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The Vertex Prime Valuation for Jahangir and Theta Graphs

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Abstract: ***A sequence of instructions which can help to solve a problem is called an algorithm. The reason for composing an algorithm is to reduce the timespan and understanding the solution of problems in simple way. In this paper, vertex prime valuation of the Jahangir graph $J_{n,m}$ for $n \geq 2$, $m \geq 3$ and generalized Theta graph θ (l_1, l_2, l_3, ..., l_n) has been investigated by using algorithms .We discuss vertex prime valuation of some graph operations on both graphs viz. Fusion, Switching and Duplication, Disjoint union and Path union.***

Keywords- ***Prime labeling; Jahangir graph; Theta graph; Fusion; Switching and Duplication; Disjoint union and Path union.***

I. INTRODUCTION

A graph *G(V, E)* is said to have a vertex prime labeling if its edges can be labeled with distinct integers from {1, 2, 3, . . . , |*E* |} such that for each vertex of degree at least 2, the greatest common divisor of the labels on its incident edges is 1. A graph that admits a vertex prime labeling is called a vertex prime graph. A description on prime labeling of Jahangir graph can be found in [1]. Throughout this paper, we consider finite simple undirected graphs.

A. Definition:

A labeling or a valuation of a graph is a map that carries the graph elements to the set of numbers, usually to the non-negative or to the positive integers. If the domain is the set of vertices, then it is called as a vertex labeling. If the domain is the set of edges, then the labeling is called the edge labeling. If the labels are assigned to the vertices and to the edges of a graph, such a labeling is called total labeling.

B. Definition:

A larger graph that is the union of two or more graphs is known as the disjoint union of graphs. The vertex set of the larger graph formed is the disjoint union of the vertex sets of the combined graphs and the edge set is the disjoint union of the edge sets of the above graphs.

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We note that disjoint union of two or more nonempty graphs is disconnected.

C. Definition:

Let G_1, G_2, G_3, …, G_n ,$n \geq 2$ be n copies of a fixed graph G. The graph which is obtained by adding an edge between G_i and G_{i+1} for i = 1, 2, ..., n −1 is known as the path union of G.

D. Definition:

The generalized Theta graph θ (l_1, l_2, l_3, ..., l_n) constitutes n ≥ 3 paired internally disjoint paths of length l_1, l_2, l_3, ..., l_n that shares a pair of common end points *x* and *y*.

II. JAHANGIR GRAPH

A. Algorithm

Procedure (vertex prime labeling of Jahangir Graph)

$V = \{v_1, v_2, \dots, v_{nm+1}\}$

$E = \{v_iv_{i+1} / 1 \leq i \leq nm - 1\} \cup v_{nm}v_1 \cup \{v_{1+jn}v_{nm+1} / 0 \leq j \leq m - 1\}$

Define $f: E \rightarrow \{1, 2, \dots, (n + 1)m\}$

for $1 \leq i \leq nm - 1$,

$f(v_iv_{i+1}) = i$

and $f(v_{nm}v_1) = nm$

end for

for $0 \leq j \leq m - 1$

$f(v_{jn+1}v_{nm+1}) = (nm+1) + j$

end for

end procedure

B. Theorem

The Jahangir graph $J_{n,m}$ for $n \geq 2$, $m \geq 3$ is a vertex prime for the edge labeling of $J_{n,m}$ defined in algorithm A.

C. Remark

The Fusion of two vertices v_i and v_j for all *i*, *j* in a Jahangir graph $J_{n,m}$ $n \geq 2$, $m \geq 3$ reduces the number of vertices by one where as the number of edges remains same. Thereby the resulting Jahangir graph $J_{n,m}$ for $n \geq 2$, $m \geq 3$ is a vertex prime by algorithm A.

D. Theorem

The Duplication of any vertex v_i for $1 \leq i \leq nm+1$ in $J_{n,m}$ for $n \geq 2$, $m \geq 3$ is a vertex prime graph.

