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Circular space curve frame (3D) with any load and spring supports by Transfer Matrix Method and Finite Element Method

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***Abstract**: In this paper, authors present a new numerical method, combining the Transfer Matrix Method and Finite Element Method (TMM - FEM), to analyze spatially circular curved bar, with general load and elastic support. Analysis space curved bar is complex problem because conventional methods will not simultaneously calculate the entire structure, or difficulty in establish the stiffness matrix, or the size of stiffness matrix is too large due to multiple elements. TMM - FEM method is proposed to promote the advantages of each method. Due to being directly generated from the parametric equations of the bar axis, the analytical results are accurate. Results are programed in Matlab and verified with SAP2000 programe.*

***Keywords**: Circular space curve frame element, Transfer Matrix Method (TMM), Finite Element Method (FEM), General load, Spring support.*

I. INTRODUCTION

The circle space continuous curved frame has helical axis on the space of Descartes coordinates Oxyz, the projection of the bar axis on Oxy (coordinate plane) is circular, the height of the bar axis increases linearly along the Oz axis (height axis). Bar is used in structures of spiral stair, path, overpass, rail, etc. Internal force and displacement of bar is often calculated by Finite Element Method, by divided into straight elements.

Now, we show the new method to analys circular space frame, with space curved element (3D), any load, any spring support, is called TMM - FEM. It was initiated from [1], concretizing, establishing and verifying with flat circular bar, flat ellipse bar in [3-6], [9].

II. THEORY

Assumtion the material is deformation elastic, ignoring the effect of shear deformation, buckling and curvature of the bar. Consider curved bar 'm' with two ends 1 and 2. At the conventional pinned,

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stress and displacement are positive when in the same direction as the separate coordianate system and vectors at the ends 1 and 2 of the element are :

$\{P_1\}=\{P_1 \quad M_1\}^T$; $\{U_1\}=\{U_1 \quad \Omega_1\}^T$; $\{P_2\}=\{P_2 \quad M_2\}^T$; $\{U_2\}=\{U_2 \quad \Omega_2\}^T$ with symbols $\{P\}$, $\{M\}$, $\{U\}$, $\{\Omega\}$ is the stress, moments, straight and rotating deformation vector at the joint (pinned).

The equation of the transfer matrix method is the relationship between the load and displacement at the ends of the m - element [1]:

$$\begin{Bmatrix} U_2 \\ P_2 \end{Bmatrix} = \begin{bmatrix} [A_{12}^U] & [A_2^{*T}]^{-1}\int [B]\,ds\,[A_1]^{-1} \\ 0 & -[A_{12}^P] \end{bmatrix} \begin{Bmatrix} U_1 \\ P_1 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} U_1 \\ P_1 \end{Bmatrix} = [T] \begin{Bmatrix} U_1 \\ P_1 \end{Bmatrix} \quad (1)$$

Improvements (1) by taken unknown factor on the same side, we have:

$$\begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} = \begin{bmatrix} -T_{12}^{-1}T_{11} & T_{12}^{-1} \\ T_{21}-T_{22}T_{12}^{-1}T_{11} & T_{22}T_{12}^{-1} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{13} & K_{14} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = [k_c]_m \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} \quad (2)$$

Equation (2) is the basic form of the equation involing the force and displacement of joint by the TMM - FEM method, where $[k_c]_m$ is the stiffness matrix of the bar element m in the global coordinate system, we need to determine this matrix first, and then, steps to analysic by TMM - FEM is carried out like in [9].

The parametric equations of space circular, with radius R, in the form:

$$x = R\cos\varphi;\ y = R\sin\varphi;\ z = Rm\varphi \quad (3)$$

where R is the radius of bar plan axis projection on Oxy coordinate system; φ is angular coordinates of the consider running point S(x, y, z) (position of S from coordinates origin); $m = \tan\theta = h/(2\pi R)$; θ is the slope of the rapids curve compared to the plan Oxy; h is the height of rapids curve after each 2π period of φ angle.

