On Soft Semi Weakly Generalized Closed Set in Soft Topological Spaces

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Abstract—Soft sets has helped the development of soft topological space and it was also applied in the field of life science, Social science and Engineering. Many researchers developed various ideas based on the properties of soft topology. The article deals with study of properties in soft topological space based on soft semi weakly generalized closed set.

Keywords—Soft swg-closed set, Softswg-open set, Soft swg-closure set, Soft swg-interior.

Subject Classification:06D72,54A05

I. INTRODUCTION

Intheyear1999, the idea of soft set theory was introduced by Molodstov[2]tofindanswertoeveral problemsinlifescience,engineering,and in practical life situationetc...Later theresthus wereappliedindifferent fieldsoft study viz., operations research, gametheoryetc... C a’gmanN[1]introduced softtopologyfromwhich manyresearchers applidatas abase toworkon softtopologicalspacesatatwaisthe beginningforsoftmathematicalconcepts.Here we introduced the idea of soft topological space in the basis of soft semi weakly generalized closed set. The collection of soft sets over $F_A$ was studied by Shahir M and Naz M, [7] and definedfewnotionsoftopologicalspace. The properties of soft topological spaces werestudiedbythe authors [3-8].

Let U be a universal set and E set of parameters; $P(U)$ the power set of U. The collection of all soft sets over Uand E denoted by $S(U)$. If $A \subseteq E$, then the pair $(F, A)$ is said to be the soft set over U and it is denoted by $F_A$ for $F_A$ where F is a mapping of A onto $P(U)$. Note that for $e \in A, F(e) = F_A$ [2]. Let $F_B$ and $G_C$ be the soft sets in a universe set $U$, $B \subseteq E$. Soft subset of $G_C$ represented $F_B \subseteq G_C$, when (i) $B \subseteq C$ and (ii) $\forall \ e \in B, F(e)=G(e)$. The relative complement of a soft set $F_A$ denoted by $F_A^c$, is being represented by the function $f_A^c(e) = f_A^c(F_A) = U - f_A(e)$ $e \in E$. In other words $(F_A^c)^c = F_A, F_B^c = F_B$ and $F_B^c = F_B$ [1]. Let $F_A \in S(U)$. A soft topology on $(F, A)$, which is a group of soft subsets of $F_A$ has the following properties(i)

$F_\emptyset, F_A \in \tau$ . (ii) $F_A \subseteq F_A$ : $i \in 1 \subseteq \Rightarrow U_{i \in 1} F_A_i \in \tau$. (iii) $[F_A_i \subseteq F_A : 1 \leq i \leq n, \ n \in N] \subseteq \tau \Rightarrow \cap_{i=1} F_A_i \in \tau$. The pair $(F, A)$ is known assoft topological spaces [7]. Let $(F_A, \tau)$ be a soft topological space and $F_B$ be a soft set over $F_A$. (i) Soft interior of $F_B$ is the soft set int($F_B$) = $U_{F_C} F_B$ and $F_B \subseteq F_C$.

(ii) Soft closure of $F_B$ is the soft set cl($F_B$) = $C_F$ : $F_B$ is soft closed set and $F_B \subseteq F_B$ [7]. (i) Soft $\alpha$-closed set, when $cl(int(cl(F_B))) \subseteq F_B$ [3]. (ii) Soft $\beta$-closed set (or) Soft semi-pre closed set, when $int(cl(int(F_B))) \subseteq F_B$ [3]. (iii) Soft semi-closed set, when $int(cl(F_B)) \subseteq F_B$ [5]. (iv) Soft pre-closed set, when $cl(int(F_B)) \subseteq F_B$ [6]. (v) Soft regular closed set, when $cl(F_B) = F_B$ [8]. (vi) Soft generalized closed set, when $cl(F_B) \subseteq F_C$ whenever $F_B \subseteq F_C$ and $F_C$ is a soft open set and Soft weakly closed set, if every $cl(F_B) \subseteq F_C$. whenever $F_B \subseteq F_C$ and $F_C$ is a soft semi-open set [8].

II. SOFT SEMI WEAKLY GENERALIZED CLOSED SET AND SOFT SEMI WEAKLY GENERALIZED CLOSED SETS IN THEIR PROPERTIES

Definition 2.1. Let $(F_A, \tau)$ is a soft topological space and $F_B \subseteq F_A$. $F_B$ is identified to be a Soft Semi Weakly Generalized closed set (briefly, soft swg-closed set) if every int($F_B$) $\subseteq F_C$, whenever $F_A \subseteq F_C$ and $F_C$ (soft semi-open set).