Proof: By Duplication of a vertex, v_i in a graph the number of vertices is increased by 1. Suppose the vertex v_i is in the cycle and adjacent to v_{nm+1}, then the number of edges increases to three. If the vertex v_i is in the cycle and not adjacent to v_{nm+1}, then the number of edges is increased by two.

Whereas the duplication of v_{nm+1} results in the increase of edges by m which are the edges incident on the new vertex $v_i^{'}$.

One can note that the edge labeling is same as in Algorithm A for the edges of the all vertices except $v_i^{'}$. If $v_i^{'}$ is the duplication of v_i which is in cycle and is not adjacent to $v_{nm+1,}$ then, the new edges formed are labeled as $(n + 1)\, m +1$ and $(n + 1)\, m+2$. But if $v_i^{'}$ is adjacent to v_{nm+1} then, the new edges formed are labeled as $(n + 1)\, m +1$, $(n +1)\, m + 2$ and $(n + 1)\, m + 3$. Now suppose that $v_i^{'}$ is the duplication of $v_{nm+1,}$ then, the new m edges formed are labeled as $(n +1)\, m + i$ for $1 \leq i \leq m$ respective of the order. Therefore, for any vertex the gcd of edge labelings is 1. Hence, the graph is a vertex prime.

E. Theorem

The switching of the vertex v_{nm+1} in $J_{n, m}$ for $n \geq 2$, $m \geq 3$ is a vertex prime graph.

Proof: The switching of a vertex in a graph does not allow any change in the number of vertices. Consider the vertex v_{nm+1} and remove all m edges incident with v_{nm+1} and add $(n - 1)\, m$ edges to the vertices which are not adjacent to v_{nm+1}. Then the total number of edges are $(2n - 1)m$ .The edge labelings are defined as $f(v_i\, v_{i+1}) = i$ for $1 \leq i \leq nm - 1$, $f(v_{nm}\, v_1) = nm$ and the new edges which are incident with v_{nm+1} are labeled by the successive natural numbers from $nm+1$to $(2n - 1)m$ irrespective of the order. One can note that for any vertex the edge labelings have atleast two successive natural numbers, which are relatively prime. Hence the switching of v_{nm+1} in $J_{n, m}$ is vertex prime.

F. Remark

The switching of a vertex in the cycle of $J_{n, m}$ is not a vertex prime, as the switching of vertices in cycle form pendent vertices. And for pendent vertices the condition of primeness fails.

G. Figure 1

The vertex prime labeling of Switching of v_{10} in $J_{3,3}$

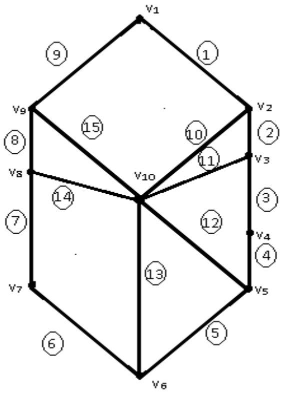

Figure.1 The vertex prime labeling of switching of v_{10} in $J_{3,3}$

H. Algorithm

Procedure (The vertex prime labeling of $NJ_{n,m}$)

The vertex set of k^{th} copy of $NJ_{n, m}$ is $\{v_1^k, v_2^k, \ldots, v_{nm+1}^k\}$

The edge set is $\{v_i^k v_{i+1}^k | 1 \leq i \leq nm - 1, 1 \leq k \leq N\} \cup \boldsymbol{v_{nm}^k v_1^k}$ $\cup \{v_{jn+1}^k v_{nm+1}^k \mid 0 \leq j \leq m - 1, 1 \leq k \leq N\}$

