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T-Span, T-Edge Span Critical Graphs

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Abstract— *Given a graph $G = (V, E)$ and a finite set T of positive integers containing 0, a T-coloring of G is a function $f : V(G) \to Z^+ \cup \{0\}$ for all $u \neq w$ in $V(G)$ such that if $uw \in E(G)$ then $|f(u) - f(w)| \notin T$. For a T-coloring f of G, the f-span $sp_T^f(G)$ is the maximum value of $|f(u) - f(w)|$ over all pairs u, w of vertices of G. The T-span $sp_T(G)$ is the minimum f-span over all T-colorings f of G. The f-edge span $esp_T^f(G)$ of a T-coloring is the maximum value of $|f(u) - f(w)|$ over all edges uw of G. The T-edge span $esp_T(G)$ is the minimum f-edge span over all T-colorings f of G. It is known that $sp_T(H) \leq sp_T(G)$ and $esp_T(H) \leq esp_T(G)$ for every graph G. In this paper we classify which graphs containing a sub graph H such that $sp_T(H) < sp_T(G)$ and $esp_T(H) < esp_T(G)$. Also we discuss the Mycielskian of T-coloring.*

Key words: T-coloring, T-span, T-edge spanAMS subject classification 05C15

I. INTRODUCTION AND DEFINITIONS

All graphs considered in this paper are finite, simple and undirected. All the definitions which are not discussed in this paper one may refer [1]. Let $G = (V, E)$ be a graph and let T be any finite set of positive integers containing 0. A T-coloring of G is a function $f : V(G) \to Z^+ \cup \{0\}$ for all $u \neq w$ in $V(G)$ such that if $uw \in E(G)$ then $|f(u) - f(w)| \notin T$. For a T-coloring f of G, the f-span $sp_T^f(G)$ is the maximum value of $|f(u) - f(w)|$ over all pairs u, w of vertices of G. The T-span $sp_T(G)$ is the minimum f-span over all T-colorings f of G. The f-edge span $esp_T^f(G)$ of a T-coloring is the maximum value of $|f(u) - f(w)|$ over all edges uw of G. The T-edge span $esp_T(G)$ is the minimum f-edge span over all T-colorings f of G. In this paper $\chi(G)$ is the chromatic number of G and $\omega(G)$ is the clique number of G. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple graphs. A graph G is called Hamiltonian-triangle graph if it has odd cycle on n vertices with at least one triangle. The Cartesian product $G_1 \square G_2$, has vertex set $V_1 \times V_2$ and two vertices (u_1, v_1) and (u_2, v_2) are adjacent in G_1, G_2 iff either $u_1 = u_2$ and v_1 is adjacent to v_2 in G_2 or $v_1 = v_2$ and u_1 is adjacent to u_2 in G_1.

The concept of T-coloring problem was introduced by Hale in [4]. Basic results on T-coloring problems are discussed in [2].

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Cozzen and Roberts [2] studied T-sets of the form $T = \{0, 1, 2, \dots, r\} \cup S$, where S is any set that doesn't contain any multiple of $(r + 1)$ and call this set as r-initial set. Raychaudhuri [10] studied T-sets of the form, $T = \{0, s, 2s, \dots, ks\} \cup S$ where $S \subseteq \{s + 1, s + 2, \dots, ks - 1\}$, and $s, k \geq 1$, calling this T-set as a k-multiple of s set. In this paper we consider these two T sets.

In their papers of Hale and Robert, defined a parameter $sp_T(G)$ which is trying to save the spectrum when the frequencies allotted to the transmitter. According to the T-coloring problems vertices are considered as transmitters. Thus removal of one transmitter may reduce more span which is related to spectrum and more edge span which is related to bandwidth in such a way that we define the removal of some verteices implies $sp_T(G - V') < sp_T(G)$ and $esp_T(G - V') < esp_T(G)$ where V' is a proper subset of $V(G)$. For more results on T-coloring one may refer [3,5,6,7,8] and survey on T-coloring as discussed in [9].

