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Strong T-Coloring of Graphs

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Abstract: *A T-coloring of a graph $G = (V, E)$ is the generalized coloring of a graph. Given a graph $G = (V, E)$ and a finite set T of positive integers containing 0, a T-coloring of G is a function $f : V(G) \to Z^{+} \cup \{0\}$ for all $u \neq w$ in $V(G)$ such that if $uw \in E(G)$ then $|f(u) - f(w)| \notin T$. We define Strong T-coloring (ST-coloring, in short), as a generalization of T-coloring as follows. Given a graph $G = (V, E)$ and a finite set T of positive integers containing 0, a ST-coloring of G is a function $f : V(G) \to Z^{+} \cup \{0\}$ for all $u \neq w$ in $V(G)$ such that if $uw \in E(G)$ then $|f(u) - f(w)| \notin T$ and $|f(u) - f(w)| \neq |f(x) - f(y)|$ for any two distinct edges uw, xy in $E(G)$. The ST-Chromatic number of G is the minimum number of colors needed for a ST-coloring of G and it is denoted by $\chi_{ST}(G)$. For a ST coloring c of a graph G we define the c_{ST}-span $sp^{c}_{ST}(G)$ is the maximum value of $|c(u) - c(v)|$ over all pairs u, v of vertices of G and the ST-span $sp_{ST}(G)$ is defined by $sp_{ST}(G) = \min sp^{c}_{ST}(G)$ where the minimum is taken over all ST-coloring c of G. Similarly the c_{ST}-edgespan $esp^{c}_{ST}(G)$ is the maximum value of $|c(u) - c(v)|$ over all edges uv of G and the ST-edge span $esp_{ST}(G)$ is defined by $esp_{ST}(G) = \min esp^{c}_{ST}(G)$ where the minimum is taken over all ST-coloring c of G. In this paper we discuss these concepts namely, ST-chromatic number, $sp_{ST}(G)$, and $esp_{ST}(G)$ of graphs.*

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I. INTRODUCTION

All graphs considered in this paper are finite, simple and undirected. For definitions not discussed in this paper one may refer [1] .In T-coloring, transmitters are represented as the vertices and if two transmitters interfere with each other then there is an edge between them. In that model, for all the interference we have one fixed T see Ref([4, 2]). T-coloring of a graph $G = (V, E)$ is the generalization coloring of a graph. Let $T \subset Z^{+} \cup \{0\}$ be any fixed set. The graph G admits a T-coloring, if there exists $f : V(G) \to Z^{+} \cup \{0\}$, such that for every edge $uw \in E(G)$, $|f(u) - f(w)| \notin T$. For more results on T-coloring one may refer [9, 6, 10] and the survey on T-coloring as discussed in [7]. The T-coloring of a graph is a good model to explain the interference between the transmitters.

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Suppose this fixed set T is allowed to vary for each interfering transmitter. In such a situation, one can have the model in terms of a new definition namely, strong T-coloring of G, which indeed is a generalization of T-coloring of a graph.

II. MAIN RESULTS

Definition 2.1. Let G be a graph and T be any finite set of non-negative integers. A ST-coloring of G is a function $f : V(G) \to Z^{+} \cup \{0\}$ such that for all $u \neq w$ in $V(G)$
(i) $uw \in E(G)$ then $|f(u) - f(w)| \notin T$ and
(ii) $|f(u) - f(w)| \neq |f(x) - f(y)|$ for any two distinct edges uw, xy in $E(G)$.

The ST-Chromatic number of G is the minimum number of colors needed for a ST coloring of G and it is denoted by $\chi_{ST}(G)$.

The following observation is immediate.

Observation 1: (i) $\chi_{ST}(G) \geq \chi(G) = \chi_{T}(G)$.

Theorem 2.1. If G is a simple connected graph then there exists a ST-coloring.

Proof.: Let G be a graph with $V(G) = \{v_1, v_2, \dots, v_n\}$ and let T be the set of positive integers containing 0 with k as its largest element. Define ST -coloring of G as follows.

$c(v_i) = (k+2)^{n+i}$ for $1 \leq i \leq n$

Now we need to prove that

$$|f(v_i) - f(v_j)| \neq |f(v_l) - f(v_m)| \qquad (1)$$

where $v_i v_j$, $v_l v_m \in E(G)$.

