

# On Soft Semi Weakly Generalized Closed Set in Soft Topological Spaces

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**Abstract**—Soft sets has helped the development of soft topological space and it was also applied in the field of life science, Social science and Engineering. Many researchers developed various ideas based on the properties of soft topology. The article deals with study of properties in soft topological space based on soft semi weakly generalized closed set.

**Keywords**—Soft swg-closed set, Softswg-open set, Soft swg-closure set, Soft swg-interior.

**Subject Classification:**06D72,54A05

## I. INTRODUCTION

In the year 1999, the idea of soft set theory was introduced by Molodstov [2] to find answers to several problems in life science, engineering, and in practical life situation etc., Later the results were applied in different fields of study viz., operations research, game theory etc., C. A. G. Mani [1] introduced soft topology from which many researchers applied it as a base to work on soft topological space and it was the beginning for soft mathematical concepts. Here we introduced the idea of soft topological space in the basis of soft semi weakly generalized closed set. The collection of soft sets over  $F_A$  was studied by Shabir M and Naz M, [7] and denoted few notions of soft topological space. The properties of soft topological spaces were studied by the authors [3-8]

Let  $U$  be an universal set and  $E$  set of parameters;  $P(U)$  the power set of  $U$ . The collection of all soft sets over  $U$  and  $E$  is denoted by  $S(U)$ . If  $A \subseteq E$ , then the pair  $(F, A)$  is said to be the soft set over  $U$  and it is denoted by  $F_A$  or  $F_E$  where  $F$  is a mapping of  $A$  onto  $P(U)$ . Note that for  $e \notin A, F(e) = F_\emptyset$  [2]. Let  $F_B$  and  $G_C$  be the soft sets in a universe set  $U$  and  $B, C \subseteq E$ . Soft subset of  $G_C$  represented  $F_B$ , symbolized by  $F_B \subseteq G_C$ , when (i)  $B \subseteq C$  and (ii)  $\forall e \in B, F(e) = G(e)$ . The relative complement of a soft set  $F_A$ , denoted by  $F_A^c$ , is being represented by the function  $f_A^c(e) = f_A^c(e)$ , that is  $f_A^c(e) = U - f_A(e) \forall e \in E$ . In other words  $(F_A^c)^c = F_A, F_\emptyset^c = F_E$  and  $F_E^c = F_\emptyset$  [1]. Let  $F_A \in S(U)$ . A soft topology on  $(F_A, \tau)$ , which is a group of soft subsets of  $F_A$  has the following properties (i)

$F_\emptyset, F_A \in \tau$ . (ii)  $\{F_{A_i} \subseteq F_A : i \in I\} \subseteq \tau \Rightarrow \bigcup_{i \in I} F_{A_i} \in \tau$ . (iii)  $\{F_{A_i} \subseteq F_A : 1 \leq i \leq n, n \in \mathbb{N}\} \subseteq \tau \Rightarrow \bigcap_{i=1}^n F_{A_i} \in \tau$ . The pair  $(F_A, \tau)$  is known as soft topological spaces [7]. Let  $(F_A,$

$\tau)$  be a soft topological space and  $F_B$  be a soft set over  $F_A$ . (i) Soft interior of  $F_B$  is the soft set  $int(F_B) = \bigcup \{F_C : F_C \text{ is soft open set and } F_B \subseteq F_C\}$ . (ii) Soft closure of  $F_B$  is the soft set  $cl(F_B) = \bigcap \{F_C : F_C \text{ is soft closed set and } F_C \subseteq F_B\}$  [7]. (i) Soft  $\alpha$ -closed set, when  $cl(int(cl(F_B))) \subseteq F_B$  [3]. (ii) Soft  $\beta$ -closed set (or) Soft semi-pre closed set, when  $int(cl(int(F_B))) \subseteq F_B$  [3]. (iii) Soft semi-closed set, when  $int(cl(F_B)) \subseteq F_B$  [5]. (iv) Soft pre-closed set, when  $cl(int(F_B)) \subseteq F_B$  [6]. (v) Soft regular closed set, when  $cl(int(F_B)) = F_B$  [8]. (vi) Soft generalized closed set, when  $cl(F_B) \subseteq F_C$ , whenever  $F_B \subseteq F_C$  and  $F_C$  is a soft open set and Soft weakly closed set, if every  $cl(F_B) \subseteq F_C$ , whenever  $F_B \subseteq F_C$  and  $F_C$  is a soft semi-open set [8].

## II. SOFT SEMI WEAKLY GENERALIZED CLOSED SET AND SOFT SEMI WEAKLY GENERALIZED CLOSED SETS IN THEIR PROPERTIES

**Definition 2.1.** Let  $(F_A, \tau)$  is a soft topological space and  $F_B \subseteq F_A, F_B$  is identified to be a Soft Semi Weakly Generalized closed set (briefly, soft swg-closed set) if every  $cl(int(F_B)) \subseteq F_C$ , whenever  $F_B \subseteq F_C$  and  $F_C$  (soft semi-open set).