PREVIOUS RESULTS:

Theorem 1.1. [6] Let W_n be a Wheel graph with n even. For a k-multiple of s set T,

$sp_T(W_n) = esp_T(W_n) = 3k + 3$ as $s = 1$
$sp_T(W_n) = esp_T(W_n) = 2k + 3$ as $s = 2$
$sp_T(W_n) = esp_T(W_n) = 3k + 3$ as $s = 3$
$sp_T(W_n) = esp_T(W_n) = 3$ as $s \geq 4$.

Theorem 1.2. [6] Let W_n be a Wheel graph with n is odd. For T is a k-multiple of s set,

$sp_T(W_n) = esp_T(W_n) = 2k + 2$ as $s = 1,2$
$sp_T(W_n) = esp_T(W_n) = 2$ as $s \geq 3$.

Theorem 1.3. [8] For any odd cycle C_n and $T = \{0,1,2,\dots,k-1\}$, $esp_T(G) = \left\lceil \frac{(n+1)}{(n-1)} \right\rceil k$.

Theorem 1.4. [5] For any odd cycle C_n and a k-multiple of s set T,

$sp_T(C_n) = esp_T(C_n) = 2k + 2$ as $s = 2$
$sp_T(C_n) = esp_T(C_n) = 2$ as $s \geq 3$.

Theorem 1.5. [2] If G is weakly γ perfect, then for all sets T, $esp_T(G) = sp_T(G) = sp_T(K_{\chi(G)})$.

II. MAIN RESULTS

Definition 2.1. A subset V' of the vertex set $V(G)$ of a connected graph G is called span altering set if $sp_T(G - V') < sp_T(G)$.

Definition 2.2. A subset V' of the vertex set $V(G)$ of a connected graph G is called edge span altering set if $esp_T(G - V') < esp_T(G)$.

Obviously, In any graph G if we remove $n - 1$ vertices reduces the span and the edge span. So the question is what is the minimum number of vertices whose removal reduces the span and the edge span.

The minimum cardinality of a span altering set is called the span critical number denoted by $\zeta(G)$.

The minimum cardinality of an edge span altering set is called the edge span critical number denoted by ζ'(G).

Theorem 2.1. For all T sets $\zeta(K_n) = \zeta'(K_n) = 1$.

Proof. It is obvious that $sp_T(K_n) = esp_T(K_n)$ for every n. Now removal of a single vertex implies that $sp_T(K_{n-1}) < sp_T(K_n)$ and $esp_T(K_{n-1}) < esp_T(K_n)$. Hence, $\zeta(K_n) = \zeta'(K_n) = 1$.

Theorem 2.2. Let W_n be a wheel graph with n even. For a k-multiple of s set T, $\zeta(W_n) = \zeta'(W_n) = 1$.

Proof. Let T be a k-multiple of s set.

Using Theorem 1.1, $sp_T(W_n) = esp_T(W_n) = 3k + 3$ as $= 1$, $sp_T(W_n) = esp_T(W_n) = 2k + 3$ as $s = 2$, $sp_T(W_n) = esp_T(W_n) = 3k + 3$ as $s = 3$ and $sp_T(W_n) = esp_T(W_n) = 3$ as $s \geq 4$. Now remove the maximum degree vertex of W_n which causes more interferences, it reduces to an odd cycle by Theorem 1.3 and Theorem 1.4, $sp_T(C_n) < sp_T(W_n)$ and $esp_T(C_n) < esp_T(W_n)$ Hence, $\zeta(W_n) = \zeta'(W_n) = 1$, n is even.

Theorem 2.3. Let W_n be a wheel graph with n odd. For a k-multiple of s set T, $\zeta(W_n) = \zeta'(W_n) =$

Proof. Let T be a k-multiple of s set. Invoking Theorem 1.2, $sp_T(W_n) = esp_T(W_n) = 2k + 2$ as $s = 1,2$ and $sp_T(W_n) = esp_T(W_n) = 3$ as $s \geq 3$. Now remove the maximum degree vertex of W_n it reduces to an odd cycle by Theorem 1.3 and Theorem 1.4, $sp_T(C_n) < sp_T(W_n)$ and $esp_T(C_n) < esp_T(W_n)$ Hence, $\zeta(W_n) = \zeta'(W_n) = 1$, n is odd.