Example 2.2. Let $U = \{a, b, c\}, E = \{e_1, e_2, e_3\}, A = \{e_1, e_2\} \subseteq E$.

$F_A = \{\{e_1\}, \{a, b, c\}, \{e_2\}, \{a, b, c\}\}$. $F_1 = \{(e_1, \{b\}), (e_2, \{b\})\}, F_2 = \{(e_1, \{e_2\}), (e_2, \{a, b\})\}, F_3 = \{e_1, \{a, c\}, e_2, \{a, b\}\} F_4 = \{e_1, \{e_2\}\}, (e_2, \{a, b\}\} F_4 = \{(e_1, \{a, c\}\), (e_2, \{a, b\}\)\} F_5 = \{e_1, \{a, c\}\}, (e_2, \{a, b\}\) F_6 = \{(e_1, \{a\}\), (e_2, \{a\}\)\}$F_7 = \{e_1, \{a\}\}$, $F_8 = \{(e_1, \{a\}\), (e_2, \{a\}\)\} F_9 = \{(e_1, \{a\}\), (e_2, \{a\}\)\}$F_10 = \{(e_1, \{a\}\), (e_2, \{a\}\)\}$F_11 = \{(e_1, \{a\}\), (e_2, \{a\}\)\}$F_12 = \{(e_1, \{a\}\), (e_2, \{a\}\)\}$F_13 = \{(e_1, \{a\}\), (e_2, \{a\}\)\}$F_14 = \{(e_1, \{a\}\), (e_2, \{a\}\)\}$F_15 = F_A, F_16 = F_A$. Let $\tau = \{\emptyset, F_A, F_1, F_2, F_3, F_4, F_5, F_6, F_7\}$. Then $(F_A, \tau)$ is a soft topological space. Soft open set are $\{F_A, F_1, F_2, F_3, F_4, F_5, F_6, F_7\}$. Soft closed set are $\{F_A, F_1, F_2, F_3, F_4, F_5, F_6, F_7\}$. Revised Manuscript Received on August 05, 2019.

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Theorem 2.3. All soft closed sets represent soft semi-closed set.

Proof. Let $F_B$ be a soft closed set in $(F_A, \tau)$. Now $int(F_B) \subseteq F_B$ and $F_B \subseteq F_C$ always, if $F_B$ is a soft closed set and soft open set. So, if $F_B \subseteq F_C$, where $F_C$ is a soft semi-open set in $F_A$, then $cl(int(F_B)) \subseteq cl(F_B)$, which implies that $F_B \subseteq cl(F_B)$, which is a contradiction. Therefore, $F_C = F_B$, and therefore, $cl(int(F_B)) = F_B$ contains empty soft semi-closed set. Conversely, Assume $cl(int(F_B)) = F_B$ contains empty soft semi-closed set. Let $F_B \subseteq F_C$, where $F_C$ is a soft semi-open set. Now $F_B \subseteq F_C$, which implies that $F_B$ is a soft closed set. Therefore, $F_B$ is a soft closed set.

Example 2.4. We have following example 2.2. Let $F_B = \{(c_1, \{c_1\}), (e_1, \{c_1\})\}$ and $F_C = \{(e_1, \{c_1\}), (e_2, \{c_1\})\}$, where $F_C$ is a soft semi-open set, then $int(F_B) = F_B$, so, $cl(int(F_B)) \subseteq F_C$, Now $F_B$ is soft semi-closed set but it is not a soft closed set.

Theorem 2.5. All soft $g$-closed sets represent soft semi-closed set.

Proof. Let $F_B$ stands soft $g$-closed set, then $cl(F_B) \subseteq F_B$ whenever $F_B \subseteq F_C$ and remains a soft open set. Each soft open set denotes soft semi-openset. $int(F_B) \subseteq F_B$, if $F_B$ is a soft open set. Now $cl(int(F_B)) \subseteq F_B \subseteq F_C$. Therefore, $F_B$ refers as soft $g$-closed set.

Example 2.6. We have following example 2.2. Let $F_B = \{(e_1, \{c_1\}), (e_2, \{a_1\})\}$ and $F_B = \{(e_1, \{c_1\}), (e_2, \{a_1\})\}$, where $F_C$ is a soft semi-open set and soft semi-closed set. Then $int(F_B) = F_B$, $cl(int(F_B)) = F_B$, and $cl(F_B) \subseteq F_C$. Therefore, $F_B$ be a soft closed set but it is not a soft $g$-closed set.

