1}{2}\{|4\omega-2r-1|+|4\omega-2r-1|\}=|4\omega-2r-1|$$

Then $v_r=(2r+1,-(2\omega-2r-1),-2\omega)$, $\alpha=(2\omega-1,2\omega,1)$

$$d(v_r,\alpha)=\frac{1}{2}\{|2r+1-2\omega+1|+|-2\omega+2r+1-2\omega|+|-2\omega-1|\}$$
$$=\frac{1}{2}\{|2r-2\omega+2|+|-4\omega+2r+1|+|2\omega+1|\}$$
$$=\frac{1}{2}\{|2\omega-2r-2|+|4\omega-2r-1|+|2\omega+1|\}$$
$$=\frac{1}{2}\{|2\omega-2r-2+2\omega+1|+|4\omega-2r-1|\}$$
$$=\frac{1}{2}\{|4\omega-2r-1|+|4\omega-2r-1|\}$$

$= |4\omega - 2r - 1| = d(u_r, \alpha)$

Using equation 1, we can find d(u_r,β)

$u_r = (2r, -(2\omega-1-2r), -(2\omega-1))$, $\beta = (2\omega-1,-1,-2\omega)$

$d(u_r,\beta) = \frac{1}{2}\{|2r-2\omega+1| + |-2\omega+1+2r+$ *1+−2ω+1+2ω}*

$= \frac{1}{2}\{|2\omega-2r-1| + |2\omega-2r-2| +$ |1|}

$= \frac{1}{2}\{|2\omega-2r-1+1| + |2\omega-2r-$ *2}*

$= \frac{1}{2}\{|2\omega-2r| + |2\omega-2r-2|\}$

Using equation 2, find d(v_r,β)

where $v_r = (2r+1, -(2\omega-2r-1), -2\omega)$, $\beta = (2\omega-1,-1,-2\omega)$

$d(v_r,\beta) = \frac{1}{2}\{|2r+1-2\omega+1| + |-2\omega+2r+1+1| + |-2\omega+2\omega| + [2]\}$

$= \frac{1}{2}\{|2r-2\omega+2| + |-2\omega+2r+$ *2+2}*

$= \frac{1}{2}\{|2\omega-2r-2| + |2\omega-2r-2| +$ [2]}

$= \frac{1}{2}\{|2\omega-2r-2+2| + |2\omega-2r-2|\}$

$= \frac{1}{2}\{|2\omega-2r| + |2\omega-2r-2|\}$

$= d(u_r,\beta)$

$\therefore \{\alpha,\beta\}$ is not a resolving set for $(u_l,v_l)\&(u_r,v_r)$. Hence proved.

Lemma 3:

Let $G = \{u_l(-2r, 2\omega-1-2r, 2\omega-1)\}$, $H = \{v_l(-(2r+1), 2\omega-1-2r, 2\omega)\}$, $I = \{u_r(-2r, -(2\omega-1), -(2\omega-1-2r))\}$, $J = \{v_r(-(2r+1), -2\omega, -(2\omega-1-2r))\}$

where $0 \le r \le \omega-2 \& \omega \ge 2$, $u_l \in G, v_l \in H, u_r \in I, v_r \in J$, then $\{\gamma,\eta\}$ is not a resolving set for $(u_l,v_l)\&(u_r,v_r)$.

Proof:

We know that if u &v not lie on the same vertex channel then

$d(u,v) = \frac{1}{2}\{|x_1-x_2| + |y_1-y_2| + |z_1-z_2|\}$ →1

and if u&v lie on the same vertex channel then

$d(u,v) = \frac{1}{2}\{|x_1-x_2| + |y_1-y_2| + |z_1-z_2| + |2|\}$→2

Using equation 1, find $d(u_l,\gamma)$& $d(v_l,\gamma)$ where

$u_l = (-2r, 2\omega-1-2r, 2\omega-1), \gamma = (-(2\omega-1), -2\omega, -1)$

$$d(u_l,\gamma) = \frac{1}{2}\{|-2r+2\omega-1| + |2\omega-1-2r+2\omega| + |2\omega-1+1|\}$$

