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Ternary Γ-SO-semirings-3

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Abstract: *"In this paper we are introducing the notions of subtractive and strong subsets in partial ternary* Γ*- semirings.*

We show that in a ternary Γ*- SO semiring satisfying that every non-zero ideal is strong and subtractive Further we will show that join of any two ideals is equal to the sum of those two ideals in a ternary* Γ *- SO semiring satisfying the decomposition property. In a ternary* Γ *- SO semiring satisfying the decomposition property then ideal(R) is a distributive lattice".*

Keywords: Subtractive, strong, austere, join, entire.

I. INTRODUCTION

The notion of ideals in SO-rings studied by G.V.S Acharyulu [1] and M.MuralikrishnaRao[8] studied ideals in Γ- semirings. Further ideals of SO-Partial Γ- semirings investigated by Sivamala.M, Siva Prasad.K [19]. Recently the study of ideals in ternary Γ- semiring [12].

II. PRELIMINARIES

Throughout this paper ternary Γ-SO –semiring refers to TΓSS. And CTΓ-SS refers to complete ternary Γ-SO –semiring. From below some important definitions is given:

Definition2.1: [8] "Let R and Γ be two additive commutative semi groups. R is said to be a ***ternary Γ-semiring*** if there exist a mapping from R ×Γ× R×Γ× R⟶ R which maps $(x_1,\alpha,x_2,\beta,x_3)\longrightarrow[x_1\alpha x_2\beta x_3]$ satisfying the conditions

(i) $(a\alpha b\beta c)\gamma d\delta e = a\alpha(b\beta c\gamma d)\delta e = a\alpha b\beta(c\gamma d\delta e)$

(ii) $[(a+b)\alpha c\beta d] = [a\alpha c\beta d]+[b\alpha c\beta d]$

(iii) $[a\alpha(b+c)\beta d] = [a\alpha b\beta d]+[a\alpha c\beta d]$

(iv) $[a\alpha b\beta(c+d)] = [a\alpha b\beta c]+[a\alpha b\beta d]$

for all $a, b, c, d\in R$ and $\alpha,\beta,\gamma,\delta\in\Gamma$".

Definition2.2:[12] "Let M be a ternary Γ-SO-semiring. A non-empty subset A of M is known as ***left (lateral, right) ternary Γ-ideal*** of M, if it satisfies the following:

(i) A is a left (lateral, right) partial ternary Γ-ideal of M.

(ii) $x\in$ M and $y\in$ A such that $x\leq y$ then $x\in$ A.

If A is left, lateral as well as right ternary Γ-ideal of M, then A is known as ternary Γ-ideal of M".

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Definition2.3: [12] "A partial ternary Γ-semiring said to have a ***left (lateral, right) unity element*** provided there exist a family $(e_i : i\in I)$ of M and $(\alpha_i,\beta_i : i\in I)$ of $\Gamma\ni$ $\sum e_i\alpha_i e_i\beta_i a = a(\sum e_i\alpha_i a\beta_i e_i = a, \sum a\alpha_i e_i\beta_i e_i = a)$ for any $a\in$ M".

Definition2.4:[12] "A TΓSS M is said to be **CTΓ-SS** (***complete ternary Γ-SO-semiring)*** if every family of elements in M is sum able".

And for more preliminaries the references [12] [13] [14] [15][16][17] and [18].

III. MAIN RESULTS

Definition3.1: U is known as non empty subset PTΓS ("partial ternary Γ - semiring") if $u_1, u_2\in$R, $u_1+u_2\in$U and $u_2\in$U $\Rightarrow u_1\in$U then R is called ***subtractive.***

Definition3.2: U is known as non empty subset of a PTΓS R is known as ***strong*** if $u_1, u_2\in$R, $u_1+u_2\in$U $\Rightarrow u_1, u_2\in$U. Since strong subset is clearly subtractive.

Example: 3.3: Let R= $\{0,u_1,u_2,u_3,u_4,u_5\}$, $\Gamma=\{\alpha,\beta\}$ by using R, Σ is described.

$$\sum_i x_i = \begin{cases} x_j \text{ if } x_i = 0\ \forall i\neq j, \text{ for some } j \\ z \text{ if } x_j = u_1, x_k = u_2 \text{ for some } j,k \text{ and } x_i = 0 \forall i\neq j,k \\ \infty, \text{ otherwise} \end{cases}$$

Γ –monoid is defined by R.