**Example 2.2.** Let  $U = \{a, b, c\}, E = \{e_1, e_2, e_3\}, A = \{e_1, e_2\} \subseteq E$ .

$F_A = \{(e_1, \{a, b, c\}), (e_2, \{a, b, c\})\}$ ,  
 $F_1 = \{(e_1, \{b, c\}), (e_2, \{b, c\})\}, F_2 = \{(e_1, \{c\}), (e_2, \{a, b\})\}, F_3 = \{(e_1, \{a, c\}), (e_2, \{b\})\}, F_4 = \{(e_1, \{c\}), (e_2, \{b\})\}, F_5 = \{(e_1, \{b, c\}), (e_2, \{a, b, c\})\}, F_6 = \{(e_1, \{a, b, c\}), (e_2, \{b, c\})\}, F_7 = \{(e_1, \{a, c\}), (e_2, \{a, b\})\}, F_8 = \{(e_1, \{a\}), (e_2, \{a\})\}, F_9 = \{(e_1, \{a, b\}), (e_2, \{c\})\}, F_{10} = \{(e_1, \{b\}), (e_2, \{a, c\})\}, F_{11} = \{(e_1, \{a, b\}), (e_2, \{a, c\})\}, F_{12} = \{(e_1, \{a\}), (e_2, \{a\})\}, F_{13} = \{(e_2, \{a\})\}, F_{14} = \{(e_1, \{b\}), (e_2, \{c\})\}, F_{15} = F_A, F_{16} = F_\emptyset$ . Let  $\tau = \{F_\emptyset, F_A, F_1, F_2, F_3, F_4, F_5, F_6, F_7\}$ . Then  $(F_A, \tau)$  is a soft topological space. Soft open set are  $\{F_A, F_\emptyset, F_1, F_2, F_3, F_4, F_5, F_6, F_7\}$ . Soft closed set are  $\{F_A, F_\emptyset, F_8, F_9, F_{10}, F_{11}, F_{12}, F_{13}, F_{14}\}$ .

(i) Let us consider  $F_B = F_8 = \{(e_1, \{a\}), (e_2, \{a\})\}, F_B \subseteq F_A$  and  $F_C = \{(e_1, \{a\}), (e_2, \{a, b\})\}$  where  $F_C$  is soft semi-open set, then  $int(F_B) = F_\emptyset, cl(int(F_B)) = F_\emptyset$ . So  $cl(int(F_B)) \subseteq F_C$ .  $F_B$  is soft semi weakly generalized closed set (soft swg-closed set) but not soft semi-open set.

(ii) Let us consider  $F_B = \{(e_1, \{c\}), (e_2, \{b, c\})\}$  and  $F_C = \{(e_1, \{a, b, c\}), (e_2, \{b, c\})\}$  where  $F_C$  is a soft semi-open set, then  $int(F_B) = F_4, cl(int(F_B)) = F_A$ . So,  $cl(int(F_B)) \not\subseteq F_C$ .  $F_B$  is a soft semi-open set but it is neither a soft semi weakly generalized closed set (soft swg-

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closed set) nor soft closed set or soft open set.

**Theorem 2.3.** All soft closed sets represent soft swg-closed set.

**Proof.** Let  $F_B$  be a soft closed set in  $(F_A, \tilde{\tau})$ . Now  $int(F_B) \subseteq F_B$  and  $F_B \subseteq F_C$  always, if  $F_B$  is a soft closed set and soft open set. So, if  $F_B \subseteq F_C$ , where  $F_C$  is a soft semi-open set in  $F_A$ , then  $cl(int(F_B)) \subseteq cl(F_B) \subseteq F_C$ . Thus  $cl(int(F_B)) \subseteq F_C$ , whenever  $F_B \subseteq F_C$ , where  $F_C$  denotes soft semi-open set. Therefore,  $F_B$  is a soft swg-closed set.  $\square$

**Example 2.4.** We have following example 2.2., Let  $F_B = \{(e_1, \{c\}), (e_2, \{a, b, c\})\}$  and  $F_C = \{(e_1, \{a, c\}), (e_2, \{a, b, c\})\}$ , where  $F_C$  is a soft semi-open set, then  $int(F_B) = F_\phi$ ,  $cl(int(F_B)) = F_\phi$ , so,  $cl(int(F_B)) \subseteq F_C$ . Now  $F_B$  is soft swg-closed set but it is not a soft closed set.

**Theorem 2.5.** All soft g-closed sets represent soft swg-closed set.

**Proof.** Let  $F_B$  stands soft g-closed set, then  $cl(F_B) \subseteq F_C$  whenever  $F_B \subseteq F_C$  and  $F_C$  remains a soft open set. Each soft open set denotes soft semi-open set.  $int(F_B) \subseteq F_B$ , if  $F_B$  is a soft open set. Now  $cl(int(F_B)) \subseteq cl(F_B) \subseteq F_C$ . Therefore,  $F_B$  refers as a soft swg-closed set.  $\square$

**Example 2.6.** We have following example 2.2., Let  $F_B = \{(e_1, \{c\}), (e_2, \{a\})\}$ ,  $F_C = \{(e_1, \{c\}), (e_2, \{a, b\})\}$  and  $F_B \subseteq F_C$  where  $F_C$  represents soft open set and soft semi-open set. Then  $int(F_B) = F_\phi$ ,  $cl(int(F_B)) = F_\phi$ ,  $cl(int(F_B)) \subseteq F_C$ . Therefore,  $F_B$  be a soft swg-closed set but it is not a soft g-closed set.