Define f: $E\,(NJ_{n, m}) \rightarrow \{1, 2, \ldots, N\,(n + 1)\, m\}$

for $1 \leq k \leq N$, $1 \leq i \leq nm - 1$

$$f(v_i^k v_{i+1}^k) = m(n+1)(k - 1)+i$$

end for

for $1 \leq k \leq N$

$$f\,(\boldsymbol{v_{nm}^k v_1^k}) = m\,((n + 1)\, k - 1$$

end for

for $1 \leq k \leq N$, $0 \leq j \leq m - 1$

$$f(\, v_{jn+1}^k v_{nm+1}^k) = m((n+1)\, k - 1) + (j+1)$$

end for

end procedure

I. Theorem

For $n \geq 2$, $m \geq 3$ and N is a finite positive integer then the graph $NJ_{n, m}$ is a vertex prime graph by algorithm F.

J. Theorem

For $n, p \geq 2$, $m, q \geq 3$ the disjoint union of $J_{n, m}$, $J_{p, q}$ is a vertex prime graph.

Proof: Let G be the disjoint union of $J_{n, m}$, $J_{p, q}$ then the vertex set is V:$\{v_1, v_2, v_3, \ldots, v_{nm+1}, v_1^{'}, v_2^{'}, \ldots, v_{pq+1}^{'}\}$and edge set is E:$\{v_i v_{i+1} | 1 \leq i \leq nm - 1\} \cup v_{nm} v_1 \cup \{v_{1+jn} v_{nm+1} \mid 0 \leq j \leq m - 1\} \cup \{v_k^{'} v_{k+1}^{'} \mid 1 \leq k \leq pq\text{-}1\} \cup v_{pq}^{'} v_1^{'} \cup \{v_{pq+1}^{'} v_{lp+1}^{'} | \; 0 \leq l \leq q - 1\}$. Define the edge labeling f:$E \rightarrow \{1,2,\ldots, |E|\}$ as $f(v_i v_{i+1}) = i$ for $1 \leq i \leq nm - 1$; $f(v_{nm}\; v_1) = nm$ and $f(v_{jn+1} v_{nm+1}) = (nm+1)+j$ for $0 \leq j \leq m - 1$ and $f(\, v_k^{'} v_{k+1}^{'}) = (n+1)\, m + k$ for $1 \leq k \leq pq - 1$; $f(v_{pq}^{'} v_1^{'}) = (n+1)m + pq$; $f(v_{pq+1}^{'} v_{lp+1}^{'}) = (n+1)\, m + pq + (l+1)$ for $0 \leq l \leq q - 1$. One can note that for any vertex v at least two edges incident on v are labeled by two successive natural numbers, and they are relatively prime. Therefore, the graph G is Vertex prime.

K. Corollary

The path union of N copies of Jahangir graph $J_{n, m}$ is vertex prime labeling.

Proof: The path union of N copies of $J_{n,m}$ can be formed by adding edges between the vertices v_1^k *and* v_1^{k+1} for $1 \leq k \leq N - 1$.The labeling for the edge $v_1^k v_1^{k+1}$ is $N(n+1)m + k$. The labelings of the remaining edges are as in theorem G. With this labeling the path union is vertex prime labeling.

L. Corollary

The path union of any two Jahangir graphs $J_{n, m}$, $J_{p, q}$ is vertex prime labeling.

Proof: The edge labeling of all edges except the edge which join the two graphs is as in theorem H and the joint edge is labeled by $(n + 1)\, m + (p + 1)\, q +1$. With this labeling, the path union is vertex prime.