Theorem 2.4. Let C_n be a cycle with n odd. For a k-multiple of s set $T, \zeta(C_n) = \zeta'(C_n) = 1$.

Proof. Let T be a k-multiple of s set. Remove any one of the vertices of C_n implies a tree T. Since tree is a weakly γ perfect using Theorem 1.5, $sp_T(T) = sp_T(K_2)$. When s = 1, $sp_T(K_2) = k + 1 < sp_T(C_n) = 2k + 2$ and $esp_T(K_2) = k + 1 < \left\lceil \frac{(n+1)}{(n-1)} \right\rceil k$ and when $s \geq 2$, $sp_T(K_2) = esp_T(K_2) = 1 < sp_T(C_n) = esp_T(C_n) = 2$. Hence, $\zeta(C_n) = \zeta'(C_n) = 1$.

Theorem 2.5. Let G be a bipartite graph. Then $\zeta(G) = \zeta'(G) = n - 1$

Proof. Since G is a bipartite graph which is weakly γ perfect. Using Theorem 1.5., $sp_T(G) = esp_T(G)$, $sp_T(K_2) = esp_T(K_2)$. Therefore the only way to reduce span and edge span is remove n−1 vertices from G. Hence, $\zeta(G) = \zeta'(G) = n - 1$.

The following observation is immediate:

(i) $\zeta(G) = 1$ or $\zeta'(G) = 1$ if and only if there is an optimal T-coloring in which largest integer vertex v is in a singleton color class.

Theorem 2.6. If G is a graph with chromatic number $k \geq 3$ then $1 \leq \zeta(G) \leq n - 2$ and $1 \leq \zeta'(G) \leq n - 2$.

Proof. In any graph G with chromatic number $k \geq 3$ contains an odd cycle. Now remove $n - 2$ vertices results a K_2. Hence, $1 \leq \zeta(G) \leq n - 2$ and $1 \leq \zeta'(G) \leq n - 2$.

Theorem 2.7. Let T be a k-multiple of s set. If G is a non-perfect graph then $1 \leq \zeta(G) \leq n - \omega(G)$. **Proof.** Let G be a non-perfect graph. Thus $\chi(G) > \omega(G)$. Label the vertices of $\omega(G)$ by $S = \{v_1, v_2, \ldots, v_\omega\}$. Let $V_0 = V(G) - S$. Hence, we have $\mathrm{sp_T}(G - V_0) < \mathrm{sp_T}(G)$.

This bound is not attained and the converse is not true.

Theorem 2.8. Not all non perfect graphs has $\zeta(G) = 1$.

Proof. Petersen graph is not perfect $\omega(G) = 2$ and $\chi(G) = 3$ and $\zeta(G) = 2$. Since $sp_T(P) = esp_T(P) = 2k + 2$ as $s = 1,2$ and $sp_T(P) = esp_T(P) = 2$ as $s \geq 3$ which is greater than $sp_T(K_2) = esp_T(K_2) = k + 1$ as s = 1 and $sp_T(K_2) = esp_T(K_2) = 1$ as $s \geq 2$.

Theorem 2.9. Let G be either unicyclic graph or hamiltonian-triangle graph. Let T be a k-multiple of s set. Then $\zeta(G) = 1$.

Proof. Let G be an unicyclic graph. Then removal of a single vertex in that cycle produces a tree. Hence $\zeta(G) = 1$. Let G be a graph that contains a triangle and an odd cycle on n vertices. Thus G has at most $\binom{k}{2}$ edges. Now removal of a maximum degree vertex v results $\chi(G - v) < \chi(G)$. Which in turn implies that $\mathrm{sp_T}(G - v) < \mathrm{sp_T}(G)$. Hence, $\zeta(G) = 1$.

Theorem 2.10. Let T be a k-multiple of s set. The span critical number of a critical graph is one.