Convention $c = \cos\theta = \dfrac{1}{\sqrt{1+m^2}}; mc = \dfrac{m}{\sqrt{1+m^2}} = \sin\theta$

From the relationship of trigonometric and geometry determine the matrix of transformations from the global coordinate system to the local coordinate system, [H'], and from the local coordinate system to the global coordinate system, [H]:

$$\left[H'\right]=\begin{bmatrix}\cos x'x & \cos x'y & \cos x'z\\ \cos y'x & \cos y'y & \cos y'z\\ \cos z'x & \cos z'y & \cos z'z\end{bmatrix}:=\begin{bmatrix}-c\sin\varphi & c\cos\varphi & mc\\ -\cos\varphi & -\sin\varphi & 0\\ mc\sin\varphi & -mc\cos\varphi & c\end{bmatrix}$$

$$[H]=\left[H'\right]^T=\begin{bmatrix}\cos x'x & \cos y'x & \cos z'x\\ \cos x'y & \cos y'y & \cos z'y\\ \cos x'z & \cos y'z & \cos z'z\end{bmatrix}=\begin{bmatrix}x_s' & -\dfrac{y_s'}{c} & -mx_s'\\ y_s' & \dfrac{x_s'}{c} & -my_s'\\ mc & 0 & c\end{bmatrix}$$

$$[H]_P=[H]_M=[H]_u=[H]_\omega$$

With :

$$x_s'=\frac{dx}{ds}=-\sin\varphi\cos\theta;\ y_s'=\frac{dy}{ds}=\cos\varphi\cos\theta;\ z_s'=\frac{dz}{ds}=m\cos\theta;\ ds=\frac{R}{c}d\varphi \tag{4}$$

Making characteristic matrices of bar elements. Cause the bar has one end on the Ox axis, one end is the running point S(x, y, z), so the operator matrix A is structured:

$$A_s=\begin{bmatrix}0 & z & -y\\ -z & 0 & x\\ y & -x & 0\end{bmatrix}=\begin{bmatrix}0 & Rm\varphi & -R\sin\varphi\\ -Rm\varphi & 0 & R\cos\varphi\\ R\sin\varphi & -R\cos\varphi & 0\end{bmatrix} \tag{5}$$

The same together matrix of two ends displacement of m - element has shape:

$$\mathsf{A}_1=\begin{bmatrix}I_3 & 0_3\\ A_1 & I_3\end{bmatrix};\ \mathsf{A}_2=\begin{bmatrix}I_3 & 0_3\\ A_2 & I_3\end{bmatrix} \tag{6}$$

Determine the structure of deformed matrices in integral matrix [Ƀ]:

$$\mathsf{M}_P=H_PM_PH_P^T=$$

$$=\frac{1}{\text{EF}}\begin{bmatrix}x_s' & -\dfrac{y_s'}{c} & -mx_s'\\ y_s' & \dfrac{x_s'}{c} & -my_s'\\ mc & 0 & c\end{bmatrix}\begin{bmatrix}x_s' & y_s' & mc\\ -\dfrac{y_s'}{c} & \dfrac{x_s'}{c} & 0\\ -mx_s' & -my_s' & c\end{bmatrix}=\begin{bmatrix}\dfrac{x_s'^2}{\text{EF}} & \dfrac{x_s'y_s'}{\text{EF}} & \dfrac{mcx_s'}{\text{EF}}\\ \dfrac{x_s'y_s'}{\text{EF}} & \dfrac{y_s'^2}{\text{EF}} & \dfrac{sy_s'}{\text{EF}}\\ \dfrac{mcx_s'}{\text{EF}} & \dfrac{mcy_s'}{\text{EF}} & \dfrac{m^2c^2}{\text{EF}}\end{bmatrix}=\begin{bmatrix}b_1 & b_2 & b_3\\ & b_4 & b_5\\ \text{DX} & & b_6\end{bmatrix}$$

$$\mathsf{M}_M=H_MM_MH_M^T=$$

$$=\begin{bmatrix}x_s' & -\dfrac{y_s'}{c} & -mx_s'\\ y_s' & \dfrac{x_s'}{c} & -my_s'\\ mc & 0 & c\end{bmatrix}\begin{bmatrix}\dfrac{1}{GI_{x'}} & 0 & 0\\ 0 & \dfrac{1}{EI_{y'}} & 0\\ 0 & 0 & \dfrac{1}{EI_{z'}}\end{bmatrix}\begin{bmatrix}x_s' & y_s' & mc\\ -\dfrac{y_s'}{c} & \dfrac{x_s'}{c} & 0\\ -mx_s' & -my_s' & c\end{bmatrix}=\begin{bmatrix}a_1 & a_2 & a_3\\ & a_4 & a_5\\ DX & & a_6\end{bmatrix}$$

