International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-8 Issue-12, October 2019

Power Dominator Chromatic Number for some Special Graphs

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Abstract: *Let G = (V, E) be a finite, connected, undirected with no loops, multiple edges graph. Then the power dominator coloring of G is a proper coloring of G, such that each vertex of G power dominates every vertex of some color class. The minimum number of color classes in a power dominator coloring of the graph, is the power dominator chromatic number $\chi_{pd}(G)$. Here we study the power dominator chromatic number for some special graphs such as Bull Graph, Star Graph, Wheel Graph, Helm graph with the help of induction method and Fan Graph. Suitable examples are provided to exemplify the results.*

Keywords: Bull graph, Coloring, Power dominator coloring, star graph, Wheel graph.

AMS Mathematics Subject Classification (2010): 05C15, 05C69

I. INTRODUCTION

In Graph theory, Domination and Coloring are two main areas of study in graph theory and these topics have been extensively explored by considering different variants, (see, [[1],[2],[3]].

While formulating a problem related to electric power system in graph theoretical terms, Haynes et al. [4] introduced the concept of power domination. A vertex set S of a graph G is defined as the power dominating set of graph if every vertex and every edge in the graph is monitored by S, with following a set of rules for power monitoring system. Based on the ideas of coloring and power domination, K.S. Kumar & N.G. David [[5],[6]] presented a new coloring variant called power dominator coloring of graph.

In this paper, we consider the graphs $G(V,E)$ that are finite, connected, undirected with no loops and multiple edges and find the power dominator chromatic numbers of some special graphs such as Bull Graph, Star Graph, Wheel Graph, Helm graph, and Fan Graph.

Revised Manuscript Received on October 30, 2019.
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II. PRELIMINARIES

Some of the basic definitions required for this paper are recalled below.

Definition 1: Proper Coloring

A proper coloring [4] of a graph G is an assignment of colors to the vertices of the graph such that no two adjacent vertices have the same color and the chromatic number χ(G) of the graph G is the minimum number of colors needed in a proper coloring of G.

Definition 2: Dominator Coloring

A dominator coloring [[5],[6],[7],[8],[9]] of a graph is a proper coloring such that each vertex dominates every vertex in at least one color class consisting of vertices with the same color. The chromatic number $\chi_d(G)$ of a graph G is the minimum number of colors needed in a dominator coloring of G.

Definition 3: Bull Graph

A bull graph is the planar undirected graph with 5 vertices and 5 edges constructed by inducting two pendent vertices to any two vertices of C_3.

Definition 4: Star Graph

A star $K_{1,n}$ is a tree with n vertices of degree 1and root vertex has degree n.

Definition 5: Wheel Graph [6]

A wheel graph $W_{1,n}$ is the join of C_n and star graph $K_{1,n}$.

Definition 6: Helm Graph [9]

The *Helm Graph* H_n is the graph obtained from a wheel graph $W_{1,n}$ by attaching a pendant edge at each vertex of the *n* – cycle.

Definition 7: Flower Graph [9]

A *flower* graph Fl_n is the graph obtained from a helm by joining each pendant vertex to the central vertex of the helm.

Definition 8: Monitoring Set

For a vertex v ∊ G, a monitoring set M(v)[6] is associated as follows:

Step(i): M(v) = N[v], the closed neighborhood on v

Step(ii) : Add a vertex u to M(v), (which is not in M(v)) whenever u has a neighbor w ∊ M(v) such that all the neighbors of w other than u, are already in M(v)

Step(iii): Repeat Step(ii) if no other vertex could be added to M(v).

Then we say that *v* power dominates the vertices in *M*(*v*).

Note that if a vertex *v* dominates [13] another vertex *u*, then *v* power dominates *u* but the converse need not be true.

Definition 9: Power Dominator Coloring

The power dominator coloring of G is a proper coloring of G, such that every single vertex of G power dominates all vertices of some color class.

The minimum number of color classes in a power dominator coloring of the graph, is the power dominator chromatic number. And it is denoted by $\chi_{pd}(G)$.

In Example 1, we explain the concept of power dominator coloring with the graph Fig 1.