III. THETA GRAPH

A. Algorithm

Procedure (The vertex prime labeling of generalized Theta graph)

$V = \{(x, y)\} \cup \{v_{ij}\; 1 \leq i \leq n, 1 \leq j \leq (l_i - 1)\}$

$E = \{v_{ij} v_{i(j+1)}\; 1 \leq i \leq n, 1 \leq j \leq (l_i - 1)\} \cup \{x v_{i1} 1 \leq i \leq n\}$ $\cup \{y v_{i(li - 1)} 1 \leq i \leq n\}$

The path lengths are $l_1, l_2, l_3, \ldots, l_n$

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The path number is i
If (i *mod* 2==1)
Choose $l_0 = 0$
Define the labelings
$f(xv_{i1}) = 1+\sum_{k=0}^{i-1} l_k$
$f(yv_{i(li-1)}) = \sum_{k=0}^{i} l_k$
for $1 \le j \le (l_i - 2)$
$f(v_{ij}v_{i(j+1)}) = (j+1) + \sum_{k=0}^{i-1} l_k$
end for
else

$f(xv_{i1}) = \sum_{k=1}^{i} l_k$
$f(yv_{i(li-1)}) = 1+ \sum_{k=1}^{i-1} l_k$
for $1 \le j \le (l_i - 2)$

$f(v_{ij}v_{i(j+1)}) = (\sum_{k=1}^{i} l_i) - j$
end for
end if
end procedure

B. Theorem

The Generalized Theta graph θ ($l_1, l_2, l_3, \ldots, l_n$), $n \ge 3$ is a vertex prime by algorithm A.

C. Figure 2

The vertex prime labeling of the Theta graph θ (6,6, 6,6),

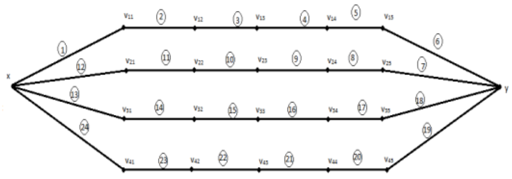

Figure. 2 The vertex prime labeling of Theta graph θ(6,6,6,6)

D. Figure 3

The vertex prime labeling of the Theta graph θ (6, 5, 3,6)

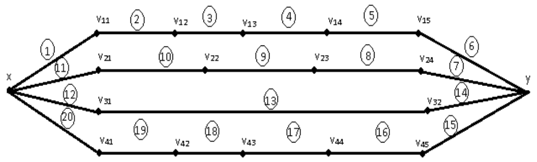

Figure. 3 The vertex prime labeling of $\theta(6, 5, 3,6)$

E. Theorem

The Duplication of any vertex in the Generalized Theta graph θ ($l_1, l_2, l_3, \ldots, l_n$), $n \ge 3$ is a vertex prime.

***Proof*:** The theorem is proved for the following two cases:

Case 1: If the vertex v_{ij} $1 \le i \le n, 1 \le j \le (l_i - 1)$ is in the path i then, the number of edges is increased by 2, and they are labeled as $\sum_{i=1}^{n} l_i + 1$ and $\sum_{i=1}^{n} l_i + 2$.

Case 2: If the vertex is either of the common end vertices x or y, then the number of edges is increased by n and are labeled as $\sum_{i=1}^{n} l_i + i$ for $1 \le i \le n$ irrespective of the order. The edge labelings of the remaining edges, which are not adjacent to the duplicated vertex, follow the same pattern as in algorithm A of this section.

F. Theorem

The Fusion of any two vertices in the Generalized Theta graph is a vertex prime.

Proof: The Fusion of any two vertices in the Generalized Theta graph reduces the number of vertices by 1, whereas the total number of edges remain same. The labeling of edges is carried out according to the algorithm A. One can note that for any vertex, the edge labelings have atleast two successive natural numbers, which are relatively prime. Hence, the resulting graph is vertex prime.

G. Remark

The switching of a vertex in Generalized Theta graph is not a vertex prime as the switching of vertices form pendent vertices. One can easily see that, for pendent vertices the condition of primeness fails.

IV.CONCLUSION

In this paper we concluded that Jahangir and generalized Theta graphs are vertex prime graphs. The Algorithms used in this paper are more efficient compare to the algorithms used in [6] for obtaining results on labelings of Jahangir and Generalized Theta Graphs. We noticed that the vertex prime valuation does not possible while switching any vertex in the cycle of a cyclic graph.

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