Theorem 2.7. All soft regular closed set represent soft semi-closed set.

Proof. Let $F_B$ refer to a soft closed regular set, if $cl(int(F_B)) = F_B$. Since $F_B \subseteq F_C$, where $F_C$ remains soft semi-openset. Now $cl(int(F_B)) = F_B \subseteq F_C$. Therefore, $F_B$ refers as a soft semi-closed set.

Example 2.8. We have following example 2.2. Let $F_B = \{(e_1, \{c_1\}), (e_2, \{b_1\})\}$, $F_C = \{(e_1, \{c_1\}), (e_2, \{b_1\})\}$ and $F_B \subseteq F_C$ where $F_C$ is a soft semi-open set, then $int(F_B) = F_B$, $cl(F_B) \subseteq F_C$. Therefore, $F_B$ refers as a soft semi-closed set but it is not a soft regular closed set.

Theorem 2.9. The intersection of two soft semi-closed and semi-closed set gives another soft semi-closed set.

Proof. Let $F_B$ and $F_C$ stay any two soft semi-closed sets and $F_D$ remains any soft semi-open set comprise $F_B$ and $F_C$. By definition of soft semi-closed set, $cl(F_B) \subseteq F_B$ and $cl(F_C) \subseteq F_C$. Hence $cl(F_B) \cap cl(F_C) \subseteq F_B \cap F_C$. Therefore, $F_B \cap F_C$ refers as sosoft semi-closed set. $\Box$

Remark 2.10. If $F_B$ and $F_C$ are soft semi-closed sets then $F_B \cup F_C$ does not need be a soft semi-closed set.

Example 2.11. From 2.2. Let $F_B = \{(e_1, \{c_1\})\}$ and $F_C = \{(e_2, \{b_1\})\}$, then $F_B$ and $F_C$ are soft semi-closed sets. $F_B \cup F_C = \{(e_1, \{c_1\}), (e_2, \{b_1\})\}$ is not a soft semi-closed set.

Theorem 2.12. Let $(F_A, \tau)$ be a soft topological space. Then a soft subset $F_B$ of $F_A$ is soft closed set in $(F_A, \tau)$, if $cl(F_B) = F_B$ and $cl(F_B) = F_B$ contains empty soft semi-closed set.

Proof. Let $F_B$ be a non empty soft semi-closed set subset $of(F_B) = F_B$. Now $F_B \subseteq F_B$, which implies that $F_B \subseteq cl(int(F_B))$, since $cl(int(F_B)) = F_B$, $F_B = cl(int(F_B))$. Then $F_B \subseteq cl(int(F_B))$. Now $F_B \subseteq cl(int(F_B))$ which implies that $F_B \subseteq F_B$. Hence, $F_B$ proved as soft $g$-closed set in $F_A$. $\Box$
Theorem 2.16. Let \((F_A, \tau)\) considered as soft topological space, then a soft subset \(F_B\) inside \(F_A\) is soft nowhere dense, then \(F_B\) is soft closed set in \(F_A\).

Proof. When a soft subset \(F_B\) of \(F_A\) is soft nowhere dense, then \(\text{int}(F_B) = \emptyset\). Let \(F_B \subseteq F_C\) where \(F_C\) represent soft semi-open set, which denotes that \(c(\text{int}(F_B)) = c(\emptyset) = F_\emptyset \subseteq F_C\). Therefore, \(F_B\) is proved as soft closed set in \(F_A\).

Theorem 2.17. Let \((F_A, \tau)\) be a soft topological space. For every soft subset \(F_A \subseteq F_A\), either \(F_A\) refers as soft semi-closed set, or \(F_A\) \(\subseteq\) soft closed set in \((F_A, \tau)\).

Proof. Assume that \(F_A\) does not belong to soft semi-closed set of \((F_A, \tau)\). Then \(F_A\) is a soft semi-open set and the only soft semi-open set containing \(F_A\) itself. Therefore, \(c(\text{int}(F_A)) \subseteq F_A\) and so \(F_A\) proved as soft closed set in \((F_A, \tau)\).

Theorem 2.18. If \(F_B\) state soft semi-open set and soft closed set in \(F_A\), both these constitute soft generalized closed set.