**Theorem 2.7.** All soft regular closed set represent soft swg-closed set.

**Proof.** Let  $F_B$  refer to a soft regular closed set, if  $cl(int(F_B)) = F_B$ . Since  $F_B \subseteq F_C$ , where  $F_C$  remains soft semi-open set. Now  $cl(int(F_B)) = F_B \subseteq F_C$ . Therefore,  $F_B$  refers as a soft swg-closed set.

**Example 2.8.** We have following example 2.2., Let  $F_B = \{(e_1, \{a\}), (e_2, \{b\})\}$ ,  $F_C = \{(e_1, \{a\}), (e_2, \{b, c\})\}$  and  $F_B \subseteq F_C$  where  $F_C$  is soft semi-open set, then  $int(F_B) = F_\phi$ ,  $cl(int(F_B)) = F_\phi$ ,  $cl(int(F_B)) \subseteq F_C$ . Therefore,  $F_B$  refers as a soft swg-closed set but it is not a soft regular closed set

**Theorem 2.9.** The intersection of two soft swg-closed sets gives another soft swg-closed set.

**Proof.** Let  $F_B$  and  $F_C$  stay any two soft swg-closed sets and  $F_D$  remains any soft semi-open set comprise  $F_B$  and  $F_C$ . By definition of soft swg-closed set,  $cl(int(F_B)) \subseteq F_D$  and  $cl(int(F_C)) \subseteq F_D$ . Hence

$cl(int(F_B \tilde{\cap} F_C)) \subseteq cl(int(F_B)) \tilde{\cap} cl(int(F_C)) \subseteq F_D$ . Therefore,  $cl(int(F_B \tilde{\cap} F_C)) \subseteq F_D$ . Therefore,  $F_B \tilde{\cap} F_C$  refers as a soft swg-closed set in  $F_A$ .  $\square$

**Remark 2.10.** If  $F_B$  and  $F_C$  are soft swg-closed sets the  $F_B \tilde{\cup} F_C$  does not need be a soft swg-closed set.

**Example 2.11.** From 2.2., Let  $F_B = \{(e_1, \{c\})\}$  and  $F_C = \{(e_2, \{b\})\}$ , then  $F_B$  and  $F_C$  are soft swg-closed sets.  $F_B \tilde{\cup} F_C = \{(e_1, \{c\}), (e_2, \{b\})\}$  is not a soft swg-closed set

**Theorem 2.12.** Let  $(F_A, \tilde{\tau})$  be a soft topological space. Then a soft subset  $F_B$  of  $F_A$  is soft swg-closed set in  $(F_A, \tilde{\tau})$  iff  $cl(int(F_B)) - F_B$  contains empty soft semi-closed set

**Proof.** Let  $F_C$  be a non empty soft semi-closed set subset of  $cl(int(F_B)) - F_B$ . Now  $F_C \subseteq cl(int(F_B)) - F_B$ . Which implies that  $F_C \subseteq cl(int(F_B)) \tilde{\cap} F_B^c$ . Since  $cl(int(F_B)) - F_B = cl(int(F_B)) \tilde{\cap} F_B^c$ . Thus  $F_C \subseteq cl(int(F_B))$ . Now

$F_C \subseteq F_B^c$  which implies that  $F_B \subseteq F_C^c$ . When  $F_C^c$  represents soft semi-open set and  $F_B$  represents soft swg-closed set, we have  $cl(int(F_B)) \subseteq F_C^c$  which implies that  $(cl(int(F_B)))^c \subseteq (F_C^c)^c$  which implies that  $F_C \subseteq cl(int(F_B))$  which implies that  $F_C \subseteq cl(int(F_B)) \tilde{\cap} (cl(int(F_B)))^c = F_\phi$  which is a contradiction. Therefore,  $F_C = F_\phi$ . Therefore,  $cl(int(F_B)) - F_B$  contains empty soft semi-closed set. Conversely, Assume  $cl(int(F_B)) - F_B$  contains empty soft semi-closed sets. Let  $F_B \subseteq F_C$ , where  $F_C$  is a soft semi-open set. Presume,  $cl(int(F_B))$  is not contained in  $F_C$ . Then  $cl(int(F_B)) \not\subseteq F_C$ , then  $cl(int(F_B)) \tilde{\cap} F_C^c$  is a non empty soft semi-closed set of  $cl(int(F_B)) - F_B$ , which is a contradiction. Then  $cl(int(F_B)) \subseteq F_C$  whenever  $F_B \subseteq F_C$ . Therefore,  $F_B$  is a soft swg-closed set.  $\square$

**Theorem 2.13.** The soft swg-closed sets be a soft regular closed set,  $\Leftrightarrow cl(int(F_B)) - F_B$  soft semi-closed set.