We consider the graph G in Fig. 1. The vertex v_4 is colored by color 1 (C_1). The vertices v_2, v_3, v_5, v_6, v_8 are assigned the color 2 (C_2) and the vertices v_1, v_7, v_9 are assigned the color 3 (C_3). Every individual vertices of $v_1, v_2, v_3, v_5, v_6, v_7, v_8, v_9$ power dominates the vertex v_4 with color C_1. And vertex v_4 power dominates $\{v_4\}$ by the definition of power dominator coloring.

Example 1

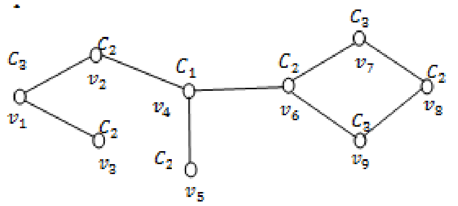

Fig. 1

For instance, consider v_3, it power dominates v_1, which power dominate v_2 and which in turn power dominate v_4 with color C_1. Hence the vertex v_3 power dominate the vertex v_1 with color C_1. In the similar way we can show that, every vertex in the Fig. 1, can power dominate v_4 with color C_1. Therefore, it is clear that, not less than 3 colors will be sufficient to offer a power dominator coloring for G *in Fig*.1.

The result relating the chromatic number, power dominator chromatic number and dominator chromatic number was proved in [11].

Theorem 1 [11]. *For any graph* G, $\chi(G) \leq \chi_{pd}(G) \leq \chi_d(G)$.

Remark: For the graph in Fig. 1, $\chi(G) = 2$, $\chi_{pd}(G) = 3$, $\chi_d(G) = 5$. Hence $(G) < \chi_{pd}(G) < \chi_d(G)$.

III. MAIN RESULTS

Theorem 1

The power dominator coloring for a Bull graph G, $\chi_{pd}(G) = 3$

Proof

Let graph $G(V, E)$ be the bull graph with 5 vertices. The vertices of the graph be v_1, v_2, v_3, v_4, v_5. A power dominator coloring of G is given as, Color the vertices v_1, v_4, v_5 with C_1, vertices v_2 and v_5 colored with C_2 and C_3 respectively. So vertices v_1, v_5 power dominate v_2 and vertices v_1 with C_2, v_4 power dominate v_3 with C_3. Both the vertices v_3 power dominate v_2 and v_2 power dominate v_3.

Thus any vertex of G power dominates some color class. The power dominator coloring for a Bull graph G is $\chi_{pd}(G) = 3$.

Example 2

In Figure 2, the power dominator coloring of bull graph G is shown.

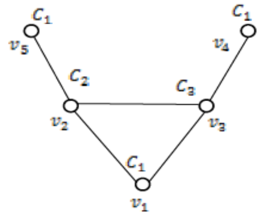

Fig. 2

In the Fig. 2 the color classes of the bull graph are $C_1 = \{v_1, v_4, v_5\} C_2 = \{v_2\}$ & $C_3 = \{v_3\}$. Then $\chi_{pd}(G) = 3$.

Theorem 2

For $n \geq 3$, the power dominator chromatic number of the star graph, $\chi_{pd}(K_{1,n}) = 2$

Proof

The vertices of the star graph be v_1 the root vertex, pendent vertices are v_{i+1}, $1 \leq i \leq n$. Assign the color C_1 to the root vertex and color C_2 to the pendent vertices v_{i+1}, $1 \leq i \leq n$. The root vertex power dominates the color class $\{v_2, v_3, v_4, \dots, v_{n+1}\}$, each pendent vertex power dominate color class $\{v_1\}$. This guarantees that the power dominator coloring is proper. The power dominator chromatic number for the star graph $\chi_{pd}(K_{1,n}) = 2, n \geq 3$. □

Example 3

In Figure 3, let us present the power dominator coloring of star graph $K_{1,8}$

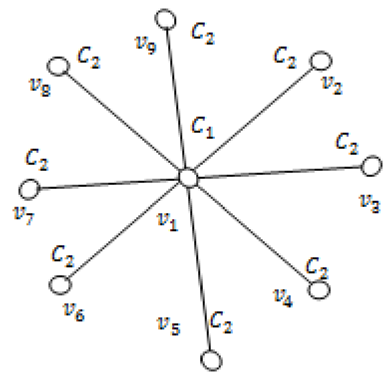

Fig. 3

In the Fig. 3 the color classes of the star graph $K_{1,8}$ are $C_1 = \{v_1\}$, $C_2 = \{v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$.Then $\chi_{pd}(K_{1,8}) = 2$.