$$a_1=\frac{x_s'^2}{GI_{x'}}+\frac{y_s'^2}{c^2EI_{y'}}+\frac{m^2x_s'^2}{EI_{z'}};\ a_2=\frac{x_s'y_s'}{GI_{x'}}-\frac{x_s'y_s'}{c^2EI_{y'}}+\frac{m^2x_s'y_s'}{EI_{z'}};\ a_3=\frac{mcx_s'}{GI_{x'}}-\frac{mcx_s'}{EI_{z'}};$$

$$a_4=\frac{y_s^{'2}}{GI_{x'}}+\frac{x_s^{'2}}{c^2EI_{y'}}+\frac{m^2y_s^{'2}}{EI_{z'}};a_5=\frac{mcy_s^{'}}{GI_{x'}}-\frac{mcy_s^{'}}{EI_{z'}};a_6=\frac{m^2c^2}{GI_{x'}}+\frac{c^2}{EI_{z'}} \tag{7}$$

$$\mathrm{A}^T\mathrm{M}_M=\begin{bmatrix}0 & -z & y\\ z & 0 & -x\\ -y & x & 0\end{bmatrix}\begin{bmatrix}a_1 & a_2 & a_3\\ a_2 & a_4 & a_5\\ a_3 & a_5 & a_6\end{bmatrix}=\begin{bmatrix}-za_2+ya_3 & -za_4+ya_5 & -za_5+ya_6\\ za_1-xa_3 & za_2-xa_5 & za_3-xa_6\\ -ya_1+xa_2 & -ya_2+xa_4 & -ya_3+xa_5\end{bmatrix} \tag{8}$$

$$\mathrm{M}_M A=\begin{bmatrix}a_1 & a_2 & a_3\\ a_2 & a_4 & a_5\\ a_3 & a_5 & a_6\end{bmatrix}\begin{bmatrix}0 & z & -y\\ -z & 0 & x\\ y & -x & 0\end{bmatrix}=\begin{bmatrix}-za_2+ya_3 & za_1-xa_3 & -ya_1+xa_2\\ -za_4+ya_5 & za_2-xa_5 & -ya_2+xa_4\\ -za_5+ya_6 & za_3-xa_6 & -ya_3+xa_5\end{bmatrix} \tag{9}$$

$$\mathrm{M}_P+\mathrm{A}^T\mathrm{M}_M A=\begin{bmatrix}\begin{pmatrix}b_1+z^2a_4\\ -2yza_5+y^2a_6\end{pmatrix} & \begin{pmatrix}b_2-z^2a_2+yza_3\\ +xza_5-xya_6\end{pmatrix} & \begin{pmatrix}b_3-y^2a_3+yza_2\\ -xza_4+xya_5\end{pmatrix}\\ & \begin{pmatrix}b_4+z^2a_1\\ -2xza_3+x^2a_6\end{pmatrix} & \begin{pmatrix}b_5-yza_1+xya_3\\ +xza_2-x^2a_5\end{pmatrix}\\ Symmetry & & \begin{pmatrix}b_6+y^2a_1\\ -2xya_2+x^2a_4\end{pmatrix}\end{bmatrix} \tag{10}$$

Thus, we have equation (11)