R ×Γ× R×Γ× R⟶ R is mapped as given below:

α	0	u_1	u_2	u_3	u_4	u_5
0	0	0	0	0	0	0
u_1	0	0	0	0	0	0
u_2	0	0	0	0	0	0
u_3	0	0	0	0	0	0
u_4	0	0	0	0	0	0
u_5	0	0	0	0	0	0

β	0	u_1	u_2	u_3	u_4	u_5
0	0	0	0	0	0	0
u_1	0	u_1	0	u_3	0	u_1
u_2	0	0	u_2	0	u_4	u_2
u_3	0	0	u_3	0	u_1	u_3
u_4	0	u_4	0	u_2	0	u_4
u_5	0	u_1	u_2	u_3	u_4	u_5

Then R is a PTΓS. Here the subset $\{0, u_1, u_2, u_5\}$ is strong. Therefore it is subtractive subset.

Example 3.4: In example 3.3 the subset U= $\{0, u_3, u_5\}$ is subtractive but not strong because $u_1+u_2=u_5$ and both $u_1, u_2\notin$U.

Theorem3.5: In a TΓSS R every non-zero ideal is strong and hence subtractive.

Proof: R is non zero which is ideal in condition. Let $x, y\in R$ such that $x+y$ exists in R and $x+y\in X$.

Since X is an ideal and $x\leq x+y$, $y\leq x+y \Rightarrow x, y\in X$. Therefore X is a strong ideal of R.

Definition3.6: "A PTΓS R is said to be ***left austere.*** R has no non-zero subtractive left partial ideals. A TΓSS R is left Austere if it has no non-zero left ideals".

Definition3.7: **A** PTTS R is said to be **entire** If it satisfies the conditions $a\alpha b\beta c = 0 \Rightarrow$either b=0 and a=0 and c=0 where $\forall$ $a, b, c \in R$, $\alpha, \beta \in \Gamma$.

Theorem3.8: If R is a left austere TΓSS with left unity then R is entire.

Proof: Let $x, y, z \in$ R. Suppose $x\alpha y\beta z = 0\ \forall \alpha, \beta \in \Gamma$.

Let us take A= $\{r \in R/r\alpha y\beta z = 0\ \forall \alpha, \beta \in \Gamma\}$

First we show that A is a left ideal of R.

Clearly $0\in A$. Suppose $x\neq0$ then $x \in A \neq\{0\}$.

Let $\{a_i{:}i \in I\}$ in R and $a_i \in A\ \forall\ i \in I$

then for all $\alpha, \beta \in \Gamma$ each $a_i\alpha y\beta z = 0$

$\Rightarrow \forall \alpha, \beta \in \Gamma\ (\sum_i a_i)\ \alpha y\beta z = 0 \Rightarrow \sum_i a_i \in A$.

Let $s\in$R, and $t\in A$ such that $s \leq t$ since $t\in A$,

$t\alpha y\beta z = 0\ \forall \alpha, \beta \in \Gamma$. Since $s \leq t$

$\Rightarrow s\alpha y\beta z \leq t\alpha y\beta z\ \forall \alpha, \beta \in \Gamma$

$\Rightarrow s\alpha y\beta z = 0\ \forall \alpha, \beta \in \Gamma \Rightarrow s \in A$.

Let q, $r\in$R, $\gamma, \delta \in \Gamma$ and $x\in A$.

Here $x\in A \Rightarrow x\alpha y\beta z = 0\ \forall \alpha, \beta \in \Gamma$

Consider

$(q\gamma r\delta x)\alpha y\beta z = q\gamma r\delta(x\alpha y\beta z) = q\gamma r\delta(0) = 0\forall \alpha, \beta \in \Gamma$

$\Rightarrow q\gamma r\delta x \in A$.

The value of R is obtained as ideal in left position if the value of A is maintained at non zero condition.

Since R has left unity, then there exist a family $(e_i{:}i \in I)$ in R and $\gamma_i, \beta_i \in \Gamma$ such that $\sum_i e_i\gamma_i e_i\beta_i r = r\ \ \forall\ r \in R$. Since A=R, and hence $(e_i{:}i \in I) \in A$. $\Rightarrow e_i\alpha y\beta z = 0$ $\forall \alpha, \beta \in \Gamma$.