If $v_i v_j$ and $v_l v_m$ are adjacent then clearly equation (1) holds. Hence, assume $v_i v_j$ and $v_l v_m$ be two non adjacent edges. Therefore i, j, l, m are distinct positive integers. W.l.g assume that i is the largest integer and m is the least integer. Then either $m \leq j \leq l \leq i$ or $m \leq l \leq j \leq i$.

Case(i) : $m \leq j \leq l \leq i$.

Suppose (1) is not true. Then we have,

$$|f(v_i) - f(v_j)| = |f(v_l) - f(v_m)|$$
$$(k+2)^{n+i} - (k+2)^{n+j} = (k+2)^{n+l} - (k+2)^{n+m}$$
$$(k+2)^{i} - (k+2)^{j} = (k+2)^{l} - (k+2)^{m}$$
$$(k+2)^{i-m} - (k+2)^{j-m} = (k+2)^{l-m} - 1$$
$$(k+2)^{i-m} + 1 = (k+2)^{l-m} + (k+2)^{j-m} \qquad (2)$$

Therefore $(k+2)^{i-m}$ and $(k+2)^{l-m} + (k+2)^{j-m}$ are consecutive integers, where $i - m > l - m > j - m > 2$.

Let $k + 2 = a, i - m = x, l - m = y, j - m = z$.

$$a^{y} + a^{z} - a^{x} \leq a^{x-1} + a^{x-2} - a^{x}$$
$$\leq a^{x-2}(a + 1 - a^{2}) \leq 0$$

Which in turn implies that $(k+2)^{l-m} + (k+2)^{j-m} - (k+2)^{i-m}$ is negative, a contradiction to (2). Similar is the case when $m \leq l \leq j \leq i$.

Theorem 2.2. Let T be any set. If H is the subgraph of a graph G then $\chi_{ST}(H) \leq \chi_{ST}(G)$.

Proof. Let T be any set and $\chi_{ST}(G) = l$. Then there exists a ST-coloring c of G that uses l colors. Obviously the coloring c is a ST-coloring of H for the same set T. Hence, $\chi_{ST}(H) \le l = \chi_{ST}(G)$.

Corollary 2.1. For every graph , $\omega(G) \le \chi_{ST}(G)$.

Theorem 2.3. Let T be any set. For any positive integer q, $\chi_{ST}(K_{1,q}) = q + 1$.

Proof. Let T be any set and k be the largest element in T. Let $v_1, v_2, \dots, v_q$ be the pendant vertices of $K_{1,q}$ and v_0 be the center of $K_{1,q}$ that is adjacent to each, $v_i, 1 \le i \le q$.

Define a function $c : V(K_{1,q}) \to Z^+ \cup 0$ by

$$c(v_i) = \begin{cases} 0 & if \quad i = 0 \\ k + i & if \quad 1 \le i \le q \end{cases}$$

It is clear that c is a ST-coloring of $K_{1,q}$. Hence, $\chi_{ST}(K_{1,q}) \le q + 1$. . Suppose that there exists a ST-coloring of $K_{1,q}$ using fewer than $q + 1$ colors say $c_1, c_2, \dots, c_q$. Without loss of generality, v_0 is assigned some c_i and $v_1, v_2, \dots, v_q$ are assigned from the remaining $q - 1$ colors. Then there are two vertices, say v_k and v_l received same color, which in turn implies that $|f(v_0) -$ $fvk\neq fv0- fvl$, a contradiction to the definition of ST-coloring. Hence, $\chi_{ST}(K_{1,q}) = q + 1$.

The following observations are immediate.

Observation 2:

(i) If G is a simple connected graph with $n \ge 3$ then $3 \le \chi_{ST}(G) \le n$.

(ii) $\chi(G) \le \Delta(G) + 1 \le \chi_{ST}(G) \le n$.

Corollary 2.2. If Wn is the wheel graph and T is any set, then $\chi_{ST}(W_n) = n + 1$.

Proof. Let T be any set and k be the largest element in T. Let $v_1, v_2, \dots, v_n$ be the corner vertices of W_n and v_0 be the center of W_n that is adjacent to each v_i ,$1 \le i \le n$.