$$[\mathrm{B}]=\left[\begin{array}{ccc|ccc}\begin{pmatrix}b_1+z^2a_4\\ -2yza_5+y^2a_6\end{pmatrix} & \begin{pmatrix}b_2-z^2a_2+yza_3\\ +xza_5-xya_6\end{pmatrix} & \begin{pmatrix}b_3-y^2a_3+yza_2\\ -xza_4+xya_5\end{pmatrix} & \begin{pmatrix}-za_2\\ +ya_3\end{pmatrix} & \begin{pmatrix}-za_4\\ +ya_5\end{pmatrix} & \begin{pmatrix}-za_5\\ +ya_6\end{pmatrix}\\ \begin{pmatrix}b_2-z^2a_2+yza_3\\ +xza_5-xya_6\end{pmatrix} & \begin{pmatrix}b_4+z^2a_1\\ -2xza_3+x^2a_6\end{pmatrix} & \begin{pmatrix}b_5-yza_1+xya_3\\ +xza_2-x^2a_5\end{pmatrix} & \begin{pmatrix}za_1\\ -xa_3\end{pmatrix} & \begin{pmatrix}za_2\\ -xa_5\end{pmatrix} & \begin{pmatrix}za_3\\ -xa_6\end{pmatrix}\\ \begin{pmatrix}b_3-y^2a_3+yza_2\\ -xza_4+xya_5\end{pmatrix} & \begin{pmatrix}b_5-yza_1+xya_3\\ +xza_2-x^2a_5\end{pmatrix} & \begin{pmatrix}b_6+y^2a_1\\ -2xya_2+x^2a_4\end{pmatrix} & \begin{pmatrix}-ya_1\\ +xa_2\end{pmatrix} & \begin{pmatrix}-ya_2\\ +xa_4\end{pmatrix} & \begin{pmatrix}-ya_3\\ +xa_5\end{pmatrix}\\ \hline -za_2+ya_3 & za_1-xa_3 & -ya_1+xa_2 & a_1 & a_2 & a_3\\ -za_4+ya_5 & za_2-xa_5 & -ya_2+xa_4 & a_2 & a_4 & a_5\\ -za_5+ya_6 & za_3-xa_6 & -ya_3+xa_5 & a_3 & a_5 & a_6\end{array}\right]$$

$$[\mathrm{B}]=\begin{bmatrix}\mathrm{M}_P+\mathrm{A}^T\mathrm{M}_M A & \mathrm{A}^T\mathrm{M}_M\\ \mathrm{M}_M A & \mathrm{M}_M\end{bmatrix} \tag{11}$$

Calculating the integral equation for each term of [B]:

$$\int_0^{\varphi}(-za_2+ya_3)ds=\frac{R^2mc}{2GI_{x'}}\left(\frac{3\sin 2\varphi}{4}+\varphi\sin^2\varphi-\frac{3\varphi}{2}\right)+\frac{R^2m}{2cEI_{y'}}\left(\frac{\sin 2\varphi}{4}+\varphi\sin^2\varphi-\frac{\varphi}{2}\right)$$
$$+\frac{R^2mc}{2EI_{z'}}\left(\frac{m^2\sin 2\varphi}{4}+m^2\varphi\sin^2\varphi-\frac{m^2\varphi}{2}+\varphi-\frac{\sin 2\varphi}{2}\right);$$

$$\int_0^{\varphi}(-za_4+ya_5)ds=\frac{R^2mc}{4GI_{x'}}\left(-\varphi\sin 2\varphi-\varphi^2+3\sin^2\varphi\right)+\frac{R^2m}{4cEI_{y'}}\left(\varphi\sin 2\varphi-\sin^2\varphi-\varphi^2\right)$$
$$-\frac{R^2mc}{4EI_{z'}}\left(m^2\varphi\sin 2\varphi-m^2\sin^2\varphi+m^2\varphi^2+2\sin^2\varphi\right);$$

$$\int_0^{\varphi}(-za_5+ya_6)ds=\frac{R^2m^2c}{GI_{x'}}\left(2-2\cos\varphi-\varphi\sin\varphi\right)+\frac{R^2c}{EI_{z'}}\left(m^2\cos\varphi+m^2\varphi\sin\varphi-m^2+1-\cos\varphi\right);$$

$$\int_0^{\varphi}(za_1+xa_3)ds=\frac{R^2mc}{2GI_{x'}}\left(\frac{3\sin^2\varphi}{2}-\frac{\varphi\sin2\varphi}{2}+\frac{\varphi^2}{2}\right)+\frac{R^2m}{4cEI_{y'}}\left(\varphi\sin2\varphi-\sin^2\varphi+\varphi^2\right)$$

$$+\frac{R^2mc}{4EI_{z'}}\left(m^2\sin^2\varphi-m^2\varphi\sin2\varphi+m^2\varphi^2-2\sin^2\varphi\right);$$

$$\int_0^{\varphi}(za_2-xa_5)ds=-\frac{R^2mc}{2GI_{x'}}\left(\frac{\varphi}{2}+\varphi\sin^2\varphi-\frac{3\sin2\varphi}{4}\right)+\frac{R^2m}{2cEI_{y'}}\left(\frac{\sin2\varphi}{4}+\varphi\sin^2\varphi-\frac{\varphi}{2}\right)$$