Theorem 3

For any $n \geq 3$, the power dominator chromatic number of the wheel graph

$$\chi_{pd}(W_{1,n}) = \begin{cases} 3, if\ n\ is\ even \\ 4, if\ n\ is\ odd \end{cases}$$

Proof

Case (i) Wheel graph $W_{1,n}$, when n is 3

Let v_1 be the central vertex and the vertices on the cycle are v_2, v_3, v_4. Let us color the central vertices v_1 with color C_1 and v_2, v_3, v_4 with color C_2, C_3 and C_4 respectively.

And color classes $\{v_2\}, \{v_3\}, \{v_4\}$ power dominates itself by definition of power dominator coloring and vertex v_1, power dominates the color classes $\{v_2\}, \{v_3\}, \{v_4\}$.

The above procedure gives us proper power dominator coloring for the wheel graph $W_{1,n}$, when n is 3. And the power dominator chromatic number for wheel graph $W_{1,n}$ is 4. i.e., $\chi_{pd}(W_{1,n}) = 4$.

Case (ii) Wheel graph $W_{1,n}$, when n is odd number greater than 3

Let v_1 be the central vertex and the vertices on the cycle are $v_2, v_3, v_4, \dots, v_{n+1}$. Let us color the central vertices v_1 with color C_1, all odd indexed vertices $v_i, 3 \le i \le n$ with color C_2, all even indexed vertices except v_{n+1} $v_i, 3 \le i \le n$ with color C_3 and vertex v_{n+1} is colored by C_4. And color classes $\{v_1\}$ power dominates itself by definition of power dominator coloring and vertex v_1, power dominates the color classes $\{v_{i+3}\}, \{v_{i+2}\}, \{v_{n+1}\}$. The above procedure gives us proper power dominator coloring for the wheel graph $W_{1,n}$, when n is odd. The power dominator chromatic number for wheel graph $W_{1,n}$ is 4. i.e., $\chi_{pd}(W_{1,n}) = 4$.

Case (iii) Wheel graph $W_{1,n}$, when n is even number greater than 3

Let v_1 be the central vertex and the vertices on the cycle are $v_2, v_3, v_4, \dots, v_{n+1}$. Let us color the central vertices v_1 with color C_1, all odd indexed vertices $v_i, 3 \le i \le n+1$ with color C_2, and all even indexed vertices $v_i, 3 \le i \le n$ with color C_3. And color classes $\{v_1\}$ power dominates itself by definition of power dominator coloring and vertex v_1, power dominates the color classes $\{v_{i+3}\}, \{v_{i+2}\}$. The above procedure gives us proper power dominator coloring for the wheel graph $W_{1,n}$, when n is odd. The power dominator chromatic number for wheel graph $W_{1,n}$ is 4. i.e., $\chi_{pd}(W_{1,n}) = 3$. □

Example 3

In Figure 4, the power dominator coloring of wheel graph $W_{1,6}$ is presented

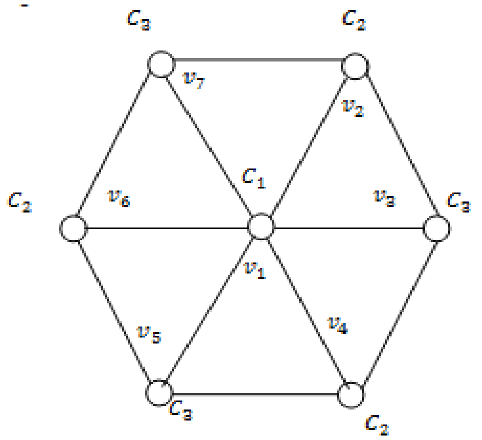

Fig. 4

In the Fig. 4 the color classes of the wheel graph of $n = 6$ are $C_1 = \{v_1\}, C_2 = \{v_2, v_4, v_6\}$ & $C_3 = \{v_3, v_5, v_7\}$. Then $\chi_{pd}(W_{1,4}) = 3$.

Theorem 4

For any $n \ge 3$, the power dominator chromatic number of the Helm graph $\chi_{pd}(H_n) = n + 1$.

Proof

We prove this theorem by method of induction.