Proof. Let \(F_B\) be both soft closed set and soft semi-open set. Let \(F_B \subseteq F_C\) where \(F_C\) is soft open set. Then by definition of soft closed set, \(c(\text{int}(F_B)) \subseteq F_C\). Since \(F_B\) is soft semi-open set, \(c(\text{int}(F_B)) \subseteq F_C\). Which implies that \(c(\text{int}(F_B)) \subseteq F_C\), where \(F_C\) shows as open set. Hence \(F_B\) proved as soft generalized closed set.

Theorem 2.19. Let \(F_B\) be soft semi-open set and \(F_B\) is soft closed set. Then \(F_B \cap F_B\) is soft semi-open set.

Proof. Let \(F_B\) be a soft semi-open set and \(F_B\) is soft closed set. Then \(F_B \cap F_B\) is soft semi-open set. Now, show that \(F_B \cap F_B\) is soft semi-open set. Let \(F_B \cap F_B\) be soft semi-open set. Since \(F_B\) is soft open set, \(F_B \cap F_B\) is soft closed set. Therefore, \(F_B \cap F_B\) is soft semi-open set in \(F_A\).

Theorem 2.20. Let \((F_A, \tau)\) be soft topological space. If \(F_B\) is soft semi-open set and \(F_B \subseteq F_C\), then \(F_B\) is soft semi-closed set. Let \(F_B \cap F_B\) be soft semi-open set where \(F_C\) is soft open set. Then by definition of soft semi-open set, \(c(\text{int}(F_B)) \subseteq F_C\). Since \(F_B\) is soft semi-closed set, \(F_B \cap F_B\) is soft closed set. Therefore, \(F_B \cap F_B\) is soft semi-closed set.

Theorem 3.2. A soft subset \(F_B\) of \((F_A, \tau)\) is soft regular open set, then it is soft open set.

Proof. Let \(F_B\) be a soft regular open set in \(\tau\). Then \(F_B\) is soft regular closed set. It implies \(F_B = c(\text{int}(F_B))\). Since \(F_B\) is soft semi-open set in \(\tau\). It implies that \(c(\text{int}(F_B)) \subseteq F_B\). Since \(F_B\) represents soft semi-open set. Therefore, \(F_B\) proved as soft semi-open set.

Theorem 3.3. A soft subset \(F_B\) of a soft topological space \((F_A, \tau)\) is soft semi-open set iff \(F_B \subseteq \text{int}(F_B)\) when \(F_C \subseteq F_B\) and \(F_C\) is a soft semi-closed set.

Proof. Assume that \(F_B\) is a soft semi-open set in \(\tau\). Then \(F_B\) is soft semi-closed set in \((F_A, \tau)\). Then \(c(F_B)\) contains \(F_B\). Then \(F_B\) is soft semi-open set containing \(F_B\), i.e., \(F_B \subseteq F_C\). It implies \(c(F_B) \subseteq c(F_C)\). Since \(F_B\) is soft semi-closed set. Conversely, suppose \(F_B \subseteq \text{int}(F_B)\) when \(F_C \subseteq F_B\) and \(F_C\) is a soft semi-closed set. Then \(F_B\) is soft semi-closed set containing \(F_B\) and \(F_B \subseteq \text{int}(F_B)\). It follows \(F_B \subseteq \text{int}(F_B)\). \(F_B\) is soft semi-open set and so, \(F_B\) proved as soft semi-open set.

III. SOFT SEMI WEAKLY GENERALIZED OPEN SET AND SOFT SEMI WEAKLY GENERALIZED OPEN SETS IN THEIR PROPERTIES

Definition 3.1. A soft subset \(F_B\) of \((F_A, \tau)\) refers as soft semi weakly generalized open set (briefly, soft semi-open set), if its compliment is soft closed set in \((F_A, \tau)\).

Theorem 3.4. Let \(F_B\) be a soft semi-open set in \(F_A\), then \(F_B\) is soft semi-open set.

Proof. Let \(F_B\) be a soft semi-open set in \(F_A\), then \(F_B\) is soft semi-open set. Since \(F_B\) is soft semi-open set, \(F_B \subseteq F_C\) and \(F_B \cap F_B\) is soft semi-closed set. Therefore, \(F_B\) is soft semi-open set.
IV. SOFT SWG-CLOSURE AND SOFT SWG-INTERIOR & RESULTS

**Definition 4.1.** Let \((F_a, A)\) be soft topological space, \(F_a \subseteq A\). The soft \(swg\)-closure of \(F_a\) (briefly, \(swg-cl(F_a)\)) is the intersection of all soft \(swg\)-closed subsets containing \(F_a\). In symbols, \(swg-cl(F_a) = \{ F_c; F_a \subseteq F_c \text{ and } F_c \text{ is a soft } swg\text{-closed set in } A \} \).