$$+\frac{R^2mc}{EI_{z'}}\left(\frac{m^2\varphi\cos2\varphi}{4}-\frac{m^2\sin2\varphi}{8}+\frac{\varphi}{2}+\frac{\sin2\varphi}{4}\right);$$

$$\int_0^{\varphi}(za_3-xa_6)ds=\frac{R^2m^2c}{GI_{x'}}\left(\varphi\cos\varphi-2\sin\varphi\right)-\frac{R^2c}{EI_{z'}}\left(m^2\varphi\cos\varphi-m^2\sin\varphi+\sin\varphi\right);$$

$$\int_0^{\varphi}(-ya_1+xa_2)ds=\frac{R^2c}{GI_{x'}}(\cos\varphi-1)+\frac{R^2m^2c}{EI_{z'}}(\cos\varphi-1);\ \int_0^{\varphi}(-ya_2+xa_4)ds=\frac{R^2c}{GI_{x'}}\sin\varphi+\frac{R^2m^2c}{EI_{z'}}\sin\varphi;$$

$$\int_0^{\varphi}(-ya_3+xa_5)ds=\frac{R^2mc}{GI_{x'}}\varphi-\frac{R^2mc}{EI_{z'}}\varphi;\ \int_0^{\varphi}a_1ds=\frac{Rc}{2GI_{x'}}\left(\varphi-\frac{\sin2\varphi}{2}\right)+\frac{R}{2cEI_{y'}}\left(\varphi+\frac{\sin2\varphi}{2}\right)+\frac{Rm^2c}{2EI_{z'}}\left(\varphi-\frac{\sin2\varphi}{2}\right);$$

$$\int_0^{\varphi}a_2ds=-\frac{Rc}{2GI_{x'}}\sin^2\varphi+\frac{R}{2cEI_{y'}}\sin^2\varphi-\frac{Rm^2c}{2EI_{z'}}\sin^2\varphi;\ \int_0^{\varphi}a_3ds=\frac{Rmc}{GI_{x'}}(\cos\varphi-1)+\frac{Rmc}{EI_{z'}}(1-\cos\varphi);$$

$$\int_0^{\varphi}a_4ds=\frac{Rc}{2GI_{x'}}\left(\varphi+\frac{\sin2\varphi}{2}\right)+\frac{R}{2cEI_{y'}}\left(\varphi-\frac{\sin2\varphi}{2}\right)+\frac{Rm^2c}{2EI_{z'}}\left(\varphi+\frac{\sin2\varphi}{2}\right);$$

$$\int_0^{\varphi}a_5ds=\frac{Rmc}{GI_{x'}}\sin\varphi-\frac{Rmc}{EI_{z'}}\sin\varphi;\ \int_0^{\varphi}a_6ds=\frac{Rm^2c}{GI_{x'}}\varphi+\frac{Rc}{EI_{z'}}\varphi;$$

$$\int_0^{\varphi}\left(b_1+z^2a_4-2yza_5+y^2a_6\right)ds=$$

$$\frac{Rc}{2EF}\left(\varphi-\frac{\sin2\varphi}{2}\right)+\frac{R^3m^2c}{GI_{x'}}\left(\frac{\varphi\cos2\varphi}{4}-\frac{5\sin2\varphi}{8}+\frac{\varphi^2\sin2\varphi}{4}-\varphi\sin^2\varphi+\frac{\varphi^3}{6}+\varphi\right)$$

$$+\frac{R^3m^2}{cEI_{y'}}\left(\frac{\sin2\varphi}{8}-\frac{\varphi\cos2\varphi}{4}-\frac{\varphi^2\sin2\varphi}{4}+\frac{\varphi^3}{6}\right)+\frac{R^3m^4c}{EI_{z'}}\left(\frac{\varphi\cos2\varphi}{4}-\frac{\sin2\varphi}{8}+\frac{\varphi^2\sin2\varphi}{4}+\frac{\varphi^3}{6}\right)$$

$$+\frac{R^3m^2c}{EI_{z'}}\left(\frac{\sin2\varphi}{4}-\frac{\varphi\cos2\varphi}{4}\right)+\frac{R^3c}{EI_{z'}}\left(\frac{\varphi}{2}-\frac{\sin2\varphi}{4}\right);$$