Proof. Let G be a critical graph and T be a k-multiple of s set. Then $\chi(H) < \chi(G)$ for every proper subgraph H of G. Thus removal of every single vertex implies that $\mathrm{sp_T}(G - v <$ spTG. Hence, *ζ(G) = 1.*

Theorem 2.11. Every graph with $\zeta(G) = 1$ contains an odd cycle but the converse is not true.

Proof. Let P be a Petersen graph with $\chi(P) = 3$ and $\zeta(G) = 2$.

Theorem 2.12. Let $\zeta(G_1) = 1$ and $\zeta(G_2) = 1$. Then the span critical number of cartesian product of G_1 and G_2 need not be equal to 1.

Proof. The cartesian product $G_1 \; \square \; G_2$of two graphs G_1 and G_2 contains sub graphs that are isomorphic to both G_1 and G_2. Also we know that every graph with $\zeta(G) = 1$ contains an odd cycle. Thus G_1 and G_2 contains an odd cycle. Suppose $G_1 = C_m$ and $G_1 = C_n$, m, n is odd. In $G_1 \; \square \; G_2$ we have G_1 copies of G_2 and G_2 copies of G_1. There are m number of vertex disjoint cycles or n number of vertex disjoint cycles will appear. Thus removal of single vertex will not reduce the span.

Theorem 2.13. Let $\zeta'(G_1) = 1$ and $\zeta'(G_2) = 1$. Then the edge span critical number of cartesian product of G_1 and G_2 need not equal to 1.

Proof. Proof is similar that of Theorem 2.12.

III. T-COLORING OF MYCIELSKIAN GRAPHS

One may refer for Mycielskian Construction[11].

The construction is as follows:

Let G be a triangle free graph with $\chi(G) = k$ and $V(G) = \{v_1, v_2, \ldots, v_n\}$. Construct a graph G' as follows. Let $V(G') = \{v_1, v_2, \ldots, v_n\} \cup \{u_1, u_2, \ldots, u_n\} \cup \{w\}, E(G') = E(G)$ and for every u_i, add edges from u_i to all of $v_i$is neighbours. In addition, take vertex w and made adjacent to all the $u_i's$. We call w is the central vertex of $\mu(G)$[11].

We denote Myceilskian of graph G by $\mu(G)$.

Previous results:

Theorem 3.1[2] For all graphs G and all T-sets, (i) $\chi_T(G) = \chi(G)$,

(ii) $sp_T(K_{\omega(G)}) \leq sp_T(G) \leq esp_T(G) \leq sp_T(K_{\chi(G)})$.

Theorem 3.2[2] If T is a k-initial set, then $sp_T(G) = sp_T(K_{\chi(G)}) = (k + 1)(\chi(G) - 1)$

Theorem 3.3 [10] If T is a k multiple of s set, then for all graphs G,

$$sp_T(G) = sp_T(K_{\chi(G)}) = \begin{cases} st + skt - sk - 1 \text{ if } \chi(G) = st \\ st + skt + m - 1 \text{ if } \chi(G) = st + m, 1 \le m \le s-1, \end{cases} \quad t \in \{1,2,3,\dots\}$$

Theorem 3.4[7] If T is *a k-multiple-of-s-set T*, and $\chi(G) \le s$ then $sp_T(G) = esp_T(G) = \chi(G) - 1$.

Theorem 3.5[5] Let H be a sub graph of a graph G. For each finite set of non-negative integers containing 0,
(i) $sp_T(H) \le sp_T(G)$, (ii) $esp_T(H) \le esp_T(G)$.

MAIN RESULTS:

Proposition 3.6 Let $\mu(G)$ be the Myceilskian of graph G. For all sets T,
(i) $\chi(\mu(G)) = \chi_T(G) + 1 = \chi(G) + 1$
(ii) $sp_T(K_2) \le esp_T(\mu(G)) \le sp_T(\mu(G)) \le sp_T(K_{\chi(G)+1})$.

Proof. Proof follows from Theorem 3.1.

Lemma 3.7. If T is a k-initial set, then $sp_T(\mu(G)) = sp_T(K_{\chi(G)+1}) = sp_T(G) + (k+1)$.