First we prove the theorem for $n = 3$

Let v_1 be the central vertex and the vertices on the cycle are v_2, v_3, v_4 and w_1, w_2, w_3 be the pendent vertices adjacent to v_2, v_3, v_4 respectively. Let us color the central vertex v_1 and pendent vertices w_1, w_2, w_3 with color C_1 and v_2, v_3, v_4 with color C_2, C_3 and C_4 respectively. And color classes $\{v_2\}, \{v_3\}, \{v_4\}$ power dominates itself by definition of power dominator coloring and vertex v_1, power dominates the color classes $\{v_2\}, \{v_3\}, \{v_4\}$. Each pendent vertices power dominate the adjacent vertex. This will ensure proper power dominator coloring for the helm graph, when n is 3. The power dominator chromatic number for helm graph, when n is 3. i.e., $\chi_{pd}(H_n) = 4$.

Next, we prove the theorem for $n = 4$

Let v_1 be the central vertex and the vertices on the cycle are v_2, v_3, v_4, v_5 and w_1, w_2, w_3, w_4 be the pendent vertices adjacent to v_2, v_3, v_4, v_5 respectively. Let us color the central vertex v_1 and pendent vertices w_1, w_2, w_3, w_4 with color C_1 and v_2, v_3, v_4, v_5 with color C_2, C_3, C_4 and C_5 respectively. And color classes $\{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}$ power dominates itself by definition of power dominator coloring and vertex v_1, power dominates the color classes $\{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}$. Each pendent vertices power dominate the adjacent vertex with the color class $\{v_i\}, 2 \le i \le 5$. This will ensure proper power dominator coloring for the helm graph, when n is 3. And the power dominator chromatic number for helm graph, when n is 4. $\chi_{pd}(H_n) = 5$, when $n = 4$.

We assume the result is true for $n = k$. We prove that the power dominator chromatic number for the helm graph H_{k+1}, $\chi_{pd}(H_{k+1}) = k + 2$

Let v_1 be the central vertex and the vertices on the rim are $v_2, v_3, v_4, \dots, v_{k+1}$ and $w_1, w_2, w_3, w_4, \dots, w_k$ be the pendent vertices adjacent to $v_2, v_3, v_4, \dots, v_{k+1}$. Let us color the central vertex v_1 and pendent vertices $w_1, w_2, w_3, w_4, \dots, w_k$ with color C_1, and $v_2, v_3, v_4, \dots v_{k+2}$ with color C_2, C_3, C_4,…, C_{k+2} respectively. The color classes $\{v_1\}$ power dominates itself by definition of power dominator coloring and a vertex v_1, power dominates the color classes $\{v_2\}, \{v_3\}, \{v_4\}, \dots, \{v_{k+2}\}$. Each pendent vertices power dominate the adjacent vertex with the color class $\{v_i\}, 2 \le i \le k+2$. The above procedure provides a proper power dominator coloring. The power dominator chromatic number for the helm graph H_{k+1}, $\chi_{pd}(H_{k+1}) = k + 2$. Hence the result.

Example 4

In Figure 5, the power dominator coloring of helm graph H_6 is presented

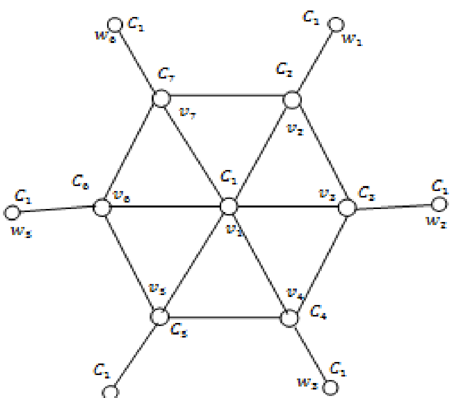

Fig. 5

In the Fig. 5 the color classes of the helm graph H_6 are $C_1 = \{v_1, w_1, w_2, w_3, w_4, w_5, w_6\}$, $C_i = \{v_i\}$ $2 \le i \le 7$. Then $\chi_{pd}(H_6) = 7$.

Theorem 5

For $n \ge 3$, the power dominator chromatic number for the Flower graph $\chi_{pd}(Fl_n) = \begin{cases} 3, if\ n\ is\ even \\ 4, if\ n\ is\ odd \end{cases}$

Proof

Let v_1 be the central vertex and the vertices on the rim be $v_2, v_3, v_4, \dots, v_{n+1}$ and $w_1, w_2, w_3, w_4, \dots, w_n$ be the outer vertices adjacent to $v_2, v_3, v_4, \dots, v_{n+1}$.