$$\int_0^{\varphi}\left(b_2-z^2a_2+yza_3+xza_5-xya_6\right)ds=$$

$$-\frac{Rc}{2EF}\sin^2\varphi+\frac{R^3m^2c}{GI_{x'}}\left(\frac{3\varphi\sin2\varphi}{4}-\frac{\varphi^2\cos2\varphi}{4}+\frac{\cos2\varphi}{8}-\sin^2\varphi-\frac{1}{8}\right)$$

$$-\frac{R^3m^2}{cEI_{y'}}\left(\frac{\varphi\sin2\varphi}{4}-\frac{\varphi^2\cos2\varphi}{4}+\frac{\cos2\varphi}{8}-\frac{1}{8}\right)+\frac{R^3m^4c}{EI_{z'}}\left(\frac{\varphi\sin2\varphi}{4}-\frac{\varphi^2\cos2\varphi}{4}+\frac{\cos2\varphi}{8}-\frac{1}{8}\right)$$

$$+\frac{R^3m^2c}{EI_{z'}}\left(\frac{\sin^2\varphi}{2}-\frac{\varphi\sin2\varphi}{2}\right)-\frac{R^3c}{2EI_{z'}}\sin^2\varphi;$$

$$\int_0^{\varphi}\left(b_3 - y^2a_3 + yza_2 - xza_4 + xya_5\right)ds =$$
$$\frac{Rmc}{EF}(\cos\varphi-1)+\frac{R^3mc}{GI_{x'}}(2-2\cos\varphi-\varphi\sin\varphi)-\frac{R^3mc}{EI_{z'}}(1-\cos\varphi)-\frac{R^3m^3c}{EI_{z'}}(\cos\varphi+\varphi\sin\varphi-1);$$
$$\int_0^{\varphi}\left(b_4 + z^2a_1 - 2xza_3 + x^2a_6\right)ds =$$
$$=\frac{Rc}{EF}\left(\frac{\varphi}{2}+\frac{\sin2\varphi}{4}\right)+\frac{R^3m^2c}{GI_{x'}}\left(\frac{5\sin2\varphi}{8}-\frac{3\varphi\cos2\varphi}{4}-\frac{\varphi^2\sin2\varphi}{4}+\frac{\varphi^3}{6}+\frac{\varphi}{2}\right)$$
$$+\frac{R^3m^2}{cEI_{y'}}\left(\frac{\varphi\cos2\varphi}{4}-\frac{\sin2\varphi}{8}+\frac{\varphi^2\sin2\varphi}{4}+\frac{\varphi^3}{6}\right)-\frac{R^3m^2c}{EI_{z'}}\left(\frac{\sin2\varphi}{4}-\frac{\varphi\cos2\varphi}{2}\right)$$
$$+\frac{R^3m^4c}{EI_{z'}}\left(\frac{\sin2\varphi}{8}-\frac{\varphi\cos2\varphi}{4}-\frac{\varphi^2\sin2\varphi}{4}+\frac{\varphi^3}{6}\right)+\frac{R^3c}{EI_{z'}}\left(\frac{\varphi}{2}+\frac{\sin2\varphi}{4}\right);$$
$$\int_0^{\varphi}\left(b_5 - yza_1 + xya_3 + xza_2 - x^2a_5\right)ds =$$
$$\frac{Rmc}{EF}\sin\varphi+\frac{R^3mc}{GI_{x'}}(\varphi\cos\varphi-2\sin\varphi)+\frac{R^3mc}{EI_{z'}}\sin\varphi-\frac{R^3m^3c}{EI_{z'}}(\sin\varphi-\varphi\cos\varphi);$$
$$\int_0^{\varphi}\left(b_6 + y^2a_1 - 2xya_2 + x^2a_4\right)ds = \frac{Rm^2c}{EF}\varphi+\frac{R^3c}{GI_{x'}}\varphi+\frac{R^3m^2c}{EI_{z'}}\varphi; \quad (12)$$

According to separated of integral limit properties, we will calculate the integral in equation (12), corresponding m - element with the limit are φ1 and φ2; then replace them into equation (11) to have [B], continue to replace into equation (2) we have the stiffness matrix of space circular curved bar, [kc]m, in global axis. Applying the same steps as FEM, we will find joint displacements and joint forces at two ends, 1 and 2, of m - element.