Define a function $c : V(W_n) \to Z^+ \cup 0$ by

$$c(v_i) = \begin{cases} 0 & if \quad i = 0 \\ k + i & if \quad 1 \le i \le n \end{cases}$$

Hence, clearly c is a ST-coloring of W_n. Since $n + 1$ colors are used by the ST-coloring c, it follows that $\chi_{ST}(W_n) \le n + 1$. Now invoking Theorem 2.2 and Theorem 2.3, $\chi_{ST}(K_{1,q}) \le \chi_{ST}(W_n)$. Hence, $\chi_{ST}(W_n) = n + 1$.

The following are some known results which will be using in the next theorems.

Theorem 2.4. [3] A nontrivial connected graph G is Eulerian if and only if every vertex of G has even degree.

Theorem 2.5. [3] An connected graph G contains an Eulerian trial if and only exactly two vertices have odd degree. Furthermore, each Eulerian trail of G begins at one of these odd vertices and ends at the other.

Theorem 2.6. [3] If G is a graph of size m, then $\sum_{v \in V(G)} \deg v = 2m$.

Lemma 2.1. Let k be a positive integer and G be a graph with more than $\binom{k}{2}$ edges. Then $\chi_{ST}(G) > k$.

Proof. Let G be a graph with more than $\binom{k}{2}$ edges. Suppose $\chi_{ST}(G) = r \le k$. In any ST-coloring of G with r colors the end vertices of G are assigned by one of the r colors. So there can be at most $\binom{r}{2}$ edges whose absolute differences of the any two distinct edges are distinct. This is a contradiction to the fact that G has more than $\binom{k}{2}$edges. Hence, $\chi_{ST}(G) > k$.

Conjecture: If $\chi_{ST}(G) > k$, then $m(G) > k$.

Theorem 2.7. Let T be any set and P_n be the path on n vertices.

(i) For every even integer $k \ge 2$, there exists a unique integer $l \ge 0$ such that whenever $n \in [2l^2 + l + 2, 2l^2 + 2l + 2]$ then $\chi_{ST}(P_n) = k$. where $k = 2l + 2$.

(ii) For every odd integer $k \ge 3$, there exists a unique integer $l \ge 0$ such that whenever $n \in [2l^2 + 2l + 3, 2l^2 + 5l + 4]$ then $\chi_{ST}(P_n) = k$ where $k = 2l + 3$.

Proof. Let T be any set and P_n be the path on n vertices.

Claim 1 : For every positive integer k there exists an integer n such that $\chi_{ST}(P_n) = k$.

We proceed by induction on k. Certainly the result is true for $k = 1$. Assume the result is true for all integers less than k, where k is a positive integer. Thus for the positive integer $k - 1$, there exists an integer l such that $\chi_{ST}(P_l) = k - 1$. Assume P_l is the largest path with $V(P_l) = v_1, v_2, \dots, v_l$ which satisfies the above property. Thus a new color needed for v_{l+1} in P_{l+1} for a ST-coloring. Which in turn implies that $\chi_{ST}(P_{l+1}) = k$, as desired.

Case(i): Let k be an even integer such that $k = 2l + 2, l \ge 0$. Certainly K_k has a perfect matching $X = \{e_1, e_2, \dots, e_{l+1}\}$. Without loss of generality let $e_1 = uv$. Let $H = K_k - \{e_2, e_3, \dots, e_{l+1}\}$. Using Theorem 2.5 [3], H has an Eulerian trail on k vertices and $\binom{k}{2} - l$ edges. Since the number of edges in H is more than $\binom{k-1}{2}$ invoking Lemma 2.1, $\chi_{ST}(H) = k$. . Now convert the Eulerian trail in H as a $u - v$ path on $\binom{k}{2} - l$ edges, by introducing new vertices whenever the vertices are repeated. Certainly, the number of edges in P_t, where $t = \binom{k}{2} - l + 1$ is $2l^2 + 2l + 1$ which is more than $\binom{k-1}{2} = 2l^2 + l$ edges. Thus $k = \chi_{ST}(H) \ge \chi_{ST}(P_t)$. Hence again using Lemma 2.1, $\chi_{ST}(P_t) = k$. Now we need to prove that P_t is the largest path with ST-chromatic number k. Suppose $\chi_{ST}(P_{t+1}) = k$. This implies that there exists a ST-partition $\Pi = \{V_1, V_2, \dots, V_l\}$ for P_{t+1} . Every vertex in the color classes is adjacent to at most only one vertex from the other color classes and each V_i is independent set. Corresponding to the color partition Π , obtain a graph G' with k vertices and t edges from P_{t+1} by identifying all the vertices which are assigned the same color as single vertex. The degree of each vertex of G' is equal to the sum of the degrees of P_{t+1} which are assigned the same color. Certainly, G' has either an Eulerian trail or Eulerian tour. Thus in G' either exactly two vertices are of odd degree or all the vertices are of even degree. Suppose all the vertices in G' are of even degree then $d(v) \le k - 2$, for every $v \in V(G')$. Invoking Theorem 2.6 [3],