Therefore in particular $e_i\gamma_i y\beta_i z = 0\ \forall\ i \in I$

$\Rightarrow \sum_i e_i\gamma_i y\beta_i z=0 \Rightarrow y=0$ or $z=0$.

Hence R is entire.

Note3.9: In general 3.8 theorem is converse which is not true for this consider the following example.

Example3.10: Let R= [0, 1] is the real number for unit interval. $(a_i : i \in I)$ is the family in R which defines $\sum_i a_i = Sup\{a_i/i \in I\}$ and after that partial ternary monoid R is defined. If we take Γ=W then R is a partial ternary Γ-monoid. Consider the mapping $(x, \alpha, y, \beta, z) \to \inf(x, \alpha, y, \beta, z)$ of R ×Γ× R×Γ× R⟶ R then R is a PTΓS. Then R is a TΓSS with usual $\leq$ of real numbers. For any non-zero $x \in R$, [0, x] is a non-zero ideal of R and hence R is not left Austere. Since $x\alpha y\beta z$=$\inf(x, \alpha, y, \beta, z)$=0 $\forall \alpha, \beta \in \Gamma \Rightarrow x$=0 or y=0 or z=0. And hence R is an entire TΓSS.

Theorem3.11: CTT-SSR is the principal idea to join the two principles of CTT-SSR.

Remark3.12: In any CTΓ-SS $\vee_i$<a_i>=<$\sum_i a_i$>.

Definition3.13: Let X, Y be two ideals of a TΓSS then X+Y={x+y/$x\in X$, $y\in Y$}.

Definition3.14: **A** TΓSS R is known as have the ***decomposition property*** iff for any a_1 , $a_2, a_3 \in$ R, $a_1 \leq a_2 + a_3$ then there exist b_1 , $b_2 \in$ R such that $0\leq b_1\leq a_2$, $0\leq b_2\leq a_3$, and a_1=b_1+b_2.

Theorem3.15: let a TΓSS R satisfying the decomposition property and P+Q is obtained for the ideal value of R.

Proof: First we show that R is an ideal value for P+Q. In R the summable family is assumed as (x_i /i $\in$ I) and $x_i \in P + Q\ \forall\ i \in I$ then x_i=a_i+b_i for some $a_i \in P$ and $b_i \in Q\ \forall\ i \in I$

$\Rightarrow \sum_i x_i$=$\sum_i a_i$+$\sum_i b_i$ where $\sum_i a_i \in P$ and $\sum_i b_i \in Q$.

$\Rightarrow \sum_i x_i \in P + Q$

Let $x \in R$, $b\in P + Q$ for some $p \in P$ and $q \in Q$ *the value* $x\leq b \Rightarrow x\leq b = p + q$.

By decomposition property there exist $0\leq p_1\leq p$, $0\leq q_1\leq q$ such that x=p_1+q_1.

Since $p_1 \leq p, p \in P \Rightarrow p_1 \in P$ and $q_1 \leq q$, $q \in Q \Rightarrow q_1 \in Q$

Therefore x=p_1+$q_1 \in P + Q$.

Let $r_1, r_2 \in R, x \in P + Q$ and $\alpha, \beta \in \Gamma$

then $r_1, r_2 \in R$, x=p + q where $p \in P, q \in Q$ and $\alpha, \beta \in \Gamma$.

Consider

$r_1\alpha r_2\beta x = r_1\alpha r_2\beta(p + q) = r_1\alpha r_2\beta p + r_1\alpha r_2\beta q$

Where $r_1\alpha r_2\beta p \in P, r_1\alpha r_2\beta q \in$Q.

$\Rightarrow r_1\alpha r_2\beta x \in P + Q$ And

$r_1\alpha x\beta r_2 = r_1\alpha(p + q)\beta r_2 = r_1\alpha p\beta r_2 + r_1\alpha q\beta r_2$

where $r_1\alpha p\beta r_2 \in P, r_1\alpha q\beta r_2 \in$Q

$\Rightarrow r_1\alpha x\beta r_2 \in P + Q$

Now

$x\alpha r_1\beta r_2 = (p + q)\alpha r_1\beta r_2 = p\alpha r_1\beta r_2 + q\alpha r_1\beta r_2$

where $p\alpha r_1\beta r_2 \in P, q\alpha r_1\beta r_2 \in$Q

$\Rightarrow x\alpha r_1\beta r_2 \in P + Q$. Hence P+Q is an ideal of R.