$= \frac{1}{2}\{|2\omega-2r-1| + |4\omega-2r-1| +$ *2ω*

$= \frac{1}{2}\{|2\omega-2r-1+2\omega| +$ *4ω−2r−1*

$= \frac{1}{2}\{|4\omega-2r-1| + |4\omega-2r-1|\}$

$= |4\omega-2r-1|$

$$d(v_l,\gamma) = \frac{1}{2}\{|-2r-1+2\omega-1| + |2\omega-2r-1+2\omega| + |2\omega+1|\}$$

where $v_l = (-(2r+1), 2\omega-2r-1, 2\omega)$, $\gamma = (-(2\omega-1), -2\omega, -1)$

$$d(v_l,\gamma) = \frac{1}{2}\{|2\omega-2r-2| + |4\omega-2r-1| + |2\omega+1|\}$$

$= \frac{1}{2}\{|2\omega-2r-2+2\omega+1| +$ *4ω−2r−1*

$= \frac{1}{2}\{|4\omega-2r-1| +$ *4ω−2r−1*

$= |4\omega-2r-1| = d(u_l,\gamma)$

Using equation 1, find $d(u_l,\eta)$ where

$u_l = (-2r, 2\omega-1-2r, 2\omega-1)$, $\eta = (-(2\omega-1), 1, 2\omega)$

$d(u_l,\eta) = \frac{1}{2}\{|-2r+2\omega-1| + |2\omega-2r-1-$ *1+2ω−1−2ω*

$= \frac{1}{2}\{|2\omega-2r-1| + |2\omega-2r-2| +$ *−1*

$= \frac{1}{2}\{|2\omega-2r-1+1| + |2\omega-2r-$ *2*

$= \frac{1}{2}\{|2\omega-2r| + |2\omega-2r-2|\}$

Using equation 2, find $d(v_l,\eta)$ where

$v_l = (-(2r+1), 2\omega-2r-1, 2\omega)$, $\eta = (-(2\omega-$ *1,1,2ω*

$d(v_l,\eta) = \frac{1}{2}\{|-2r-1+2\omega-1| +$ *2ω−2r−1−1+2ω−2ω+2*

$= \frac{1}{2}\{|2\omega-2r-2| + |2\omega-2r-2| +$ *2*

$= \frac{1}{2}\{|2\omega-2r-2+2| +$ *2ω−2r−2*

$= \frac{1}{2}\{|2\omega-2r| + |2\omega-2r-2|\} = d(u_l,\eta)$

Using equation 1, find $d(u_r,\gamma)$ where

$u_r = (-2r, -(2\omega-1), -(2\omega-1-2r))$, $\gamma = (-(2\omega-1), -2\omega, -1)$

$d(u_r,\gamma) = \frac{1}{2}\{|-2r+2\omega-1| + |-2\omega+1+$ *2ω+−2ω+1+2r+1*

$= \frac{1}{2}\{|2\omega-2r-1| + |1| +$ *−2ω+2r+2*

$= \frac{1}{2}\{|2\omega-2r-1+1| +$ *2ω−2r−2*

$= \frac{1}{2}\{|2\omega-2r| + |2\omega-2r-2|\}$

Using equation 2, find $d(v_r,\gamma)$ where

$v_l = (-(2r+1), -2\omega, -(2\omega-1-2r)), \gamma = (-(2\omega-1), -2\omega, -1)$

$d(v_r,\gamma) = \frac{1}{2}\{|-2r-1+2\omega-1| +$ *−2ω+2ω+−2ω+1+2r+1+2*

$= \frac{1}{2}\{|2\omega-2r-2| + |-2\omega+2r+$ *2+2*

$= \frac{1}{2}\{|2\omega-2r-2+2| +$ *−2ω+2r+2*

$= \frac{1}{2}\{|2\omega-2r| + |2\omega-2r-2|\} = d(u_r,\gamma)$

Using equation 1, find $d(u_r,\eta)$& $d(v_r,\eta)$ where

$u_r = (-2r, -(2\omega-1), -(2\omega-1-2r))$, $\eta = (-(2\omega-1), 1, 2\omega)$

$d(u_r,\eta) = \frac{1}{2}\{|-2r+2\omega-1| + |-2\omega+1-1| + |-2\omega+1+2r-2\omega|\}$

$= \frac{1}{2}\{|2\omega - 2r - 1| + |-2\omega| +$
$-4\omega+2r+1$
$= \frac{1}{2}\{|2\omega - 2r - 1 + 2\omega| +$
$4\omega-2r-1$
$= \frac{1}{2}\{|4\omega - 2r - 1| + |4\omega - 2r - 1|\}$
$= |4\omega - 2r - 1|$
$d(v_r, \eta) = \frac{1}{2}\{|-2r - 1 + 2\omega - 1| +$
$-2\omega-1+-2\omega+1+2r-2\omega$
where $v_r = (-(2r+1), -2\omega, -(2\omega - 1 - 2r)$, $\eta = -2\omega-1,1,2\omega$
$d(v_r, \eta) = \frac{1}{2}\{|2\omega - 2r - 2| + |2\omega + 1| +$
$-4\omega+2r+1$
$$= \frac{1}{2}\{|2\omega - 2r - 2 + 2\omega + 1| + |4\omega - 2r - 1|\}$$
$= \frac{1}{2}\{|4\omega - 2r - 1| + |4\omega - 2r -$
1
$= |4\omega - 2r - 1| = d(u_r, \eta)$

Hence the lemma 3.