$$k(k - 2) \ge \sum d(v) = 2t$$

$$K^2 - 2k \ge 2\binom{k}{2} - l + 1$$

$$K^2 - 2l + 2$$

$$\le 0$$

$4 \le 0$, which is a contradiction.

Hence, $\chi_{ST}(P_{t+1}) > k$ and the path P_t is the largest path with $\chi_{ST}(P_t) = k$.

To prove the lower bound, obtain a graph H from K_{k-1} by adding a vertex w with any one of the vertices of K_{k-1}. Invoking Theorem 2.5 [3], H contains an Eulerian trial with $\binom{k-1}{2} + 1$ edges. Using Lemma 2.1, $\chi_{ST}(H) = k$. Now convert the Eulerian trail in H as a path P_t where $t = \binom{k-1}{2} + 2$ by introducing new vertices whenever the vertices are repeated. Hence, $\chi_{ST}(P_t) = k$, and this is the least path with ST-chromatic number k.

Case(ii): Let k be an odd integer such that $k = 2l + 3, l \geq 0$. Now we prove that for every k there exists an integer $n \in [2l^2 + 2l + 3, 2l^2 + 5l + 4]$ such that $\chi_{ST}(P_n) = k$. Consider K_k, obviously every vertex of K_k has even degree. Hence, invoking Theorem 2.4 [3], K_k contains an Eulerian tour on k vertices and $\binom{k}{2}$ edges. Now convert this Eulerian tour as a path by introducing new vertices whenever the vertices are repeated. Thus we have, $k = \chi_{ST}(K_k) \geq \chi_{ST}(P_t)$, where $t = \binom{k}{2} + 1$ Using Lemma 2.1, $\chi_{ST}(P_t) = k$ and this is the largest path with the ST-chromatic number k and $n \leq t = 2l^2 + 5l + 4$. Now we prove that $n \geq 2l^2 + 2l + 3$. Since $k = 2l + 3, k - 1 = 2l + 2$. Thus $k - 1$ is even. Invoking Case (i), P_t, where $t = \binom{k}{2} - l + 1$ is the largest path with ST-chromatic number $k - 1$. Hence, $\chi_{ST}(P_{t+1}) = k$ and $n \geq 2l^2 + 2l + 3$.

Theorem 2.8. Let T be any set and C_n be the cycle on n vertices.

(i) For every even integer $k \geq 4$, there exists a unique integer $l \geq 1$ such that whenever $n \in [2l^2 + l + 1, 2l^2 + 2l]$ then $\chi_{ST}(C_n) = k$ where $k = 2l + 2$.

(ii) For every odd integer $k \geq 5$, there exists a unique integer $l \geq 1$ such that whenever $n \in [2l^2 + 2l + 1, 2l^2 + 5l + 3]$ then $\chi_{ST}(C_n) = k$. where $k = 2l + 3$.

Proof:

Claim 2. Let T be any set and $C_n$be the cycle on n vertices. First we need to show that for every positive integer $k \geq 3$ there exists an integer n such that $\chi_{ST}(C_n) = k$. Proof is similar that of claim 1.