Proof. Let T be a k-initial set. Using Theorem 3.2, $sp_T(\mu(G)) = sp_T(K_{\chi(G)+1}) = (\chi(G) + 1 - 1)(k+1)$. Hence, $sp_T(\mu(G)) = sp_T(G) + (k+1)$.

Lemma 3.8 For all graphs and all T-sets,
(i) $sp_T(\mu(G) - w) = sp_T(G)$,
(ii) $esp_T(\mu(G) - w) = esp_T(G)$.

Proof. Invoking Theorem 3.5, $sp_T(\mu(G) - w) \ge sp_T(G)$ and $esp_T(\mu(G) - w) \ge esp_T(G)$. Let f be a T coloring of G such that $sp_T^f(G) = sp_T(G)$. Define $f'(u_i) = f'(v_i) = f(v_i), 1 \le i \le n-1$. Hence, f' is a T coloring of $\mu(G) - w$ in which f'-span is same as the f-span. Hence, $sp_T(\mu(G) - w) \le sp_T{}^{f'}(\mu(G) - w) \le sp_T^f(G) \le sp_T(G)$. Hence, $sp_T(\mu(G) - w) \le sp_T(G)$. Similarly Let g be a T coloring of G such that $esp_T^f(G) = esp_T(G)$. Define $g'(u_i) = g'(v_i) = g(v_i), 1 \le i \le n-1$. Hence, g' is a T coloring of $\mu(G) - w$ in which g'-edge span is same as the g-edge span. Hence, $esp_T(\mu(G) - w) \le esp_T{}^{g'}(\mu(G) - w) \le esp_T^g(G) \le esp_T(G)$. Hence,$esp_T(\mu(G) - w) \le esp_T(G)$.

Theorem 3.9. If T is a k-initial set, then $esp_T(\mu(G)) = esp_T(G) + (k+1)$.

Proof. Let T be a k-initial set. Using Lemma 3.8, $esp_T(\mu(G) - w) = esp_T(G)$. Since w is adjacent to all $\{u_1, u_2, \dots, u_n\}$ in $\mu(G)$, in any T coloring of $\mu(G)$ w must receive a new color different from u_i's and that should make difference at least $k+1$. Therefore color the vertex $esp_T(G) + ((k+1)$. Hence, $esp_T(\mu(G)) \le esp_T(G) + ((k+1)$. Suppose $esp_T(\mu(G)) < esp_T(G) + ((k+1)$ for some f. Then Then $|f(w) - f(u_i)| \in T$, a contradiction to the definition of T coloring. Hence, $esp_T(\mu(G)) = esp_T(G) + ((k+1)$.

Corollary 3.10 For a complete graph K_n and T is a k-initial set, $sp_T(\mu(K_n)) = esp_T(\mu(K_n)) = n(k+1)$.

Corollary 3.11 Let P_n be a path graph. For a k-initial set T, $sp_T(\mu(P_n)) = esp_T(\mu(P_n)) = 2(k+1)$.

Corollary 3,12 Let C_n be a cycle graph. For a k-initial set T, $sp_T(\mu(C_n)) = 3(k+1)$, $esp_T(\mu(K_n)) = \left\lceil \frac{n+1}{n-1}(k+1) \right\rceil + (k+1)$ if n is odd $sp_T(\mu(C_n)) = esp_T(\mu(C_n)) = 2(k+1)$.

Theorem 3.13. If T is a k-multiple of s set, then for all graphs G,

$$sp_T(\mu(G)) = sp_T(K_{\chi(G)+1}) = \begin{cases} st + skt - sk - 1 \text{ if } \chi(G) + 1 = st \\ st + skt + m - 1 \text{ if } \chi(G) + 1 = st + m, 1 \le m \le s-1, \end{cases} \quad t \in \{1,2,3,\dots\}.$$

Proof. Proof is immediate from Theorem 3.3.