The spring support has the straight elastic coefficients denoted by kux, kuy, kuz, and rotation elastic coefficients denoted by kωx, kωy, kωz, arrange corresponding position displacement into the spring support matrix, [8].

$$[K]_{cdh} = [K]_{dh} + [K]_c \quad (13)$$

Here, [K]dh is the spring (elastic) support matrix, it has terms on the diagonal are the elastic coefficients, the remaining terms are zero; [K]c is the global stiffness matrix; [K]cdh is the global stiffness matrix after mentioning the elasticity of the joint support.

III. RESULT AND DISCUSSION

A. Analysis program

Analysis program name SCA-V1 *(Analysis Space Circular Frame – Version 1)* is programmed by Matlab 2010a software, to linear analysis the space circular curved bar (frame) with any number of elements, spatial loads and any spring support.

B. Example analysis space circular frame by SCA-V1 and verification by SAP2000 program

Use SCA-V1 program to analysis space circular curved bar as shown in Figure 1. Radius of the bar in Oxy R = 9,0 m; the height of rapids curved h = 12,0 m. Steel materials have E = $2{,}1.10^8$ kN/m^2; G = $0{,}808.10^8$ kN/m^2. Bar section shape I 1000×350×20×10 mm, has geometric characterictic I_x = 2,117e-06 m^4; I_y = 4,1e-03 m^4, I_z = 1,43e-04 m^4; A = 0,0236 m^2. Value ofspring support $k_{ux}=k_{uy}=10^5$ kN/m, $k_{uz}=5\times10^4$ kN/m, $k_\omega=\infty$. Load and spring support layout on the bar given in Table 1. The analysis results, and comparison as show in Table 2, Table 3 and Table 4.

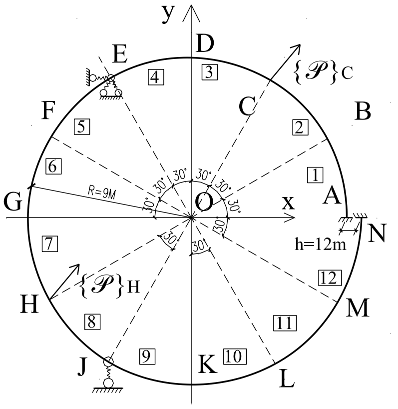

Figure 1. Analysis diagram in SCA-V1

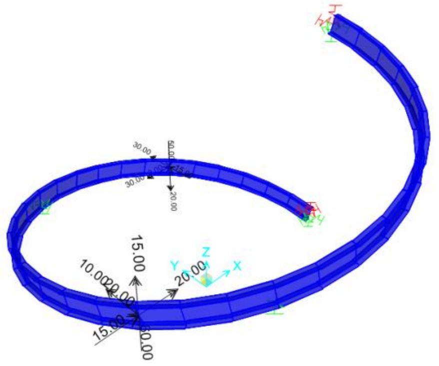

Figure 2. Analysis diagram in SAP2000

Table 1. Joint load and joint spring support on global coordinate system

P	Joint name		U	Joint name			
	C	H		A	E	J	N
P_x [kN]	10	15	U_x	k_{ux}	k_{ux}	0	k_{ux}
P_y [kN]	15	-20	U_y	k_{uy}	k_{uy}	0	k_{uy}
P_z [kN]	-100	-150	U_z	k_{uz}	k_{uz}	k_{uz}	k_{uz}
M_x [kNm]	-30	20	Ω_x	∞	0	0	∞
M_y [kNm]	30	10	Ω_y	∞	0	0	∞
M_z [kNm]	-20	15	Ω_z	∞	0	0	∞

Table 2: Joint displacement in global coordinate system

	SCA-V1			SAP 2000			Comparison results		
Joint	A	H	J	19(A)	110 (H)	125(J)	19 (A)	110 (H)	125(J)
U_x [m]	-0.00138	0.416853	-0.1258	-0.00136	0.425643	-0.13565	1.7%	2.1%	7.3%
U_y [m]	0.000762	-1.2065	-0.60541	0.000766	-1.24336	-0.6114	0.6%	3.0%	1.0%
U_z [m]	-0.00076	-3.82445	-0.00281	-0.00079	-3.98409	-0.00281	3.0%	4.0%	0.1%
Ω_x [rad]	0	0.429646	-1.07061	0	0.456038	-1.1291	-	5.8%	5.2%
Ω_y [rad]	0	-1.82473	-0.35972	0	-1.88995	-0.33571	-	3.5%	7.2%
Ω_z [rad]	0	0.38074	-0.12927	0	0.397193	-0.14058	-	4.1%	8.0%

