



MHD Flow between Two Parallel Plates under the Influence of Inclined Magnetic Field by Finite Difference Method

Aruna Sharma, A.V .Dubewar

Abstract: This study investigates the MHD flow between two parallel infinite plates. Here the upper plate is moving with constant velocity and the lower plate is held stationary and a constant pressure gradient is applied to the system which is under the influence of an inclined magnetic field. The governing equations are formulated and transformed into non dimensional form. Numerical solution of the transformed governing equations are obtained using the Finite Difference method. The fluid velocity at different inclinations of the magnetic field and different strengths of magnetic field have been shown graphically and it has been observed that the increase in the angle of inclination leads to a decrease in fluid velocity and also an increase in magnetic strength leads to a decrease in the velocity profile.

Keywords: MHD flow, magnetic field, pressure gradient, finite difference method.

I. INTRODUCTION

Magneto- hydrodynamic (MHD) flow is the study of motion of electrically conducting fluid in the presence of a magnetic field. When an electrically conducting fluid flows under the influence of a magnetic field, it gives rise to induced electric currents. This induced electric current flows in a direction perpendicular to both the magnetic field and the direction of motion of the fluid, and generates its own magnetic field which affects the original magnetic field. The interaction of electric current and magnetic field gives rise to Lorentz force which affects the velocity of fluid. The magnetohydrodynamic flow between two parallel plates is a classical problem that has a number of applications in various fields such as MHD generators, MHD pumps, petroleum industries, crude oil purification, polymer technology. It is also used in extrusion of plastics.

Hannas Alfvan [20] a Swedish electrical engineer initiated the study of MHD.

Shercliff [19] considered the steady motion of an electrically conducting fluid in pipes under transverse magnetic fields. Singh and Ram [6] had studied the laminar flow of an electrically conducting fluid through a channel. Singh [14] investigated on hydromagnetic steady flow of viscous incompressible fluid between two parallel infinite plates under the influence of inclined magnetic field.

Singh and Okwoyo [17] carried out a study of couette flow between two parallel infinite plates in the presence of a transverse magnetic field. Krishna [18] carried out an investigation on unsteady incompressible free convective flow of an electrically conducting fluid between two heated parallel horizontal plates under the action of magnetic field applied transversely to the flow. Singh [10] studied the hydromagnetic steady flow of liquid between two parallel infinite plates under applied pressure gradient when upper plate is moving with constant velocity under the influence of inclined magnetic field by using the solution of linear differential equations with constant coefficients. In the present paper, the laminar steady MHD flow of liquid between two parallel infinite plates under constant pressure gradient is considered, when the upper plate is moving with constant velocity under the influence of inclined magnetic field, the lower plate is held stationary and solved numerically by finite difference method.

II. MATHEMATICAL FORMULATION

The fluid is assumed to be viscous, laminar and incompressible and flowing between two infinite parallel plates located at $y=\pm h$ and extends from $x=-\infty$ to $x=\infty$ and from $z=-\infty$ to $z=\infty$. The lower plate is stationary and upper plate is moving with constant velocity U .

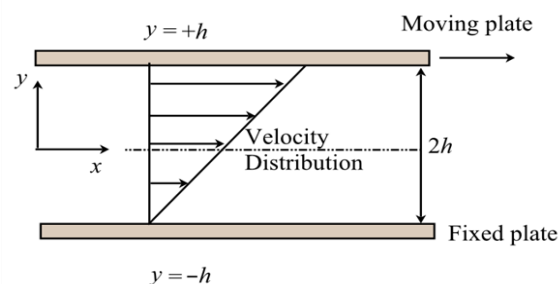


Fig 1. Geometrical Configuration of the flow.