Since $P\subseteq P + Q$, $Q\subseteq P + Q$.

P+Q⊆K is obtained based on P⊆K & Q⊆K, when R is the ideal value. Here P and Q is obtained as the smallest value of R which is given as P+Q.

Note3.16: A TΓSS R which does not satisfy the decomposition property then there exist two ideals P and Q such that R is independent of P+Q.

Example3.17: "Let R= $\{0, r_1, r_2, r_3, r_4, r_5\}$, Γ = $\{\alpha, \beta\}$ define Σ on R as

$$\sum_i x_i = \begin{cases} x_j \text{ if } x_i = 0\ \forall\ i \neq j \text{ , for some } j \\ \quad \text{undefined, otherwise} \end{cases}$$

Then R is a ternary SO -monoid.

Define the mapping R ×Γ× R×Γ× R⟶ R as follows:

α	0	r_1	r_2	r_3	r_4	r_5
0	0	0	0	0	0	0
r_1	0	0	0	0	0	0
r_2	0	0	0	0	0	0
r_3	0	0	0	0	0	0
r_4	0	0	0	0	0	0
r_5	0	0	0	0	0	0

β	0	r_1	r_2	r_3	r_4	r_5
0	0	0	0	0	0	0
r_1	0	0	0	0	0	r_1
r_2	0	0	0	0	0	r_2
r_3	0	0	0	0	0	r_3
r_4	0	0	0	0	0	r_4
r_5	0	r_1	r_2	r_3	r_4	r_5

Then R is a TΓSS. In R we have $r_3 \leq r_4 = r_1 + r_2$ and there exist no *x*, *y*$\in R$ such that $0 \leq x \leq r_1, 0 \leq y \leq r_2$ and $r_3 = x + y$ implies that the decomposition property fails. Take P= {0,r_1}, Q= {0,r_2}. Then P, Q are ideals of R and P+Q= {0, r_1,r_2, r_4} is not ideal. Since $r_3 \leq r_4$ and $r_3 \notin${0, r_1,r_2, r_4}".

Theorem3.18: "Let R be a TΓSS satisfying the decomposition property then ideal(R) is a distributive lattice".

Proof: Note that ideal(R) together with set inclusion forms a lattice where inf{I, J, K}=I∧J∧K, sup{I, J}=I∨J. Let J, K, L, M are the ideal values of R.