Table 3: Joint forces inglobal coordinate system

		SCA – V1			SAP 2000			Comparison results		
Element		2	7	12	20-35	95-110	170-185	2(20-35)	7(95-110)	12(170-185)
Joint 1	P_x[kN]	138.5	-53.2	-38.2	136.2	-54.3	-39.3	1.7%	2.2%	3.0%
	P_y[kN]	-76.2	54.1	34.1	-76.6	54.5	34.5	0.6%	0.8%	1.2%
	P_z[kN]	38.2	22.4	12.9	39.3	22.1	12.6	3.0%	1.1%	2.7%
	M_x [kNm]	153.6	81.9	-70.2	158.6	80.5	-70.3	3.1%	1.7%	0.1%
	M_y[kNm]	64.1	95.1	76.4	68.5	95.3	77.2	6.4%	0.3%	1.1%
	M_z[kNm]	-121.1	171.3	-170.6	-116.7	173.6	-170.0	3.8%	1.3%	0.3%
Joint 2	P_x[kN]	-138.5	53.2	38.2	-136.2	54.3	39.3	1.7%	2.2%	3.0%
	P_y[kN]	76.2	-54.1	-34.1	76.6	-54.5	-34.5	0.6%	0.8%	1.2%
	P_z[kN]	-38.2	-22.4	-12.9	-39.3	-22.1	-12.6	3.0%	1.1%	2.7%
	M_x[kNm]	48.3	-236.6	94.4	47.7	-234.5	92.5	1.2%	0.9%	2.0%
	M_y[kNm]	200.1	-175.2	-130.1	197.3	-176.3	-131.7	1.4%	0.6%	1.2%
	M_z[kNm]	-84.2	-345.4	383.4	-79.5	-352.5	388.6	5.8%	2.0%	1.3%

Table 4: Joint forces inlocal coordinate system

		SCA – V1			SAP 2000			Comparison results		
Element		1	10	12	9-20	140-155	170-185	1(20-35)	10(140-155)	12(170-185)
Joint 1	P_x [kN]	-66.6	-34.6	12.9	78.1	32.8	-16.9	14.7%	5.7%	23.9%
	P_y[kN]	-138.5	34.1	50.1	-129.0	37.8	50.2	7.4%	9.8%	0.3%
	P_z[kN]	53.1	20.6	10.5	56.8	19.8	9.3	6.5%	3.7%	13.1%
	M_x[kNm]	107.7	-23.1	-5.1	-78.1	31.7	-4.0	37.9%	27.2%	26.0%
	M_y[kNm]	-401.6	-100.7	99.0	-432.6	-92.0	96.3	7.2%	9.5%	2.8%
	M_z[kNm]	-689.8	277.5	-173.3	667.9	-288.6	174.6	3.3%	3.9%	0.8%
Joint 2	P_x[kN]	124.3	13.0	-36.0	116.0	18.0	-32.9	7.1%	27.9%	9.6%
	P_y[kN]	81.8	-48.6	-38.2	-91.0	47.9	42.2	10.1%	1.5%	9.5%
	P_z[kN]	-65.4	-16.0	-5.6	64.9	16.7	5.9	0.8%	4.3%	5.3%
	M_x[kNm]	46.0	40.4	-47.7	29.1	42.4	-39.7	58.2%	4.7%	20.2%
	M_y[kNm]	165.1	-27.1	-94.4	-172.7	23.5	103.6	4.4%	15.2%	8.9%
	M_z[kNm]	114.0	-77.5	402.0	113.1	-83.8	405.7	0.8%	7.6%	0.9%

Compare the results in Table 2 and Table 3, finding that joint displacement and joint force are coindidence, in local coordinate system, Table 4, some of joint forces have more deviation due to the effect of straight element division in SAP2000, therefore, SCA-V1 analysis results are guaranteed and accurate

IV. CONCLUSION

This paper presents a new method, TMMFEM, to linear analysis space circular curved bar with any load and spring support. This is a useful tool to calculating, verifying and researching, as well as the basic for advanced analysis. This method can also be applied to other types of curves such as ellipse, hyperbola, cycloid,… if we know the parametric equation of the bar.

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