III. GOVERNING EQUATIONS

The equations that describe MHD flow are a combination of continuity equation, Navier Stoke's equation of fluid dynamics and Maxwell's equation of electromagnetism. Consider an electrically conducting fluid having a velocity vector \mathbf{V} . We apply a magnetic field \mathbf{B} at right angle to this assuming steady state flow conditions, the interaction of two fields give rise to an electric field \mathbf{E} which is at right angles to \mathbf{B} and \mathbf{V} . Assuming conducting fluid is isotropic, we denote its electrical conductivity by σ .

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By Ohm's law the density of current induced in the conducting field is given by $\mathbf{J} = \sigma \mathbf{E}$ for stationary condition. The induced electromotive force called as Lorentz force is $\mathbf{F} = \mathbf{J} \times \mathbf{B}$.

The equation of continuity for the incompressible fluid flow is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

where u, v, w are the components of fluid in the x, y, z directions.

Momentum equations are given as follows:

x-momentum equation is

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \frac{F_x}{\rho} \quad (2)$$

y-momentum equation is

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\mu}{\rho} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + \frac{F_y}{\rho} \quad (3)$$

z-momentum equation is

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\mu}{\rho} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + \frac{F_z}{\rho} \quad (4)$$

where F_x, F_y, F_z are the components of $\mathbf{F} = \mathbf{J} \times \mathbf{B}$ in x, y and z directions respectively.

We are considering a two dimensional flow, therefore equation (1) reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (5)$$

The plates are of infinite length, so we assume the flow is along x -axis only and depends on y

$$\text{Therefore, } \frac{\partial u}{\partial x} = 0 \quad (6)$$

Since we have assumed steady flow, i.e flow variables are independent of time,

$$\frac{\partial u}{\partial t} = 0 \quad (7)$$

Therefore, equations (2), (3) and (4) can be written as

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{F_x}{\rho} \quad (8)$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\mu}{\rho} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{F_y}{\rho} \quad (9)$$

where, F_x, F_y and F_z are the components of the Lorentz force \mathbf{F} , as the body force is neglected and replaced by Lorentz force.

Since there is no flow in the y -direction, using (5) and (6), we can rewrite the equations (8), (9) as

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial x^2} \right) + \frac{F_x}{\rho} \quad (10)$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{F_y}{\rho} \quad (11)$$

There is no component of body force in y direction, $F_y = F_z = 0$ as $v=w=0$, the equation of motion becomes

$$\text{Now, } \frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} \right) = \frac{\partial}{\partial y} \left(\frac{v}{h^2} \frac{\partial u^*}{\partial y^*} \right)$$

$$\therefore \frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y^*} \left(\frac{v}{h^2} \frac{\partial u^*}{\partial y^*} \right) \frac{\partial y^*}{\partial y} \quad (18)$$

$$= \frac{v}{h^2} \frac{\partial^2 u^*}{\partial y^{*2}} \left(\frac{1}{h} \right)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{v}{h^3} \frac{\partial^2 u^*}{\partial y^{*2}} \quad (19)$$

$$\text{Next, } \frac{\partial p}{\partial x} = \frac{\partial}{\partial x} \left(\frac{\rho v^2}{h^2} p^* \right)$$

$$= \frac{\rho v^2}{h^2} \left(\frac{\partial p^*}{\partial x} \right) \left(\frac{\partial x^*}{\partial x} \right)$$

$$= \frac{\rho v^2}{h^2} \left(\frac{\partial p^*}{\partial x^*} \right) \left(\frac{1}{h} \right)$$

$$\therefore \frac{\partial p}{\partial x} = \frac{\rho v^2}{h^3} \left(\frac{\partial p^*}{\partial x^*} \right) \quad (20)$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial x^2} \right) + \frac{F_x}{\rho} \quad (12)$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad (13)$$

Since we are considering the flow in x -direction then the flow will be affected by the magnetic flux which is perpendicular to the flow. Since we want to study the effect of different angles of inclination of the magnetic field then the velocity and magnetic flux profiles will be

$$\mathbf{V} = \mathbf{V}(u, 0, 0)$$

$$\mathbf{B} = \mathbf{B}(0, B \sin \theta, 0)$$

Where θ is the angle between \mathbf{V} and \mathbf{B}

Equation (13) implies p , does not depend on y .