Let $x \leq p + q$ *is obtained for*
$x \in (\mathrm{J} \wedge \mathrm{K} \wedge \mathrm{L}) \vee (\mathrm{J} \wedge \mathrm{K} \wedge \mathrm{M})$
where $q \in (\mathrm{J} \wedge \mathrm{K} \wedge \mathrm{M})$, p$\in (\mathrm{J} \wedge \mathrm{K} \wedge \mathrm{L}) \Rightarrow x \leq p + q$
where $p, q \in \mathrm{J}, p, q \in \mathrm{K}, p \in \mathrm{W}$ and $q \in \mathrm{Y}$. Since *p*, *q* are in J, K and J, K implies the ideals of R that is given as $p + q \in \mathrm{J}$ &$p + q \in \mathrm{K}$. So $x \in \mathrm{J}, x \in \mathrm{K}$.
Since $x \leq p + q, p \in \mathrm{W}$Land $q \in \mathrm{M}$
$\Rightarrow x \leq p + q \in \mathrm{L} + \mathrm{M} = \mathrm{L} \vee \mathrm{M}$.
$\Rightarrow x \in \mathrm{L} \vee \mathrm{M}$. Therefore $x \in \mathrm{J} \wedge \mathrm{K} \wedge (\mathrm{L} \vee \mathrm{M})$.
Hence $(\mathrm{J} \wedge \mathrm{K} \wedge \mathrm{L}) \vee (\mathrm{J} \wedge \mathrm{K} \wedge \mathrm{M}) \subseteq \mathrm{J} \wedge \mathrm{K} \wedge (\mathrm{L} \vee \mathrm{M})$.
Let $x \in \mathrm{J} \wedge \mathrm{K} \wedge (\mathrm{L} \vee \mathrm{M}) \Rightarrow x \in \mathrm{J}, x \in \mathrm{K}$ and
$x \in \mathrm{L} \vee \mathrm{M}$ then by theorem3.11 $\mathrm{L} \vee \mathrm{M} = \mathrm{L} + \mathrm{M}$
And the value $x = p + q$ for some $p \in \mathrm{L}$ and $q \in \mathrm{M}$,
Since we have $p, q \in \mathrm{J}, p, q \in \mathrm{K}$ for $p \leq p + q$,
$q \leq p + q$. Therefore for
p$\in (\mathrm{J} \wedge \mathrm{K} \wedge \mathrm{L})$,
$q \in (\mathrm{J} \wedge \mathrm{K} \wedge \mathrm{M})$ *the value* $x = p + q$
$\Rightarrow x \in (\mathrm{J} \wedge \mathrm{K} \wedge \mathrm{L}) \vee (\mathrm{J} \wedge \mathrm{K} \wedge \mathrm{M})$
Thus $\mathrm{J} \wedge \mathrm{K} \wedge (\mathrm{L} \vee \mathrm{M}) \subseteq (\mathrm{J} \wedge \mathrm{K} \wedge \mathrm{L}) \vee (\mathrm{J} \wedge \mathrm{K} \wedge \mathrm{M})$
Hence $\mathrm{J} \wedge \mathrm{K} \wedge (\mathrm{L} \vee \mathrm{M}) = (\mathrm{J} \wedge \mathrm{K} \wedge \mathrm{L}) \vee (\mathrm{J} \wedge \mathrm{K} \wedge \mathrm{M})$

Note3.19: The decomposition property in a TΓSS fails then the lattice of ideals is not distributive for this considers the following example. Consider the TΓSS R given in example 3.17 take A= {0,r_1}, B= {0,r_2}, C= {0,r_3}, Q= {0,r_4}. Then A∧B∧ (C∨D) = {0,r_1}, whereas
(A∧B∧C)∨(A∧B∧D)={0}.

Theorem3.20: **"Let R be a CTΓ-SS R then ideal(R) forms a complete lattice with supremum as ∨ and infimum as ∧".**

Proof: Obviously, ideal(R) with set inclusion forms a lattice with {0} as the least element and R as the greatest element $\{\mathrm{R}_i / i \in \mathrm{I}\}$ family of ideals of R, $\underset{i \in \mathrm{I}}{\wedge} \mathrm{R}_i$ and $\underset{i \in \mathrm{I}}{\vee} \mathrm{R}_i$ are ideals of R. So inf$\{\mathrm{R}_i / i \in I\}$ and sup$\{\mathrm{R}_i / i \in \mathrm{I}\}$ are in ideal(R) and hence ideal (R) is a complete lattice.

Theorem3.21: "For any ideals A, B, C, D of CTΓ-SS R, AΓBΓ (C∨D) = (AΓBΓC) ∨ (AΓBΓD)".