Lemma 4

Let $S_1 = \{u_i(r+1, r, -1)/2 \le r \le 2\omega - 1\}$ $S_2 = \{v_i(r+1, 1, -r)/2 \le r \le 2\omega - 1\}$ where $u_i \in S_1$ and $v_i \in S_2$ then $\{\gamma, \eta\}$ is not a resolving set for (u_i, v_i) where $\gamma = (-(2\omega - 1), -2\omega, -1)$ and $\eta = (-(2\omega - 1), 1, 2\omega)$

Proof:

The distance between u_i and γ is
$$d(u_i, \gamma) = \frac{1}{2}\{|r + 1 + 2\omega - 1| + |r + 2\omega| + |-1 + 1|\}$$
$= \frac{1}{2}\{|2\omega + r| + |2\omega + r|\}$
$= |2\omega + r|$

The distance between v_i and γ is
$$d(v_i, \gamma) = \frac{1}{2}\{|r + 1 + 2\omega - 1| + |1 + 2\omega| + |-r + 1|\}$$
$= \frac{1}{2}\{|2\omega + r| + |2\omega + 1| +$
$r-1$
$= \frac{1}{2}\{|2\omega + 1| + |2\omega + 1|\}$
(since $r - 1 \ge 1$ implies r is positive)
$= d(u_i, \gamma)$

The distance between u_i and η is
$$d(u_i, \eta) = \frac{1}{2}\{|r + 1 + 2\omega - 1| + |r - 1| + |-1 - 2\omega|\}$$
$= \frac{1}{2}\{|2\omega + r| + |r - 1| +$
$2\omega+1$
$= \frac{1}{2}\{|2\omega + r| + |2\omega + r|\} =$
$|2\omega + r|$

The distance between v_i and η is
$d(v_i, \eta) = \frac{1}{2}\{|r + 1 + 2\omega - 1| +$
$1-1+-r-2\omega$
$= \frac{1}{2}\{|2\omega + r| + |2\omega + r|\}$
$= |2\omega + r| = d(u_i, \eta)$

Hence the Lemma 4.

Lemma 5:

If $\{\alpha, \beta\}$ is not resolving u and v then both $\{\gamma\}$ and $\{\eta\}$ must resolve u and v.

Proof:

By using lemma 2, $\{\alpha, \beta\}$ is not a resolving set for
(i) $u = u_l$ and $v = v_l$ for the same value of r.
(ii) $u = u_r$ and $v = v_r$ for the same value of r

Now we have to show that $\{\gamma\}$ and $\{\eta\}$ must resolve u and v. From lemma 2 we have
$u_l = (2r, 2\omega - 1, 2\omega - 1 - 2r)$; $v_l = (2r + 1, 2\omega, 2\omega -$ $2r-1$
$u_r = (2r, -(2\omega - 1 - 2r), -(2\omega - 1))$; $v_r = (2r + 1, -(2\omega - 1 - 2r), -2\omega)$
where $0 \le r \le \omega - 2$ & $\omega \ge 2$. Also we have $= (-(2\omega - 1), -2\omega, -1)$, $\eta = (-(2\omega - 1), 1, 2\omega)$

The distance between u_l and γ is
$$d(u_l, \gamma) = \frac{1}{2}\{|2r + 2\omega - 1| + |2\omega - 1 + 2\omega| + |2\omega - 1 - 2r + 1|\}$$
$= \frac{1}{2}\{|2\omega + 2r - 1| + |4\omega - 1| +$
$2\omega-2r$
$= \frac{1}{2}\{|4\omega - 1| + |4\omega - 1|\} =$
$|4\omega - 1|$

The distance between v_l and γ is
$$d(v_l, \gamma) = \frac{1}{2}\{|2r + 1 + 2\omega - 1| + |2\omega + 2\omega| + |2\omega - 1 - 2r + 1|\}$$
$= \frac{1}{2}\{|2\omega + 2r| + |4\omega| + |2\omega - 2r|\}$
$= \frac{1}{2}\{|4\omega| + |4\omega|\} = |4\omega| \ne d(u_l, \gamma)$

The distance between u_l and η is
$d(u_l, \eta) = \frac{1}{2}\{|2r + 2\omega - 1| + |2\omega - 1 - 1| +$
$2\omega-1-2r-2\omega$
$= \frac{1}{2}\{|2r + 2\omega - 1| + |2\omega - 2| + |2r + 1|\}$
$= \frac{1}{2}\{|2r + 2\omega - 1| + |2\omega + 2r - 1|\} =$
$|2\omega + 2r - 1|$

The distance between v_l and η is
$$d(v_l, \eta) = \frac{1}{2}\{|2r + 1 + 2\omega - 1| + |2\omega - 1| + |2\omega - 2r - 1 - 2\omega|\}$$
$= \frac{1}{2}\{|2r + 2\omega| + |2\omega - 1| +$
$2r+1=122\omega+2r$
$\ne d(u_l, \eta)$.