Since, $\mathbf{J} = \sigma \mathbf{E}$ and $\mathbf{E} = \mathbf{V} \times \mathbf{B}$, where \mathbf{V} is the fluid velocity along x -axis, the direction of fluid flow.

$$\text{Now, } \mathbf{E} = \mathbf{V} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u & 0 & 0 \\ 0 & B \sin \theta & 0 \end{vmatrix} = u B \sin \theta \hat{k} \quad (14)$$

$$\text{Now, } \mathbf{J} = \sigma \mathbf{E} = \sigma u B \sin \theta \hat{k}$$

$$\mathbf{J} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \sigma u B \sin \theta \\ 0 & B \sin \theta & 0 \end{vmatrix} = -\sigma u B^2 \sin^2 \theta \hat{i} \quad (15)$$

$$\text{Therefore, } F_x = -\sigma B^2 \sin^2 \theta u$$

Using (15), the equation of motion reduces to,

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial x^2} \right) - \frac{\sigma B^2 \sin^2 \theta}{\rho} u \quad (16)$$

IV. NON-DIMENSIONALIZATION

To simplify (16) we non dimensionalize to reduce the parameters in the equation by using the following non dimensional quantities Singh (1993).

$$x^* = \frac{x}{h}, \quad y^* = \frac{y}{h}, \quad p^* = \frac{p h^2}{\rho \nu^2}, \quad u^* = \frac{u h}{\nu} \quad \text{where } \nu = \frac{\mu}{\rho}$$

$$\text{Now, } \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} \left(\frac{u^* h}{h} \right) = \frac{\partial}{\partial y^*} \left(\frac{u^* h}{h} \right) \frac{\partial y^*}{\partial y}$$

$$= \frac{v}{h} \frac{\partial u^*}{\partial y^*} \frac{1}{h}$$

$$\therefore \frac{\partial u}{\partial y} = \frac{v}{h^2} \frac{\partial u^*}{\partial y^*} \quad (17)$$

$$\text{where } M^2 = \frac{\sigma B^2 h^2}{\rho \nu} \quad \text{i.e. } M = B h \sqrt{\frac{\sigma}{\rho \nu}}$$

M is called as the Hartmann number and is directly proportional to the Magnetic field B

From equation (26) we get

$$\text{Similarly, } \frac{\partial p}{\partial y} = \frac{\rho \nu^2}{h^3} \left(\frac{\partial p^*}{\partial y^*} \right) \quad (21) \quad \text{Substituting}$$

in equation (13) and (16) we get,

$$\left(\frac{\partial p^*}{\partial y^*} \right) = 0 \quad (22)$$

$$0 = -\frac{1}{\rho} \frac{\rho \nu^2}{h^3} \left(\frac{\partial p^*}{\partial x^*} \right) + \frac{\mu}{\rho} \frac{v}{h^3} \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B^2 \sin^2 \theta}{\rho} \left(\frac{u^* h}{h} \right) \quad (23)$$

For convenience we drop the star (*) to get

$$\frac{\partial p}{\partial y} = 0 \quad (24)$$

$$0 = -\frac{1}{\rho} \frac{\rho \nu^2}{h^3} \left(\frac{\partial p}{\partial x} \right) + \frac{\mu}{\rho} \frac{v}{h^3} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2 \sin^2 \theta}{\rho} \left(\frac{u v}{h} \right) \quad (25)$$

We can write equation (25) as

$$0 = -\frac{v^2}{h^3} \left(\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2 h^2 \sin^2 \theta u}{\rho v} \right) \text{ where } v = \frac{\mu}{\rho}$$

$$\text{i. e } 0 = -\frac{v^2}{h^3} \left(\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - M^2 \sin^2 \theta \right) \quad (26)$$

$$\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - M^2 \sin^2 \theta u = 0 \quad (27)$$