**Proof.** Presume that  $F_B$  be soft regular closed set. Since  $cl(int(F_B)) = F_B$ . Since  $cl(int(F_B)) - F_B = F_\phi$  be soft regular closed set and it is soft semi-closed set. Conversely, Assume  $cl(int(F_B)) - F_B$  is a soft semi-closed set [From Theorem 2.12]. Since,  $cl(int(F_B)) - F_B = F_\phi$  contains empty soft semi-closed set. Thus  $F_B$  proved as soft regular closed set.  $\square$

**Theorem 2.14.** Soft topological space  $(F_A, \tilde{\tau})$  and  $F_C \subseteq F_B \subseteq F_A$ . If  $F_C$  is a soft swg-closed set in  $F_A$ , then  $F_C$  is a soft swg-closed set in relation to  $F_B$  and mutually soft open set and soft swg-closed are subset of  $F_A$ , then  $F_C$ , soft swg-closed set relative to  $F_A$

**Proof.** Assume  $F_C \subseteq F_D$  and  $F_D$  be the soft open set over  $F_A$ . From the known result  $F_C \subseteq F_B \subseteq F_A$ . Then  $F_C \subseteq F_B$  and  $F_C \subseteq F_D$  which implies that  $F_C \subseteq F_B \tilde{\cap} F_D$ . Since  $F_C$  denoted as a soft swg-closed set relative to  $F_B$ ,  $cl(int(F_C)) \subseteq F_B \tilde{\cap} F_D$  which implies that  $F_B \tilde{\cap} cl(int(F_C)) \subseteq F_B \tilde{\cap} F_D$  which implies that

$F_B \tilde{\cap} cl(int(F_C)) \subseteq F_D$ . Thus  $F_B \tilde{\cap} cl(int(F_C)) \tilde{\cup} (cl(int(F_C)))^c \subseteq F_D \tilde{\cup} (cl(int(F_C)))^c$  which implies that  $F_B \tilde{\cup} (cl(int(F_C)))^c \subseteq F_D \tilde{\cup} (cl(int(F_C)))^c$ . Since  $F_B$  is a soft swg-closed set in  $F_A$ . We have  $cl(int(F_B)) \subseteq F_D \tilde{\cup} (cl(int(F_C)))^c$ . Also,  $F_C \subseteq F_B$  which implies that  $cl(int(F_C)) \subseteq cl(int(F_B))$ . Thus  $cl(int(F_C)) \subseteq cl(int(F_B)) \subseteq F_D \tilde{\cup} (cl(int(F_C)))^c$ . Hence,  $F_C$  proved as soft swg-closed set in relation to  $F_A$

**Theorem 2.15.** When soft topological space  $(F_A, \tilde{\tau})$  and  $F_C \subseteq F_B \subseteq F_A$ . If  $F_C$  is soft swg-closed set relative to  $F_B$  and  $F_B$  is soft swg-closed set  $(F_A, \tilde{\tau})$ . Then  $F_C$ , soft swg-closed set in relation to  $(F_A, \tilde{\tau})$

**Proof.** Let  $F_C \subseteq F_D$ , where  $F_D$  is soft semi-open set in  $F_A$ . Then  $F_C \subseteq F_B \tilde{\cap} F_D$ . Since  $F_C$  is soft swg-closed set relative to  $F_B$ , then  $cl(int(F_C)) \subseteq F_B \tilde{\cap} F_D$ . ie.,  $F_B \tilde{\cap} cl(int(F_C)) \subseteq F_B \tilde{\cap} F_D$ . We have  $F_B \tilde{\cap} cl(int(F_C)) \subseteq F_D$  and then  $F_B \tilde{\cap} cl(int(F_C)) \tilde{\cup} (cl(int(F_C)))^c \subseteq F_D \tilde{\cup} (cl(int(F_C)))^c$ . Since  $F_B$  is a soft swg-closed set in  $F_A$ , we obtain  $cl(int(F_B)) \subseteq F_D \tilde{\cup} (cl(int(F_C)))^c$ . Also,  $F_C \subseteq F_B$  which implies that  $cl(int(F_C)) \subseteq cl(int(F_B))$ , therefore  $cl(int(F_C)) \subseteq F_D$ . Since  $cl(int(F_C))$  is not contained in  $(cl(int(F_C)))^c$ . Hence,  $F_C$  proved as soft swg-closed set in  $F_A$ .



**Theorem 2.16.** Let  $(F_A, \tilde{\tau})$  considered as soft topological space, then a soft subset  $F_B$  inside  $F_A$  is soft nowhere dense, then  $F_B$ , soft *swg*-closed set in  $F_A$

**Proof.** When a soft subset  $F_B$  of  $F_A$  is soft nowhere dense, then  $int(F_B) = F_\phi$ . Let  $F_B \subseteq F_C$  where  $F_C$  represent soft semi-open set, which denotes that  $cl(int(F_B)) = cl(F_\phi) = F_\phi \subseteq F_C$ . Therefore,  $F_B$  proved as soft *swg*-closed set in  $F_A$ .

**Theorem 2.17.** Let  $(F_A, \tilde{\tau})$  be a soft topological space. For every soft subset  $f_A \in F_A$ , either  $\{f_A\}$  refers as soft semi-closed set, or  $\{f_A\}^c$ , soft *swg*-closed set in  $(F_A, \tilde{\tau})$

**Proof.** Assume that  $\{f_A\}$  does not belongs to soft semi-closed set of  $(F_A, \tilde{\tau})$ . Then  $\{f_A\}^c$  is a soft semi-open set and the only soft semi-open set containing in  $F_A$  itself. Therefore,  $cl(int(\{f_A\}^c)) \subseteq F_A$  and so  $\{f_A\}^c$  proved as soft *swg*-closed set in  $(F_A, \tilde{\tau})$

**Theorem 2.18.** If  $F_B$  state soft semi-open set and soft *swg*-closed set in  $F_A$ , both these constitute soft *g*-closed set

**Proof.** Let  $F_B$  be both soft *swg*-closed set and soft semi-open

set. Let  $F_B \subseteq F_C$  where  $F_C$  is soft open set. Then by definition of soft *swg*-closed set,  $cl(int(F_B)) \subseteq F_C$ . Since  $F_B$  is soft semi-open set  $cl(F_B) \subseteq cl(int(F_B)) \subseteq F_C$ . Which implies that  $cl(F_B) \subseteq F_C$ , where  $F_C$  shows as soft open set. Hence  $F_B$  proved as soft *g*-closed set.