The distance between u_r and γ is
$d(u_r, \gamma) = \frac{1}{2}\{|2r + 2\omega - 1| + |2\omega - 1 - 1| +$
$-2\omega+1+2r+2\omega$
$= \frac{1}{2}\{|2\omega + 2r - 1| + |2r + 1| + |2\omega - 2|\}$
$= \frac{1}{2}\{|2\omega + 2r - 1| + |2\omega + 2r - 1|\} =$
$|2\omega + 2r - 1|$

The distance between v_r and γ is
$$d(v_r, \gamma) = \frac{1}{2}\{|2r + 1 + 2\omega - 1| + |-2\omega + 2r + 1 + 2\omega| + |-2\omega + 1|\}$$
$= \frac{1}{2}\{|2\omega + 2r| + |2r + 1| + |2\omega - 1|\}$
$= \frac{1}{2}\{|2\omega + 2r| + |2\omega + 2r|\}$
$= |2\omega + 2r| \ne d(u_r, \gamma)$

The distance between u_r and η is
$d(u_r, \eta) = \frac{1}{2}\{|2r + 2\omega - 1| + |-2\omega + 1 +$
$2r-1+-2\omega+1-2\omega$
$= \frac{1}{2}\{|2r + 2\omega - 1| + |2\omega - 2r| +$
$4\omega-1$
$=$
$\frac{1}{2}\{|4\omega - 1| + |4\omega - 1|\} =$
$|4\omega - 1|$

The distance between v_r and η is

$$d(v_r,\eta) = \frac{1}{2}\{|2r+1+2\omega-1| + |-2\omega+2r+1-1| + |-2\omega-2\omega|\}$$
$$= \frac{1}{2}\{|2\omega+2r| + |2\omega-2r| + |4\omega|\}$$
$$= \frac{1}{2}\{|4\omega| + |4\omega|\} = |4\omega| \neq d(u_r,\eta)$$

Hence the Lemma 5.

Lemma 6:

If $\{\gamma,\eta\}$ is not resolving u and v then both $\{\alpha\}$ and $\{\beta\}$ will resolve u and v.

Proof:

By using Lemma 3 we will get $\{\gamma,\eta\}$ is not resolve by u and v.

Enough To Prove: $\{\alpha\}$ and $\{\beta\}$ will resolve u and v. Where u and v denoted by

$u_l = (-2r, 2\omega-1-2r, 2\omega-1)$, $v_l = (-(2r+$ *1,2ω−2r−1,2ω*

$u_r = (-2r, -(2\omega-1), -(2\omega-1-2r))$, $v_r = (-(2r+1)-2\omega, -(2\omega-1-2r))$

$\alpha = ((2\omega-1),\ 2\omega,\ 1)$ and $\beta = ((2\omega-1), -1, -2\omega)$

The distance between u_l and α is

$$d(u_l,\alpha) = \frac{1}{2}\{|-2r-2\omega+1| + |2\omega-1-2r-2\omega| + |2\omega-1-1|\}$$
$$= \frac{1}{2}\{|2\omega+2r-1| + |2r+1| +$$

2ω−2

$$= \frac{1}{2}\{|2\omega+2r-1| + |2\omega+2r-$$

1

$$= \{|2\omega+2r-1|\}$$

The distance between v_l and α is

$$d(v_l,\alpha) = \frac{1}{2}\{|-2r-1-2\omega+1| + |2\omega-1-2r-2\omega| + |2\omega-1|\}$$
$$= \frac{1}{2}\{|2\omega+2r| + |2r+1| +$$

2ω−1

$$= \frac{1}{2}\{|2\omega+2r| + |2\omega+2r|\} \neq$$

$d(u_l,\alpha)$

The distance between u_l and β is

$$d(u_l,\beta) = \frac{1}{2}\{|-2r-2\omega+1| + |2\omega-1-2r+1| + |2\omega-1+2\omega|\}$$
$$= \frac{1}{2}\{|2\omega+2r-1| + |2\omega-2r| +$$

4ω−1

$$= \frac{1}{2}\{|4\omega-1| + |4\omega-1|\} = |4\omega-1|$$

The distance between v_l and β is

$$d(v_l,\beta) = \frac{1}{2}\{|-2r-1-2\omega+1| +$$

2ω−2r−1+1+2ω+2ω

$$= \frac{1}{2}\{|2\omega+2r| + |2\omega-2r| + |4\omega|\}$$
$$= \frac{1}{2}\{|4\omega| + |4\omega|\} = |4\omega| \neq$$

$d(u_l,\beta)$.