Differentiating equation (27) w.r.t x, we get

$$\frac{\partial^2 p}{\partial x^2} = 0 \quad (28)$$

Since, 'p' does not depend on 'y', therefore we can use total derivative instead of partial derivative

Therefore, $\frac{d^2 p}{dx^2} = 0$, i.e $\frac{dp}{dx} = -P$ (constant) (29). Equation (27) reduces in terms of total derivatives to

$$\frac{d^2 u}{dy^2} - M^2 \sin^2 \theta u = -P \quad (30)$$

with boundary conditions $u=0$, when $y=-1$

$u=1$, when $y=1$

Equation (30) is the dimensionless equation that has been obtained.

Solving this equation numerically by finite difference method we get the solution.

V. METHODOLOGY

Finite difference Method (FDM) are numerical methods for solving differential equations by approximating them with difference equations, in which finite differences approximate the derivatives. FDM is a discretization method which converts linear (nonlinear) differential equations into system of linear (nonlinear) equations which can be solved by matrix method techniques.

1) Here we generate a grid $y_i = ih$, $i=0,1,2, \dots, n$, $h = \frac{1}{n}$

A grid is a finite set of points at which we want to find an approximate solution.

2) We substitute the derivatives with finite difference formula at every grid point where the solution is unknown to get an algebraic system of equations i. e for twice differentiable function

$$\frac{d^2 u}{dy^2} = \frac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta y)^2}$$

3) We get a set of algebraic equations which is solved by using matrix algebra.

VI. SOLUTION

Consider equation (30)

$$\frac{d^2 u}{dy^2} - M^2 \sin^2 \theta u = -P$$

Using central difference approximation for second order derivative and assuming $P=U=1$, we get,

$$\frac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta y)^2} - M^2 \sin^2 \theta u_i = -1 \quad (31)$$

Simplifying we get,

$$u_{i+1} - (2 + M^2 \sin^2 \theta (\Delta y)^2) u_i + u_{i-1} = -(\Delta y)^2 \quad (32)$$

We take $N=10$ intervals so that the vertical distance (y direction) between the plates is divided into 10 equal increments of length

$\Delta y = .2$ by distributing 11 grid points over the height $H=2$

i. e $\Delta y = \frac{2}{10}$.

The boundary conditions are $u_1=0$ and $u_{11}=1$.

For $i=2$ to 9 in equation (32) we get a system of equations which in matrix form is

$$\begin{bmatrix} B & A & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ A & B & A & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A & B & A & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A & B & A & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A & B & A & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & A & B & A & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A & B & A & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A & B & A \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A & B \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \\ u_{10} \end{bmatrix} = \begin{bmatrix} b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \\ b_8 \\ b_9 \\ b_{10} \end{bmatrix}$$

i. e in the form of $AU=B$.

The matrix A is a tridiagonal matrix and system can be solved by using Thomas algorithm.

Using MATLAB the system has been solved for different angles of inclination and different values of the strength of magnetic field M and the results have been represented graphically.

TABLE I A M=1 ANALYTIC METHOD

y	theta=30	theta=45	theta=60	theta=90
-1	0	0	0	0
-0.8	0.2504	0.2261	0.2059	0.1888
-0.6	0.4632	0.4166	0.3778	0.345
-0.4	0.6407	0.5754	0.521	0.4749
-0.2	0.7845	0.7057	0.6398	0.5838
0	0.8961	0.8101	0.7377	0.676
0.2	0.9767	0.8906	0.8177	0.7551
0.4	1.027	0.9489	0.8822	0.8245
0.6	1.0476	0.9861	0.9331	0.8867
0.8	1.0386	1.003	0.972	0.9445
1	1	1	1	1