**Theorem 2.19.** Let  $F_B$  be soft *swg*-closed set and  $F_D$  is soft closed set, then  $F_B \cap F_D$  is soft *swg*-closed set

**Proof.** Let  $F_B$  be a soft *swg*-closed set and  $F_D$  is soft closed set. Now, show that  $F_B \cap F_D$  is soft *swg*-closed set. Let  $F_B \cap F_D \subseteq F_C$  where  $F_C$  is soft semi-open set. Since  $F_D$  is soft closed set,  $F_B \cap F_D$  is soft closed set in  $F_B$  which implies that  $F_B \cap F_D \subseteq F_B$ . It implies that  $cl(int(F_B \cap F_D)) \subseteq cl(F_B \cap F_D) = F_B \cap F_D \subseteq F_C$ . It implies that  $cl(int(F_B \cap F_D)) \subseteq F_C$ . Therefore,  $F_B \cap F_D$  proved as soft *swg*-closed set.

**Theorem 2.20.** Let  $(F_A, \tilde{\tau})$  be soft topological space. If  $F_B$  is soft *swg*-closed set in  $(F_A, \tilde{\tau})$  and  $F_B \subseteq F_C \subseteq cl(int(F_B))$ , then  $F_C$  is soft *swg*-closed set in  $(F_A, \tilde{\tau})$ .

**Proof.** Given that:  $F_C \subseteq cl(int(F_B))$ , then  $cl(int(F_C)) \subseteq cl(int(F_B))$ . Which implies that  $cl(int(F_C)) - F_C \subseteq cl(int(F_B)) - F_B$ . Since  $F_B \subseteq F_C$  and  $F_B$  is soft *swg*-closed set in  $(F_A, \tilde{\tau})$  [From Theorem 2.12]. Let  $cl(int(F_B)) - F_B$  contains empty soft closed set and  $cl(int(F_C)) - F_C$  contains no non empty soft semi-closed set. Therefore,  $F_C$  proved as soft *swg*-closed set.

### III. SOFT SEMI WEAKLY GENERALIZED OPEN SET AND SOFT SEMI WEAKLY GENERALIZED OPEN SETS IN THEIR PROPERTIES

**Definition 3.1.** A soft subset  $F_B$  of  $(F_A, \tilde{\tau})$  refers as soft semi weakly generalized open set (briefly, soft *swg*-open set), if its compliment is soft *swg*-closed set in  $(F_A, \tilde{\tau})$ .

**Theorem 3.2.** A soft subset  $F_B$  of  $(F_A, \tilde{\tau})$  is soft regular open set, then it is soft *swg*-open set.

**Proof.** Let  $F_B$  be a soft regular open set in  $\tilde{\tau}$ , then  $F_B^c$  is soft regular closed set. It implies  $F_B^c = cl(int(F_B^c))$ . Let  $F_B \subseteq F_C$ , since  $F_C$  is soft semi-open set in  $\tilde{\tau}$ . It implies that  $cl(int(F_B^c)) \subseteq F_C$  where  $F_C$  represents soft semi-open set. Since  $F_B^c$  represents soft *swg*-closed set. Therefore,  $F_B$  proved as soft *swg*-open set.

**Theorem 3.3.** A soft subset  $F_B$  of a soft topological space  $(F_A, \tilde{\tau})$  is soft *swg*-open set iff  $F_C \subseteq int(cl(F_B))$  when  $F_C \subseteq F_B$  and  $F_C$  is a soft semi-closed set.

**Proof.** Presume that  $F_B$  is a soft *swg*-open set in  $F_A$ , then  $F_B^c$  represent soft *swg*-closed set. Let  $F_C$  be a soft semi-closed set in  $(F_A, \tilde{\tau})$  contained in  $F_B$ . Then  $F_C^c$  is soft semi-open set containing  $F_B^c$ . i.e.,  $F_B^c \subseteq F_C^c$ . It implies  $cl(int(F_B^c)) \subseteq F_C^c$ . Since  $F_B^c$  is soft *swg*-closed set.  $\therefore F_C \subseteq int(cl(F_B))$ . Conversely, suppose  $F_C \subseteq int(cl(F_B))$  when  $F_C \subseteq F_B$  and  $F_C$  is a soft semi-closed set. Then  $F_C^c$  is soft semi-open set containing  $F_B^c$  and  $F_C^c \subseteq (int(cl(F_B)))^c$ . It follows  $F_C^c \subseteq (cl(int(F_B^c)))^c$ .  $F_B^c$  is soft *swg*-closed set and so,  $F_B$  proved as soft *swg*-open set.  $\square$

**Theorem 3.4.** If  $int(cl(F_B)) \subseteq F_C \subseteq F_B$  and  $F_B$  is soft *swg*-open set in  $F_A$ , then  $F_C$  is soft *swg*-open set

**Proof.** Let  $int(cl(F_B)) \subseteq F_C \subseteq F_B$  it implies  $(F_B)^c \subseteq (F_C)^c \subseteq (int(cl(F_B)))^c$  which implies that  $(F_B)^c \subseteq (F_C)^c \subseteq (cl(int(F_B^c)))$  where  $F_B^c$  is soft *swg*-closed set in  $F_A$  and also  $F_C^c$  is soft *swg*-closed set. Therefore,  $F_C$  proved as soft *swg*-open set in  $F_A$ .