Proof: Let *x*$\in \mathrm{A\Gamma B\Gamma}(\mathrm{C} \vee \mathrm{D})$ then $x \leq \sum_i a_i\ \alpha_i b_i \beta_i c_i$ for some $a_i \in \mathrm{A}, b_i \in \mathrm{B}$, $c_i \in \mathrm{C} \vee \mathrm{D}$ and α_i, $\beta_i \in \Gamma$. Since $c_i \in \mathrm{C} \vee \mathrm{D}$, $c_i \leq d_i + e_i$ for some $d_i \in \mathrm{C}$ and $e_i \in \mathrm{D}$. So $x \leq \sum_i a_i\ \alpha_i b_i \beta_i (d_i + e_i)$
$\Rightarrow x \leq \sum_i a_i\ \alpha_i b_i \beta_i d_i + \sum_i a_i\ \alpha_i b_i \beta_i e_i$
where$a_i \in \mathrm{A}, b_i \in \mathrm{B}, d_i \in \mathrm{C}$, $e_i \in \mathrm{D}$ and $\alpha_i, \beta_i \in \Gamma$.
$\Rightarrow x \in (\mathrm{A\Gamma B\Gamma C}) \vee (\mathrm{A\Gamma B\Gamma D})$.
Conversely if $x \in$(AΓBΓC)∨(AΓBΓD) then $x \leq x_1 + x_2$ where $x_1 \in$AΓBΓC and $x_2 \in$AΓBΓD
Since $x_1 \in$AΓBΓC, $x_1 \leq \sum_i a_i\ \alpha_i b_i \beta_i c_i$ for some $a_i \in \mathrm{A}, b_i \in \mathrm{B}, c_i \in \mathrm{C}$ and α_i, $\beta_i \in \Gamma$
Since $x_2 \in$AΓBΓD, $x_2 \leq \sum_j d_j\ \gamma_j e_j \delta_j f_j$
where $d_j \in \mathrm{A}, e_j \in \mathrm{B}, f_j \in \mathrm{C}$ and γ_j, $\delta_j \in \Gamma$.
Therefore $x \leq \sum_i a_i\ \alpha_i b_i \beta_i c_i + \sum_j d_j\ \gamma_j e_j \delta_j f_j \leq$
$\sum_i \sum_j (a_i + d_j)(\alpha_i + \gamma_j)(c_i + f_j)(\beta_j + \delta_j)(b_i + e_j)$
Where $(a_i + d_j) \in \mathrm{A}$, $(b_i + e_j) \in \mathrm{B}$, $(c_i + f_j) \in \mathrm{C} \vee \mathrm{D}$ and $(\alpha_i + \gamma_j), (\beta_i + \delta_j) \in \Gamma$
$\Rightarrow x \in \mathrm{A\Gamma B\Gamma}(\mathrm{C} \vee \mathrm{D})$ and hence
$\mathrm{A\Gamma B\Gamma}\ (\mathrm{C} \vee \mathrm{D})$= (AΓBΓC)∨(AΓBΓD)

IV. CONCLUSION

Mainly we introduced in this paper about regular TΓSS and characterized TΓSS. In this paper we introduce the notions of subtractive and strong subsets in partial ternary Γ- semirings. We conclude that join of any two ideals is equal to the sum of those two ideals in a TΓSS

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REFERENCES

1. G.V.S. Acharyulu, Matrix representable So-rings, Semigroup Forum, Springer-Verlag,46(1993), 31-47, doi: 10.1007/BF02573542.
2. G.V.S. Acharyulu, A Study of Sum-Ordered Partial Semirings, Doctoral thesis, AndhraUniversity, 1992.
3. M.A. Arbib, E.G. Manes, Partially Additive Categories and Flow-diagram Semantics,Journal of Algebra, 62(1980), 203-227.
4. T.K. Dutta, S.K. Sardar, Semiprime ideals and irreducible ideals of Γ -semirings, NoviSad J. Math., 30(1)(2000), 97-108.
5. T.K. Dutta, S.K. Sardar, On the Operator Semirings of a
6. Γ -semiring, Southeast AsianBulletin of Mathematics, 26(2)(2002), 215-225.
7. Jonathan S. Golan. : Semirings and their Applications, Kluwer Academic Publishers,1999.