TABLE I B M=1 FINITE DIFFERENCE METHOD

y	theta=30	theta=45	theta=60	theta=90
-1	0	0	0	0
-0.8	0.2503	0.2259	0.2056	0.1885
-0.6	0.463	0.4163	0.3774	0.3446
-0.4	0.6404	0.5751	0.5206	0.4745
-0.2	0.7842	0.7053	0.6393	0.5834
0	0.8958	0.8097	0.7372	0.6756
0.2	0.9764	0.8902	0.8173	0.7548
0.4	1.0268	0.9485	0.8818	0.8242
0.6	1.0474	0.9859	0.9328	0.8866
0.8	1.0385	1.0029	0.9718	0.9444
1	1	1	1	1

MHD Flow between Two Parallel Plates under the Influence of Inclined Magnetic Field by Finite Difference Method

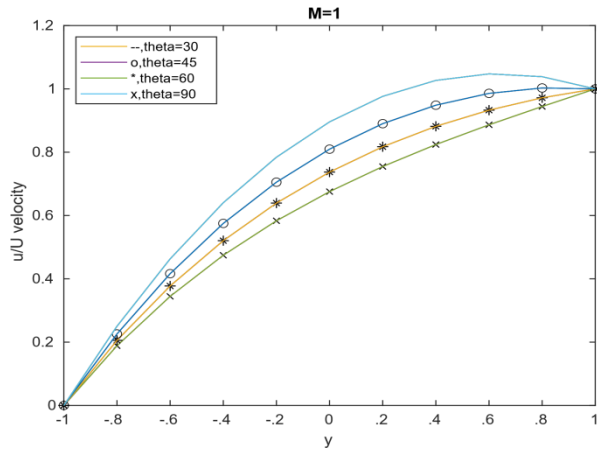


Fig. 2

. Velocity Profile (M=1, Theta= 30⁰,45⁰,60⁰, 90⁰)

TABLE II A M=1.5
ANALYTIC METHOD

Y	theta=30	theta=45	theta=60	theta=90
-1	0	0	0	0
-0.8	0.2207	0.1812	0.1533	0.1328
-0.6	0.4063	0.3305	0.2769	0.2372
-0.4	0.5609	0.4545	0.379	0.3229
-0.2	0.6882	0.5589	0.4665	0.3976
0	0.7908	0.6484	0.5456	0.4681
0.2	0.8713	0.7271	0.6214	0.5406
0.4	0.9312	0.7984	0.6992	0.6219
0.6	0.9722	0.8657	0.7842	0.7193
8	0.9949	0.9319	0.8823	0.8417
1	1	1	1	1

TABLE II B

M=1.5 FINITE DIFFERENCE METHOD

y	theta=30	theta=45	theta=60	theta=90
-1	0	0	0	0
-0.8	0.2205	0.181	0.1531	0.1325
-0.6	0.4059	0.3301	0.2765	0.237
-0.4	0.5605	0.4541	0.3786	0.3227
-0.2	0.6877	0.5585	0.4663	0.3976
0	0.7904	0.6481	0.5454	0.4682
0.2	0.8709	0.7268	0.6214	0.5409
0.4	0.9309	0.7982	0.6993	0.6223
0.6	0.9719	0.8655	0.7844	0.7198
8	0.9948	0.9318	0.8824	0.842
1	1	1	1	1

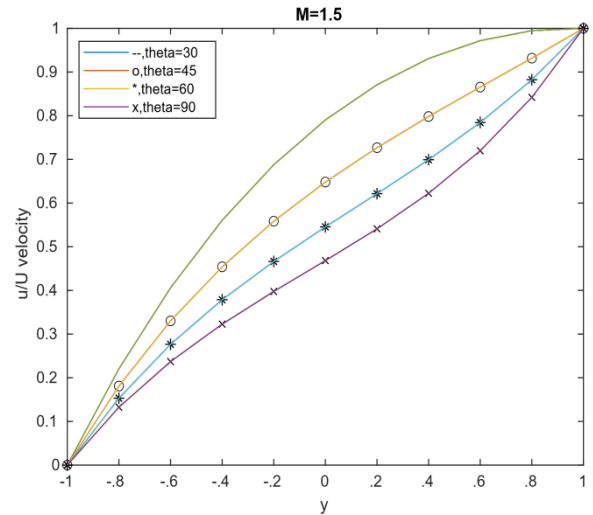


Fig 3

Velocity Profile (M=1.5, Theta =30⁰,45⁰,60⁰, 90⁰)