**Theorem 3.5.** While  $F_B$  and  $F_C$  are soft *swg*-open sets in  $F_A$ . Let  $F_B, F_C \subseteq F_A$ . If  $F_C$  is soft *swg*-open set and  $int(cl(F_C)) \subseteq F_B$ , then  $F_B \cap F_C$  is soft *swg*-open set

**Proof.** Let  $F_B$  and  $F_C$  are soft *swg*-open sets in  $F_A$  and  $F_B^c$  and  $F_C^c$  are soft *swg*-closed sets in  $F_A$ . Since  $F_B \supseteq int(cl(F_C))$  it implies  $int(cl(F_C)) \subseteq F_B \cap F_C$  [By Theorem 3.4], then  $F_B \cap F_C$  proved as soft *swg*-open set

**Theorem 3.6.** A soft set  $F_B$  is a soft *swg*-open sets in  $F_A$ ,  $\Leftrightarrow F_C = F_A$ , when  $F_C$  is soft semi-open set and  $int(cl(F_B)) \cup F_B^c \subseteq F_C$

**Proof.** Let  $F_B$  be soft *swg*-open set.  $F_C$  be soft semi-open set and  $int(cl(F_B)) \cup F_B^c \subseteq F_C$ . Which implies that  $(F_C)^c \subseteq (int(cl(F_B)) \cup F_B^c)^c = (int(cl(F_B)))^c \cap F_B = (int(cl(F_B)))^c - F_B^c = cl(int(F_B^c)) - F_B^c$ . Which implies that  $F_C^c \subseteq cl(int(F_B^c)) - F_B^c$ . Since  $F_B^c$  represent soft *swg*-closed set and  $F_C^c$  represent soft semi-closed set, based on that  $F_C^c = F_\phi$ . Therefore,  $F_C = F_A$ . Conversely, assume that  $F_D$  is a soft semi-closed set in  $F_A$  and  $F_D \subseteq F_B$ . Then  $int(cl(F_B)) \cup F_B^c \subseteq int(cl(F_B)) \cup F_D^c$ . Since both  $int(cl(F_B))$  and  $F_D^c$  are soft semi-open set, their union  $int(cl(F_B)) \cup F_D^c$  is also a soft semi-open set. It follows by hypothesis that  $int(cl(F_B)) \cup F_D^c = F_A$  and hence  $F_D \subseteq int(cl(F_B))$ . Therefore,  $F_B$  proved as soft *swg*-open sets in  $F_A$

**Theorem 3.7.** If  $F_B \subseteq F_C \subseteq F_A$ , where  $F_B$  is soft *swg*-open set is in relation to  $F_C$  and  $F_C$  is soft *swg*-open set is in relation to  $F_A$ , then  $F_B$  is a soft *swg*-open set is in relation to  $F_A$ .

**Proof.** Let  $F_D$  be a soft semi-closed set and suppose  $F_D \subseteq F_B$ . Then  $F_D$ , a soft semi-closed set is in relation to  $F_C$  and hence  $F_D \subseteq int(cl(F_B))$ . Therefore, a soft semi-open



set  $F_E$  exists such that  $F_D \subseteq F_E \cap F_C \subseteq F_B$ . But  $F_D \subseteq F_E^* \subseteq F_C$  for soft semi-open set  $F_E^*$ , since  $F_C$  is soft *swg*-open set in  $F_A$ . Thus  $F_D \subseteq F_E^* \cap F_C \subseteq F_C \cap F_E \subseteq F_B$ . It follows that  $F_D \subseteq \text{int}(cl(F_B))$ . Because soft set  $F_B$  is soft *swg*-open set. It implies  $F_D \subseteq \text{int}(cl(F_B))$  whenever  $F_D$  is a soft semi-closed set and  $F_D \subseteq F_B$ . Therefore,  $F_B$  is soft *swg*-open set in  $F_A$ .

IV. SOFT SWG-CLOSURE And SOFT SWG-INTERIOR & RESULTS

**Definition 4.1.** Let  $(F_A, \tau)$  be soft topological space,  $F_B \subseteq F_A$ . The soft *swg*-closure of  $F_B$  (briefly, *swg-cl*( $F_B$ )) to be the intersection of all soft *swg*-closed subsets containing  $F_B$ . In symbols,  $\text{swg-cl}(F_B) = \{F_C : F_B \subseteq F_C \text{ and } F_C \text{ is a soft } \text{swg}\text{-closed set in } F_A\}$

**Theorem 4.2.** While any soft subset  $F_B$  of soft topological space  $F_A$ ,  $F_B \subseteq \text{swg-cl}(F_B) \subseteq cl(\text{int}(F_B))$

**Proof.** It is based on Theorem 2.3.

**Example 4.3.** We have following example 2.2.,  $F_B = \{(e_1, \{c\}), (e_2, \{a, b\})\}$ , then  $\text{swg-cl}(F_B) = \{(e_1, \{b, c\}), (e_2, \{a, b\})\}$  and  $\text{int}(F_B) = F_2$ ,  $cl(\text{int}(F_B)) = F_A$ . Therefore,  $F_B \subseteq \text{swg-cl}(F_B) \subseteq cl(F_B)$  in  $F_A$ .