The distance between u_r and α is

$$d(u_r,\alpha) = \frac{1}{2}\{|-2r-2\omega+1| + |-2\omega+1-2\omega| + |-2\omega+1+2r-1|\}$$
$$= \frac{1}{2}\{|2\omega+2r-1| + |4\omega-1| +$$

2ω−2r

$$= \frac{1}{2}\{|4\omega-1| + |4\omega-1|\} = |4\omega-1|$$

The distance between v_r and α is

$$d(v_r,\alpha) = \frac{1}{2}\{|-2r-1-2\omega+1| +$$

−2ω−2ω+−2ω+1+2r−1

$$= \frac{1}{2}\{|2\omega+2r| + |4\omega| + |2\omega-2r|\}$$
$$= \frac{1}{2}\{|4\omega| + |4\omega|\} = |4\omega| \neq$$

$d(u_r,\alpha)$

The distance between u_r and β is

$$d(u_r,\beta) = \frac{1}{2}\{|-2r-2\omega+1| + |-2\omega+1+$$

1+−2ω+1+2r+2ω

$$= \frac{1}{2}\{|2\omega+2r-1| + |2\omega-2| +$$

2r+1

$$= \frac{1}{2}\{|2\omega+2r-1| + |2\omega+2r-$$

1=2ω+2r−1

The distance between v_r and β is

$$d(v_r,\beta) = \frac{1}{2}\{|-2r-1-2\omega+1| +$$

−2ω+1+−2ω+1+2r+2ω

$$= \frac{1}{2}\{|2\omega+2r| + |2\omega-1| +$$

2r+1

$$= \frac{1}{2}\{|2\omega+2r| + |2\omega+2r|\}$$

$$= |2\omega+2r| \neq d(u_r,\beta)$$

Hence the Lemma 6.

Therefore any pair of vertices in the oxide interconnection can be resolved by $P = \{\alpha,\beta,\gamma\}$. Hence the metric index of oxide interconnection is 2 for $\omega = 1$ and metric index of oxide interconnection is 3 for $\omega \geq 2$.

Remark: The metric basis of Oxide interconnection is not unique. By the symmetric property of Oxide interconnection, the following sets $P = \{\eta,\beta,\gamma\}$, $Q = \{\alpha,\gamma,\eta\}$ and $R = \{\alpha,\beta,\eta\}$ are also metric basis for $OX(n)$.

Theorem 1: Let $G = OX(n)$, then $fdim(G) = \begin{cases} 3 & n = 1 \\ 4 & n \geq 2 \end{cases}$

Proof: For $n \geq 1$, the fault tolerant metric basis of G is $W = \{\alpha,\beta,\gamma,\eta\}$. If we delete any one vertex in W, then it is clear that the remaining vertices in W will resolve G. That is in the absence of α, $P = \{\eta,\beta,\gamma\}$ will resolve G, in the absence of β, $Q = \{\alpha,\gamma,\eta\}$ will resolve G, and in the absence of γ, $R = \{\alpha,\beta,\eta\}$ will resolve G. Hence $W = \{\alpha,\beta,\gamma,\eta\}$ is a fault-tolerant metric basis for G.

II. CONCUSION

In this paper we have determined the fault-tolerant metric index of oxide interconnections.

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AUTHORS PROFILE

M.Somasundari received B.Sc, M.Sc and M.Phil degrees in 2010, 2012, and 2014 respectively from Madras University and Kodaikanal University . He joined Hindustan group of Institutions in July 2017. He has 5 years of teaching experience in Engineering Colleges. Currently he is doing Ph.D in Graph Theory at Hindustan Institute of Technology and Science.

F. Simon Raj received B.Sc, M.Sc and M.Phil degrees in 2002, 2004, and 2006 respectively from Loyola College , Chennai, Affiliated to University of Madras. He joined in Hindustan group of Institutions in September 2005. He has 14 years of teaching experience in Engineering Colleges. His research interests are Graph Theory applications and Graph algorithms. Currently he is guiding 4 research scholars in HITS.