Table III A
M=2 ANALYTIC METHOD

Y	theta=30	theta=45	theta=60	theta=90
-1	0	0	0	0
-0.8	0.1888	0.1412	0.1126	0.0938
-0.6	0.345	0.2535	0.1984	0.1622
-0.4	0.4749	0.3459	0.2679	0.2164
-0.2	0.5838	0.4259	0.3294	0.2652
0	0.676	0.5	0.3905	0.3165
0.2	0.7551	0.5741	0.4585	0.3785
0.4	0.8245	0.6541	0.5417	0.4613
0.6	0.8867	0.7465	0.6501	0.5784
0.8	0.9445	0.8588	0.797	0.7488
1	1	1	1	1

Table III B

M=2 FINITE DIFFERENCE METHOD

y	theta=30	theta=45	theta=60	theta=90
-1	0	0	0	0
-0.8	0.1885	0.141	0.1124	0.0936
-0.6	0.3446	0.2532	0.1982	0.1621
-0.4	0.4745	0.3457	0.2679	0.2166
-0.2	0.5834	0.4258	0.3296	0.2657
0	0.6756	0.5	0.391	0.3173
0.2	0.7548	0.5742	0.4592	0.3797
0.4	0.8242	0.6543	0.5426	0.4628
0.6	0.8866	0.7468	0.6511	0.58
8	0.9444	0.859	0.7977	0.75
1	1	1	1	1

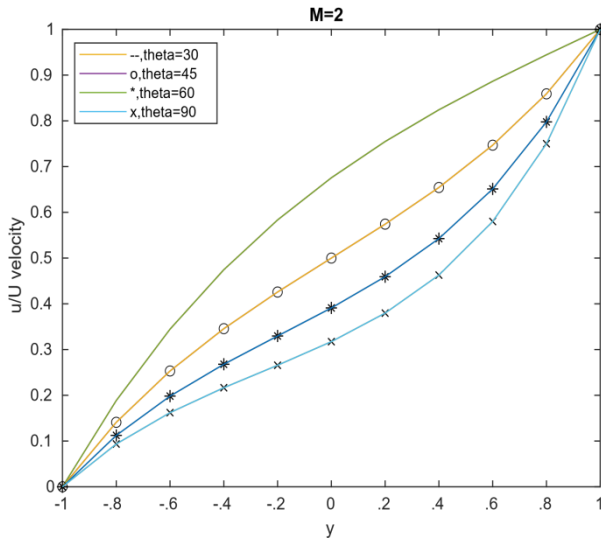


Fig.4

Velocity Profile ($M=2$, $\Theta=30^\circ, 45^\circ, 60^\circ, 90^\circ$)

In Table I A, I B, II A, II B and III A, III B for different values of the angle of inclination taking the values of $M=1, 1.5$ and 2 the values obtained by using finite difference method are shown and these values are found to be in agreement with the values calculated by analytic method (Singh 2014). We can observe that for each value of magnetic field strength, as the angle of inclination of the magnetic field (Θ) increases the velocity of the fluid decreases which can be observed in the graph shown in Fig. 2, 3 and 4.