Theorem 3.14. If T is a k-multiple of s set, then for all graphs G,
$sp_T(\mu(G)) = sp_T(K_{\chi(G)+1}) = sp_T(G) + (k+1)$

Proof. We consider two cases.
Case 1. $\chi(G) + 1 = st$.
$sp_T(\mu(G)) = sp_T(K_{\chi(G)+1}) = (\chi(G) + 1 - s)k + \chi(G) = (\chi(G) - s)k + \chi(G) + k = sp_T(G) + k + 1$
Case 2. $\chi(G) + 1 = st + m$.
$sp_T(\mu(G)) = sp_T(K_{\chi(G)+1}) = (\chi(G) + 1 - m)k + \chi(G) = (\chi(G) - m)k + \chi(G) + k = sp_T(G) + k + 1$.

Corollary 3.15. If T is a k-multiple of s set and $\chi(G) + 1 \le s$, then $sp_T(\mu(G)) = esp_T(\mu(G)) = \chi(G)$.

Proof. Proof is immediate from Theorem 3.4.

Lemma 3.16. Let G be any graph with $\chi(G) = 2$ and $T = \{0,2,4,\dots,2k\}$ then, Then $sp_T(\mu(G)) = esp_T(\mu(G)) = 2(k+1)$.

Proof. Invoking Theorem 3.14, $sp_T(\mu(G)) = 2(k+1)$. Hence, $esp_T(\mu(G)) \le 2(k+1)$ It is clear that, $esp_T(G) = esp_T(\mu(G) - w) = 1$. Without loss of generality, the smallest color assigned is 0 and the largest color assigned is 1. Suppose $esp_T(\mu(G)) < 2(k+1)$. Consider the sets $A = \{1,3,5,\dots,2k-1,2k+1\}$ and $B = \{0,2,4,\dots,2k\}$. Suppose $f(w) \in A$. Since w is adjacent to every vertices of u_i's, w must be adjacent to the vertex assigned the color 1, say u_j. Then we have $|f(w) - f(u_j)| \in T$, which is a contradiction to the definition of T-coloring. Hence $f(w) \notin$ A. Suppose $f(w) \in B$. Since w is adjacent to every vertices of u_i's, w must be adjacent to the vertex assigned the color 0, say u_k. Then we have $|f(w) - f(u_k)| \in T$, which is a contradiction to the definition of T-coloring. Hence $f(w) \notin$ B. Thus, $f(w) \ge 2k+2$. Hence, $esp_T(\mu(G)) \ge 2k+2$.

Theorem 3.17. Let G be any graph with $\chi(G) = 2$. If T is a k-multiple of 2 set, then $sp_T(\mu(G)) = esp_T(\mu(G)) = 2(k+1)$.

Proof. By Theorem 3.14, $sp_T(\mu(G)) = 2k+2$. By Theorem 3.1 (ii) we have $esp_T(\mu(G)) \le 2k+2$.By Lemma 3.16, we have if T=$\{0,2,4,\dots,2k\}$, there is no T-coloring function f of G satisfying the edge span $< 2(k+1)$. Then for any *k-multiple of 2 sets T* there is no T-coloring function f with edge span $< 2(k+1)$. Therefore $esp_T(\mu(G)) \ge 2k+2$. Hence $esp_T(\mu(G)) = sp_T(\mu(G)) = 2k+2$.

We end up the sections with the following open questions and a conjecture

(i) Prove or disprove: The edge span critical number of a critical graph is one.
(ii) For all graphs $G, 1 \le \zeta(G) \le$ number of disjoint odd cycles of G.
(iii) Classify the graphs, which have $\zeta(G) = \zeta'(G)$?

(iv) *Prove or dis prove* : $\zeta(G)$ = the number of vertex disjoint odd cycle of a graph G.
(v) Let G be any graph. Compute $\zeta(\mu(G))$ and $\zeta'(\mu(G))$.
Conjecture: Let G be any graph and $sp_T(G) = esp_T(G)$ if and only if $\zeta(G) = \zeta'(G)$.

IV. CONCLUSION

In this paper, we classify which graphs containing a sub graph $\boldsymbol{H}$ such that $\boldsymbol{sp_T(H)} < \boldsymbol{sp_T(G)}$ and $\boldsymbol{esp_T(H)} < \boldsymbol{esp_T(G)}$ .Also we discuss the Mycielskian of $\boldsymbol{T}$-coloring.

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