Case(i): Let k be an even integer and $k = 2l + 2, l \geq 1$. Certainly K_k has a perfect matching $X = \{e_1, e_2, \ldots, e_{l+1}\}$. Without loss of generality let $e_1 = uv$. Let $H = K_k - \{e_1, e_2, \ldots, e_{l+1}\}$. Using Theorem 2.4 [3], H has an Eulerian tour on k vertices and $\binom{k}{2} - (l + 1)$ edges. Since the number of edges in H is more than $\binom{k-1}{2}$ invoking Lemma 2.1, $\chi_{ST}(H) = k$. . Now convert the Eulerian trail in H as a cycle C_t where $t = \binom{k}{2} - (l + 1)$ edges, by introducing new vertices whenever the vertices are repeated. Certainly, the number of edges in C_t, where $t = \binom{k}{2} - (l + 1)$ is $2l^2 + 2l$ which is more than $\binom{k-1}{2} = 2l^2 + l$ edges. Thus $k = \chi_{ST}(H) \geq \chi_{ST}(C_t)$. Hence again using Lemma 2.1, $\chi_{ST}(C_t) = k$. Now we need to prove that C_t is the largest cycle with ST-chromatic number k. Suppose $\chi_{ST}(C_{t+1}) = k$. This implies that there exists a ST-partition $\Pi = \{V_1, V_2, \ldots, V_l\}$ for C_{t+1} . Every vertex in the color class is adjacent to at most only one vertex from the other color class and each V_i is independent set. Corresponding to the color partition Π , obtain a graph G' with k vertices and $t + 1$ edges from C_{t+1} by identifying all the vertices which are assigned the same color as single vertex. The degree of each vertex of G' is equal to the sum of the degrees of C_{t+1} which are assigned the same color. Certainly, G' contains an Eulerian tour. Thus G' has all the vertices are of even degree. Suppose all the vertices in G' are of even degree then $d(v) \leq k - 2$, for every $v \in V(G')$. Invoking Theorem 2.6 [3],

$$k(k - 2) \geq \sum d(v) = 2(t + 1)$$
$$K^2 - 2k \geq 2(2l^2 - 2l + 1)$$
$$l^2 + 2l + 1 \leq 0$$

where $l \geq 1$, which is a contradiction.

Hence, $\chi_{ST}(C_{t+1}) > k$ and the cycle C_t is the largest cycle with $\chi_{ST}(C_t) = k$.

To prove the lower bound of n, consider K_{k-1}. Obtain a graph H from K_{k-1} by introducing a new vertex w that is adjacent to some vertex say u in K_{k-1}. Invoking Theorem 2.5 [3], H contains an Eulerian trial with $\binom{k-1}{2} + 1$ edges. Using Lemma 2.1, $\chi_{ST}(H) = k$. Now convert the Eulerian trail in H as a cycle C_t where $t = \binom{k-1}{2} + 1$ by introducing new vertices whenever the vertices are repeated also replace the last vertex w by u. This produces a ST-coloring of C_t. Hence $\chi_{ST}(C_t) = k$, and this is the least cycle with ST-chromatic number k.

Case(ii): Let k be an odd integer such that $k = 2l + 3, l \geq 1$. Now we prove that for every k there exists an integer $n \in [2l^2 + 2l + 1, 2l^2 + 5l + 3]$ such that $\chi_{ST}(C_t) = k$. Consider K_k, obviously every vertex of K_k has even degree. Hence, invoking Theorem 2.4 [3], K_k contains an Eulerian tour on k vertices and $\binom{k}{2}$ edges. Now convert this Eulerian tour as a cycle by introducing new vertices whenever the vertices are repeated. Thus we have, $k = \chi_{ST}(K_k) \geq \chi_{ST}(C_t)$, where $t = \binom{k}{2}$. Using Lemma 2.1, $\chi_{ST}(C_t) = k$ and this is the largest cycle with the ST-chromatic number k and $n \leq t = 2l^2 + 5l + 3$. Now we prove that $n \geq 2l^2 + 2l + 1$. Since $k = 2l + 3, k - 1 = 2l + 2$. Thus $k - 1$ is even. Invoking Case (i), C_t, where $t = \binom{k-1}{2} - (l + 1)$ is the largest cycle with ST-chromatic number $k - 1$. Hence, $\chi_{ST}(C_{t+1}) = k$ and $n \geq 2l^2 + 2l + 1$.

Theorem 2.9. $\chi(G) = \chi_{ST}(G)$ if and only if G is the complete graph K_n.

Proof: Let $\chi(G) = \chi_T(G)$. By observation $2(ii)$, $\chi(G) = \chi_T(G) = \Delta(G) + 1$. Using Brooks' theorem, G is either complete graph or an odd cycle. Since $\chi_{ST}(C_{2k+1}) > 3, k \geq 2$. Hence $G = K_n$. The converse is obvious.