TABLE IV $\Theta=30^\circ$

Y	M=1	M=1.5	M=2	M=2.5
-1	0	0	0	0
-0.8	0.2503	0.2205	0.1885	0.1586
-0.6	0.463	0.4059	0.3446	0.287
-0.4	0.6404	0.5605	0.4745	0.3934
-0.2	0.7842	0.6877	0.5834	0.4844
0	0.8958	0.7904	0.6756	0.5657
0.2	0.9764	0.8709	0.7548	0.6423
0.4	1.0268	0.9309	0.8242	0.719
0.6	1.0474	0.9719	0.8866	0.8007
0.8	1.0385	0.9948	0.9444	0.8925
1	1	1	1	1

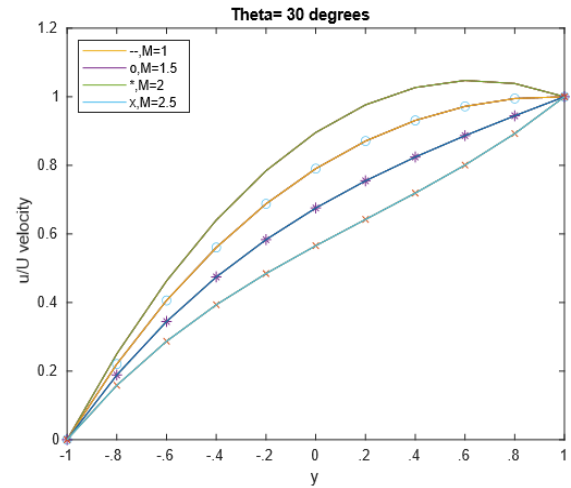


Fig 5.

Velocity Profile for $\theta=30^\circ$ ($M=1, M=1.5, M=2, M=2.5$)

In Table IV. for fixed $\Theta=30^\circ$, the velocity of fluid has been calculated for different values of the magnetic field strength M by using finite difference method and it is observed that as the magnetic field strength increases, the velocity profile decreases which can be observed in the graph shown in Fig 5.

TABLE V

	M=1.154700 5	M=1.414213 6	M=2	M=1
Y	$\Theta=60^\circ$	$\Theta=45^\circ$	$\Theta=30^\circ$	$\Theta=90^\circ$
-1	0	0	0	0
-0.8	0.188549	0.188549	0.188549	0.188549
-0.6	0.344639	0.344639	0.344639	0.344639
-0.4	0.474515	0.474515	0.474515	0.474515
-0.2	0.583372	0.583372	0.583372	0.583372
0	0.675563	0.675563	0.675563	0.675563
0.2	0.754777	0.754777	0.754777	0.754777
0.4	0.824183	0.824183	0.824183	0.824183
0.6	0.886555	0.886555	0.886555	0.886555
0.8	0.94439	0.94439	0.94439	0.94439
1	1	1	1	1

From Table V, we observe that by changing the values of magnetic field strength M and the angle of inclination ' Θ ' the velocity profile can be maintained so that velocity remains the same at all points.

VII. RESULTS AND DISCUSSION

The problem of the effect of inclined magnetic field on MHD flow between two infinite parallel plates with upper plate moving with constant velocity and lower stationary along with applied pressure gradient has been investigated.

From the analysis the following results were observed

7.1 As the angle of inclination of magnetic field increases it is found that there is a decrease in the velocity profile.

7.2 As the strength of the magnetic field increases, there is a decrease in the velocity profile.

7.3 By changing the values of the magnetic field strength M and the angle of inclination of the magnetic field, we can maintain the same velocity profile.

VIII. CONCLUSION

The study concerns the effect of inclination of magnetic field on MHD flow between parallel plates under applied pressure gradient. The results obtained using finite difference method are in agreement with the analytic method and show that for an increased angle of inclination of magnetic field the velocity decreases. An increase in the magnetic field strength M also results in decrease in the velocity. Further, it has been observed that for a given angle of inclination of the magnetic field, the magnetic field strength (M) can be chosen so that the velocity at each point remains the same, which can be used in controlling the flow in some engineering problems. Finite Difference method is found to be fast, accurate and converges faster to the exact solution.

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