Case (i) When n is odd,

The vertex v_1 is colored by color C_1. the vertices v_i, $2 \le i \le n$ on the rim are colored by colors C_2 and C_3 alternatively and assign color C_4 to vertex v_{n+1}. The outstanding vertices $w_1, w_2, w_3, w_4, \dots, w_n$ are colored with C_3 and C_2 alternatively. The vertex v_1 power dominates itself. The vertex v_i, $2 \le i \le n+1$ and w_i, $1 \le i \le n$ power dominates the color class C_1, for, it is adjacent to v_1. Hence in this case the power dominator chromatic number for flower graph Fl_n $\chi_{pd}(Fl_n) = 4$, if n is odd.

Case (ii) When n is even

The vertex v_1 is colored by color C_1. Assign colors C_2 and C_3 alternatively to the vertices v_i, $2 \le i \le n+1$ on the rim. The outstanding vertices $w_1, w_2, w_3, w_4, \dots, w_n$ are colored by colors C_3 and C_2 alternatively. the vertex v_1 power dominates itself.

The vertex v_i, $2 \le i \le n+1$ and w_i, $1 \le i \le n$ power dominates the color class C_1, for, it is adjacent to v_1. The power dominator chromatic number for flower graph Fl_n is 3, when n is even. i.e., $\chi_{pd}(Fl_n) = 3$, if n is even. □

Example 6:

In Figure 6, let us present the power dominator coloring of flower graph Fl_6

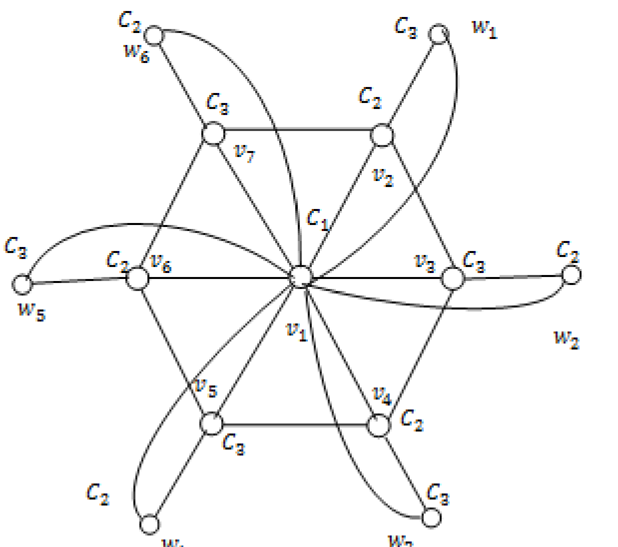

Fig. 6

In the Fig. 6 the color classes of the flower graph Fl_6 are $C_1 = \{v_1\}$. $C_2 = \{v_2, v_4, v_6, w_2, w_4, w_6\}$, $C_3 = \{v_3, v_5, v_7, w_1, w_3, w_5\}$. Then $\chi_{pd}(Fl_6) = 3$.

The results which are investigated are presented in the table below.

Name of the Graph	Number of Vertices	Power Dominator Chromatic Number $\chi_{pd}(G)$
Bull Graph	5	$\chi_{pd}(G) = 3$
Star Graph	*n + 1*	$\chi_{pd}(K_{1,n}) = 2$
Wheel Graph	*2n*	$\chi_{pd}(W_{1,n}) = \begin{cases} 3, if\ n\ is\ even \\ 4, if\ n\ is\ odd \end{cases}$
Helm Graph	*2n + 1*	$\chi_{pd}(H_n) = n + 1$
Fan Graph	*2n + 1*	$\chi_{pd}(Fl_n) = \begin{cases} 3, if\ n\ is\ even \\ 4, if\ n\ is\ odd \end{cases}$

Table 1

IV. CONCLUSION

The concept of power dominator coloring relates coloring problem with power dominating sets in graphs. Computing the power dominator chromatic number for some special graphs such as Bull Graph, Star Graph, Wheel Graph, Helm graph, and Fan Graph has been the main focus of this paper and the results are presented in the table 1. There is the scope for finding the power dominator chromatic number for product graph.

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