Lemma 2.2: Given any two integers a and b, $a \leq b$, will there exist a graph such that $\chi(G) = a$ and $\chi_{ST}(G) = b$?

Proof: Yes. Consider a graph G obtained from K_a by attaching $(b - a)$ pendant vertices to a common vertex of K_a. Clearly $\chi_{ST}(G) = a + (b - a) = b$ and $\chi(G) = a$.

Theorem 2.10: Let G be a bipartite graph with partite set t and l. G is complete iff $\chi_{ST}(G) = n$ where $t + l = n$.

Proof: Let G be a complete bipartite graph with bipartition (X, Y) such that $|X| + |Y| = n$ and $|X| = t, |y| = l$. Let $\chi_{ST}(G) = n$. This implies that all the n vertices of G are assigned distinct colors in any ST-coloring of G. We need to prove that $G \simeq K_{t,l}$. Suppose not. Then there exist a vertex $u \in X$ and $v \in Y$ such that u is not adjacent to v. Since all the colors are distinct and $d(u,v) \geq 3, u$ and v may be assigned the same color. Therefore $\chi_{ST}(G) \leq n-1$, which is a contradiction. Hence every vertex in X is adjacent to a vertex in Y and viceversa. The converse is obvious.

Conjecture: Let $\mathcal{T}$ be a tree with maximum degree Δ and T be any set. $\chi_{ST}(\mathcal{T}) = \Delta + 1$ if and only if $m(\mathcal{T}) \leq \binom{\Delta+1}{2}$. .

III. THE ST-SPAN AND THE ST-EDGE SPAN

Let c be a ST-coloring of G. If k is the largest color assigned to a vertex of G by the ST-coloring c then the coloring $\bar{c}$ of G defined by $\bar{c}(v) = k + 1 - c(v)$ for each vertex v of G is also a ST-coloring of G, called the complementary coloring of c.

For a ST-coloring c of a graph G we define the $\mathrm{c_{ST}}$-span $\mathrm{sp^c_{ST}(G)}$ is the maximum value of $|\mathrm{c(u)} - \mathrm{c(v)}|$ over all pairs $\mathrm{u, v}$ of vertices of G and the ST-span $\mathrm{sp_{ST}(G)}$ is defined by $\mathrm{sp_{ST}(G)} = \min \mathrm{sp^c_{ST}(G)}$ where the minimum is taken over all ST-coloring c of G. Similarly the $\mathrm{c_{ST}}$-edgespan $\mathrm{esp^c_{ST}(G)}$ is the maximum value of $|\mathrm{c(u)} - \mathrm{c(v)}|$ over all edges uv of G and the ST-edge span $\mathrm{esp_{ST}(G)}$ is defined by $\mathrm{esp_{ST}(G)} = \min \mathrm{esp^c_{ST}(G)}$ where the minimum is taken over all ST-coloring c of G..

For each ST-coloring of a graph G, we may assume that some vertex of G is assigned the color 0. For example c' is a ST-coloring of a graph G in which $t \geq 1$ is the smallest color assigned to any vertex of G, then the coloring c of G defined by $c(v) = c(v') - a$ for each $v \in V(G)$ is a ST-coloring of G in which some vertex of G is assigned the color 0 by c and in which $\mathrm{c_{ST}}$-span of G is the same as the $\mathrm{c'_{ST}}$-span of G. Hence for a given finite set of non negative integers $\mathrm{sp_{ST}(G)} = \min \max c(v)$ where the maximum is taken over all vertices v of G and the minimum is taken over all ST -coloring of G. Thus if $\mathrm{sp_{ST}(G)} = l$ then there exist a ST -coloring of $c : V(G) \to \{0,1,2,\ldots,l\}$ of G in which at least one vertex of G is colored 0 and at least one vertex is colored l. It is also true that $\chi_{ST}(G) \leq \mathrm{sp_{ST}(G)}$ for every graph G.

Theorem 3.1. For all graphs G,

$(i)\mathrm{sp_T(G)} \leq \mathrm{sp_{ST}(G)}$, $(ii) esp_{\mathrm{T}}(\mathrm{G}) \leq \mathrm{esp_{ST}(G)}$

Proof: Let T be any finite set of non negative integers containing 0. Every ST-coloring of G is also a T-coloring of G. Hence, $\mathrm{sp_T(G)} \leq \mathrm{sp_{ST}(G)}$, $esp_{\mathrm{T}}(\mathrm{G}) \leq \mathrm{esp_{ST}(G)}$.