**Theorem 4.4.** The soft *swg*-closure operator is soft Kuratowski closure operator on  $F_A$ .

**Proof.** (i)  $\text{swg-cl}(F_\phi) = F_\phi$ .

(ii)  $F_B \subseteq \text{swg-cl}(F_B)$  by Theorem 4.2.,

(iii) Let  $F_{B_1} \cup F_{B_2} \subseteq F_C$  and  $F_C$  belongs to soft *swg*-closed set in  $F_A$ , then  $F_{B_i} \subseteq F_C$  and by Definition 3.1.  $\text{swg-cl}(F_{B_i}) \subseteq F_C$  for each  $i=1, 2$ . Therefore,  $\text{swg-cl}(F_{B_1}) \cup \text{swg-cl}(F_{B_2}) \subseteq \cap \{F_C : F_{B_1} \cup F_{B_2} \subseteq F_C \text{ and } F_C \text{ is soft } \text{swg}\text{-closed set in } F_A\} = \text{swg-cl}(F_{B_1} \cup F_{B_2})$ . For the reverse insertion, let  $f_A \in \text{swg-cl}(F_{B_1} \cup F_{B_2})$  and suppose that  $f_A \notin \text{swg-cl}(F_{B_1}) \cup \text{swg-cl}(F_{B_2})$ . Then there exist soft *swg*-closed sets  $F_{C_1}$  and  $F_{C_2}$  with  $F_{B_1} \subseteq F_{C_1}$ ,  $f_A \notin F_{C_1}$  and  $F_{B_2} \subseteq F_{C_2}$ ,  $f_A \notin F_{C_2}$ . We have  $F_{B_1} \cup F_{B_2} \subseteq F_{C_1} \cup F_{C_2}$  and  $F_{C_1} \cup F_{C_2}$  is a soft *swg*-closed set such that  $f_A \notin F_{C_1} \cup F_{C_2}$ . Thus  $f_A \notin \text{swg-cl}(F_{B_1} \cup F_{B_2})$ . Which is a contradiction to  $f_A \in \text{swg-cl}(F_{B_1} \cup F_{B_2})$ . Hence  $\text{swg-cl}(F_{B_1}) \cup \text{swg-cl}(F_{B_2}) = \text{swg-cl}(F_{B_1} \cup F_{B_2})$ .

(iv) Let  $F_B \subseteq F_C$  and  $F_C$  is soft *swg*-closed set within  $F_A$ . Then by Definition 3.1.  $\text{swg-cl}(F_B) \subseteq F_C$  and  $\text{swg-cl}(\text{swg-cl}(F_B)) \subseteq F_C$ , we have  $\text{swg-cl}(\text{swg-cl}(F_B)) \subseteq \cap \{F_C : F_B \subseteq F_C \text{ and } F_C \text{ is a soft } \text{swg}\text{-closed set in } F_A\} = \text{swg-cl}(F_B)$ . [By Theorem 3.2.],  $\text{swg-cl}(F_B) \subseteq \text{swg-cl}(\text{swg-cl}(F_B))$  and therefore,  $\text{swg-cl}(F_B) = \text{swg-cl}(\text{swg-cl}(F_B))$ . Hence soft *swg*-closure operator is a soft Kuratowski closure operator on  $F_A$

**Theorem 4.5.** While any soft subset  $F_B$  of soft topological space in  $F_A$ .

(i)  $\text{swg-cl}(F_B)$  is smallest soft *swg*-closed set containing  $F_B$ .

(ii)  $F_B$  is soft *swg*-closed set  $\Leftrightarrow \text{swg-cl}(F_B) = F_B$ .

**Proposition 4.6.** While two soft subsets  $F_{B_1}$  and  $F_{B_2}$  of soft topological space in  $F_A$ .

(i)  $F_{B_1} \subseteq F_{B_2}$ , then  $\text{swg-cl}(F_{B_1}) \subseteq \text{swg-cl}(F_{B_2})$ .

(ii)  $\text{swg-cl}(F_{B_1} \cap F_{B_2}) \subseteq \text{swg-cl}(F_{B_1}) \cap \text{swg-cl}(F_{B_2})$ .

**Theorem 4.7.** For any  $f_A \in F_A$ ,  $f_A \in \text{swg-cl}(F_B) \Leftrightarrow F_C \cap F_B \neq F_\phi$  for every soft *swg*-open set  $F_C$  containing  $f_A$ .

**Proof.** Let  $f_A \in \text{swg-cl}(F_B)$  for any  $f_A \in F_A$ . Suppose there exists a soft *swg*-open set  $F_C$  containing  $f_A$  such that  $F_C \cap F_B = F_\phi$ . Then  $F_B \subseteq F_C^c$ . Since  $F_C^c$  is a soft *swg*-closed set containing  $F_B$ , we obtain  $cl(\text{int}(F_B)) \subseteq F_C^c$  which implies that  $f_A \notin \text{swg-cl}(F_B)$ , which is a contradiction. On the contrary, assume that  $f_A \notin \text{swg-cl}(F_B)$ . From Definition 4.1. there exists a soft *swg*-closed set  $F_D$  containing  $F_B$  such that  $f_A \notin F_D$ . Thereafter  $f_A \in F_D^c$  and  $F_D^c$  is a soft *swg*-open set in  $F_A$ . Also,  $F_D^c \cap F_B = F_\phi$  which is a contradiction to the hypothesis. Therefore,  $f_A \in \text{swg-cl}(F_B)$ .