8. E.G. Manes, D.B. Benson, The Inverse Semigroup of a Sum-Ordered Partial Semiring,Semigroup Forum, 31(1985), 129-152, doi: 10.1007/BF02572645.
9. M. Murali Krishna Rao, Γ-semirings-I, Southeast Asian Bulletin of Mathematics,19(1)(1995), 49-54.
10. M. Murali Krishna Rao, Γ-semirings-II, Southeast Asian Bulletin of Mathematics,21(1997), 281-287.
11. M. Murali Krishna Rao, The Jacobson Radical of a Γ-semiring, Southeast Asian Bulletin of Mathematics, 23(1999), 127-134.
12. M. Murali Krishna Rao, Γ-Semirings, Doctoral thesis, Andhra University, 1995.
13. SajaniLavanya. M., MadhusudhanaRao. D., and V. Syam Julius Rajendra, *On Quasi-Ternary **Γ**-Ideals and Bi-Ternary**Γ**-Ideals in Ternary **Γ**-Semirings*-International Journal of Mathematics and Statistics Invention, Volume 6, Issue 3, (September-2015), PP 05-14.
14. SajaniLavanya. M., MadhusudhanaRao. D., and V. Syam Julius Rajendra, *On Lateral Ternary Γ-Ideals of Ternary Γ-Semirings*-American International Journal of Research in Science, Technology, Engineering &Mathematics (AIJRSTEM), 12(1), September-November, 2015, pp: 11-14.
15. SajaniLavanya. M., MadhusudhanaRao. D., and V. Syam Julius Rajendra, *Prime Bi-ternary**Γ**-Ideals in Ternary **Γ**-Semirings*-British Journal of Research, Volume 2, Issue 6, November-December, 2015, pp: 156-166.
16. SajaniLavanya. M., MadhusudhanaRao. D., and V. Syam Julius Rajendra, *A Study on Jacobson Radical of a Ternary* Γ-*Semieing*-International Journal of Mathematics and Computer Applications Research, Volume 6, Issue 1, (Feb-2016), PP 17-30.
17. SajaniLavanya. M., MadhusudhanaRao. D., and VB. SubrahmanyeswaraRaoSeetamraju, *On Right Ternary* Γ-*ideals of Ternary* Γ-*Semiring*, IMPACT-International Journal of Applied, Natural and Social Sciences, Vol. 4, Issue 5, May 2016, 107-114.
18. SajaniLavanya. M., MadhusudhanaRao.D., and Vasantha. M, *A Study on Inverses Strongly Ternary Gamma Semiring*-Accepted for publication in Associated Asia Research Foundation.
19. SajaniLavanya. M., MadhusudhanaRao. D., and VB. SubrahmanyeswaraRaoSeetamraju, Properties of Right Strongly Prime Ternary Gamma Semiring- Accepted for publication in International Organization of Scientific Research Journal of Mathematics.
20. M. Siva Mala, K. Siva Prasad, Partial Γ-Semirings, Southeast Asian Bulletin of Mathematics, 38(6)(2014), 873-885.
21. M. Siva Mala, K. Siva Prasad, (ϕ, ρ)-Representation of -So-Rings, IranianJournal of Mathematical Sciences and Informatics, 10(1)(2015), 103-119, :10.7508/ijmsi.2015.01.008.
22. M. Siva Mala, K. Siva Prasad, Ideals of Sum-Ordered partial □-Semirings, SoutheastAsian Bulletin of Mathematics, 40(2016), 413-426.
23. K. Siva Prasad, M. Siva Mala, P.V. SrinivasaRao, Green's Relations in Partial Γ-Semirings, International Journal of Algebra and Statistics(IJAS), 2(2)(2013), 21-28, doi:10.20454/ijas.2013.755.
24. M. Siva Mala, K. Siva Prasad, Prime Ideals of Γ-So-rings, International Journal of Algebra and Statistics(IJAS), 3(1)(2014), 1-8, doi: 10.20454/ijas.2014.774.
25. M. Siva Mala, K. Siva Prasad, Semiprime Ideals of □-So-rings, International Journal ofAlgebra and Statistics(IJAS), 3(1)(2014), 26-33, doi: 10.20454/ijas.2014.814.
26. M.E. Streenstrup, Sum-Ordered Partial Semirings, Doctoral thesis, Graduate school ofthe University of Massachusetts, Feb 1985 (Department of Computer and Information Science).
27. P.V. SrininvasaRao, Ideals of Sum Ordered Partial semirings, International Journal ofComputational Cognition(IJCC), 7(2)(2009), 59-64.
28. P.V. SrinivasaRao, Ideal Theory of Sum-Ordered Partial Semirings, Doctoral thesis,AcharyaNagarjuna University, 2011.
29. Syamjuliusrajendra. V, Dr. Madhusudhanarao. D, Sajanilavanya. M-A Study on Completely Regular po-Ternary □-Semiring, Asian Academic Research Journal of Multidisciplinary, Volume 2, Issue 4, September 2015, PP: 240-246.
30. Syam Julius Rajendra. V, Dr.MadhusudhanaRao. D and SajaniLavanya. M-On Pure PO-Ternary Γ-Ideals in Ordered Ternary Γ-Semirings, IOSR Journal of Mathematics (IOSR-JM), Volume 11, Issue 5 Ver. IV (Sep. - Oct. 2015), PP 05-13.

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