Theorem 3.2. Let H be a subgraph of a graph G. For each finite set T of nonnegative integers containing 0,

(i) $\mathrm{sp_{ST}(H)} \leq \mathrm{sp_{ST}(G)}$ $(ii)\mathrm{esp_{ST}(H)} \leq esp_{\mathrm{ST}}(\mathrm{G})$

Proof.

(i) Let f and g be ST-colorings of G and H respectively such that $\mathrm{sp_{ST}(G)} = sp^f_{ST}(G)$, $\mathrm{sp_{ST}(H)} = sp^g_{ST}(H)$. If f is a ST-coloring of G, then f/H is a ST-coloring of H. Therefore $sp^f_{ST}(G) \geq sp^{f/H}_{ST}(H)$. Now $\mathrm{sp_{ST}(H)} = sp^g_{ST}(H) \leq sp^{f/H}_{ST}(H) \leq sp^f_{ST}(G) = \mathrm{sp_{ST}(G)}$. Hence, $\mathrm{sp_{ST}(H)} \leq \mathrm{sp_{ST}(G)}$

(ii)Proof is similar that of (i).

Corollary 3.1. If G is weakly γ-perfect then $\mathrm{sp_{ST}(G)} = \mathrm{esp_{ST}(G)} = \mathrm{sp_{ST}}(K_k)$

In closing this paper, we mention some most important questions which remain.

Conjecture: Let T be a finite set of non negative integers containing 0. If G is a graph with $\chi_{ST}(G) = k$ and $\omega(G) = l$, then $\mathrm{sp_{ST}}(K_l) \leq esp_{\mathrm{ST}}(\mathrm{G}) \leq \mathrm{sp_{ST}(G)} \leq \mathrm{sp_{ST}}(K_k)$.

Open problems:

(i) For certain families of graphs, determine $\chi_{ST}(G)$.

(ii) For which graph , $\chi_{ST}(G) = |V(G)|$?

(iii) Find the values of $\mathrm{sp_{ST}}(K_n)$ and $esp_{\mathrm{ST}}(K_n)$ when T is a k-initial set.

(iv) For certain families of graphs, compute $\mathrm{sp_{ST}(G)}$ and $esp_{\mathrm{ST}}(\mathrm{G})$.

IV. CONCLUSION

T coloring and Strong T-coloring are generalized graph coloring problems which are variants of the channel assignment problems in the broadcast networks.We may introduce the class of distance graphs as a device for studying the complete ST-coloring problem, and study about the complexity of this Strong T-coloring problem. Further studies of the structure of distance graphs may well give an additional insight to the ST-coloring problem.

REFERENCES

1. Balakrishnan.R, Ranganathan.K; *A Text Book of Graph Theory*; Springer, 2000.
2. Cozzens.M.B. and Roberts.F.S., T-Coloring of graphs and the channel assignment problem, *Congressus Numerantium,* 35 (1982), 191-208.
3. Gary Chartrand, Ping Zhang , *Chromatic graph theory* , Discrete Mathematics and its applications, CRC Press, Taylor and Francis Group,2009.
4. Hale.W.K., Frequency assignment: Theory and applications, *Proceedings of the IEEE; 68(1980), 1497-1514.*
5. Jai Roselin.S, Benedict Michael Raj.L ., T-Coloring of certain non perfect Graphs,*Journal of Applied Science and Computations,*Vol VI, Issue II, February/2019
6. Justie Su-Tzu Juan, *I-fan Sun and Pin-Xian Wu, T-Coloring on Folded Hypercubes, *Taiwanese journal of mathematics*, 13(4) (2009), 1331-1341.
7. Robert A Murphey, Panos M Paradalos, Mauricio GC Resende, Frequency Assignments Problems, *Handbook of Combinatorial optimization*, 295-377, 1999.
8. Raychaudhuri.A., Further results on T-Coloring and Frequency assignment problems, *SIAM J. Disc. Math.*, 7(1994), 605-613.
9. Liu.D.D.F., T-colorings of graphs, *Discrete Mathematics*, 101(1992), 203-212.
10. Tesman.B.A., List T-colorings of graphs, *Discrete Applied Mathematics*, 45(1993), 277-289.

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