**Theorem 4.8.** For any  $F_B \subseteq F_A$ , soft *swg*-interior of  $F_B$  (briefly, *swg-int*( $F_B$ )) is illustrated as the union of the entire soft *swg*-open sets containing in  $F_B$ . In symbols,  $\text{swg-int}(F_B) = \cup \{F_C : F_C \subseteq F_B \text{ and } F_C \text{ is a soft } \text{swg}\text{-open set in } F_A\}$

**Remark 4.9.** For any soft subset  $F_B$  of soft topological space  $F_A$ ,

(i)  $\text{swg-int}(F_B)$  is soft *swg*-open set in  $F_A$ , Since arbitrary union of every soft *swg*-open sets in  $F_A$  is in fact soft *swg*-open sets in  $F_A$ .

(ii)  $\text{swg-int}(F_B)$  is the largest soft *swg*-open set in  $F_A$  contained in  $F_B$ .

**Theorem 4.10.** Let  $F_B$  represent soft *swg*-open set in  $F_A$ ,  $\Leftrightarrow F_B = \text{swg-int}(F_B)$ .

**Proof.** Let  $F_B$  is soft *swg*-open set. Now,  $F_B$  being soft *swg*-open set in  $F_A$ ,  $F_B$  is the largest soft *swg*-open set in  $F_A$ . Therefore,  $F_B = \text{swg-int}(F_B)$ . Conversely, let  $\text{swg-int}(F_B) = F_B$  and by definition,  $\text{swg-int}(F_B)$  is a soft *swg*-open set. Then it follows that,  $F_B$  is also a soft *swg*-open set.

**Proposition 4.11.** For the two soft subset  $F_B$  and  $F_C$  of soft topological space  $F_A$ , then the following holds.

(i)  $\text{int}(cl(F_B)) \subseteq \text{swg-int}(F_B) \subseteq F_B$ .

(ii) If  $F_B \subseteq F_C$ , then  $\text{swg-int}(F_B) \subseteq \text{swg-int}(F_C)$ .

(iii)  $\text{swg-int}(F_B \cap F_C) \subseteq \text{swg-int}(F_B) \cap \text{swg-int}(F_C)$ .

(iv)  $\text{swg-int}(F_B \cup F_C) \subseteq \text{swg-int}(F_B) \cup \text{swg-int}(F_C)$ .

(v)  $\text{swg-int}(F_A) = F_A$ .

(iv)  $\text{swg-int}(F_\phi) = F_\phi$ .

**Remark 4.12.** In any soft topological space  $F_A$ , if  $\text{swg-int}(F_B) = \text{swg-int}(F_C)$  for subsets  $F_B$  and  $F_C$  of  $F_A$ , then it does not imply that  $F_B = F_C$ .

**Example 4.13.** We have following example 2.2., Let  $F_B = \{(e_1, \{c\})\}$ ,  $F_C = \{(e_2, \{b\})\}$ , then  $\text{swg-cl}(F_B) = F_\phi$  and  $\text{swg-cl}(F_C) = F_\phi$ . Therefore,  $\text{swg-int}(F_B) = \text{swg-int}(F_C)$  but  $F_B \neq F_C$ .

**Theorem 4.14.** While soft subset  $F_B$  of soft topological space  $F_A$ , then the following holds.

(i)  $(\text{swg-int}(F_B))^c = \text{swg-cl}(F_B^c)$ .

(ii)  $\text{swg-int}(F_B) = (\text{swg-cl}(F_B^c))^c$ .

(iii)  $\text{swg-cl}(F_B) = (\text{swg-cl}(F_B^c))^c$ .

**Proof.** Let  $f_A \in (\text{swg-int}(F_B))^c$ . Then  $f_A \notin \text{swg-int}(F_B)$  and so every soft *swg*-open set  $F_C$  containing  $f_A$  is such that  $F_C \cap F_B = F_\phi$ . That is every soft *swg*-open set  $F_C$  containing  $f_A$  is such that  $F_C \cap F_B = F_\phi$  [By Theorem 4.7.],  $f_A \in \text{swg-cl}(F_B^c)$  and therefore,  $(\text{swg-int}(F_B))^c \subseteq \text{swg-cl}(F_B^c)$ . Conversely, let  $f_A \in \text{swg-cl}(F_B^c)$ . Then [by Theorem 3.4.], every soft *swg*-open set  $F_D$  containing  $f_A$  is such that  $F_D \cap F_B^c \neq F_\phi$  and so



every soft  $swg$ -open set  $F_D$  containing  $f_A$  is such that  $F_D \not\subseteq F_B$ . This implies by definition 4.8.,  $f_A \notin swg-int(F_B)$  implies  $f_A \in (swg-int(F_B))^c$  and  $swg-cl(F_B^c) \subseteq (swg-int(F_B))^c$ . Thus  $(swg-int(F_B))^c = swg-cl(F_B^c)$ . (ii) and (iii) follows from (i).

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