**Theorem 4.2.** While any soft subset \(F_b\) of soft topological space \(F_a, F_b \supseteq \text{swg-cl} (F_b) \subseteq \text{cl} (\text{int}(F_b)) \).

**Example 4.3.** We have following example 2.2., \(F_b = ([e_2, \{ c \}], (e_2, \{ a, b \}))\) then \(\text{swg-cl}(F_b) = \{ ([e_1, \{ b, c \}], (e_2, \{ a, b \}) \} \text{ and } \text{int}(F_b) = F_2, \text{ cl}(\text{int}(F_b)) = F_2 \). Therefore, \(F_b \supseteq \text{swg-cl}(F_b) \subseteq \text{cl}(\text{int}(F_b)) \) in \(A \).

**Theorem 4.4.** The soft \(swg\)-closure operator is soft Kuratowski closure operator on \(F_a\).

**Proof.** (i) \(\text{swg-cl}(F_b) = F_b \).

(ii) \(F_b \supseteq \text{swg-cl}(F_b)\) by Theorem 4.2.

(iii) Let \(F_b \subseteq F_c \subseteq A\); and \(F_b\) belongs to soft \(swg\)-closed set in \(F_a\), then \(F_c \subseteq A\) and by Definition 3.1. \(\text{swg-cl}(F_b) \subseteq \text{cl}(\text{int}(F_b)) \subseteq \text{cl}(F_c)\) for each \(i = 1, 2\). Therefore, \(\text{swg-cl}(F_b) \subseteq \text{cl}(F_c)\).

(iv) By the reverse insertion, let \(F_a \in \text{swg-cl}(F_b) \subseteq A\) and suppose that \(F_a \in \text{swg-cl}(F_b) \subseteq A\).

Then there exist soft \(swg\)-closed sets \(F_{C_1}\) and \(F_{C_2}\) with \(F_b \subseteq F_{C_1} \subseteq F_{C_2} \subseteq F_a \subseteq C_1 \subseteq C_2\). We have \(F_b \subseteq \text{swg-cl}(F_{C_1}) \subseteq F_{C_2} \subseteq C_1 \subseteq C_2\) and \(F_a \in \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_{C_2})\). Hence \(\text{swg-cl}(F_b) \subseteq \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_{C_2}) \subseteq \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_{C_2}) \subseteq \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_{C_2}) \subseteq \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_{C_2}) \subseteq \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_{C_2}) \).

(v) Let \(F_b \subseteq F_{C_1} \subseteq A\) and \(F_{C_1}\) is soft \(swg\)-closed set within \(F_a\). Then by Definition 3.1. \(\text{swg-cl}(F_b) \subseteq \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_{C_2}) \subseteq \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_b)\).

(vi) \(\text{swg-cl}(F_b) \subseteq \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_{C_2}) \subseteq \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_{C_2}) \subseteq \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_{C_2}) \subseteq \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_{C_2}) \subseteq \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_{C_2}) \).

**Theorem 4.5.** While any soft subset \(F_b\) of soft topological space in \(F_a\),

(i) \(\text{swg-cl}(F_b)\) is smallest soft \(swg\)-closed set containing \(F_b\).

(ii) \(F_b \subseteq \text{swg-cl}(F_b) \subseteq \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_{C_2}) \subseteq \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_{C_2}) \subseteq \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_{C_2}) \subseteq \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_{C_2}) \subseteq \text{swg-cl}(F_{C_1}) \subseteq \text{swg-cl}(F_{C_2}) \).

**Proposition 4.6.** While two soft subsets \(F_{A_1}\) and \(F_{A_2}\) of soft topological space in \(F_a\),

(i) \(F_{A_1} \subseteq F_{A_2}\), then \(\text{swg-cl}(F_{A_1}) \subseteq \text{swg-cl}(F_{A_2})\).

(ii) \(\text{swg-cl}(F_{A_1}) \subseteq \text{swg-cl}(F_{B_1}) \subseteq \text{swg-cl}(F_{B_2}) \subseteq \text{swg-cl}(F_{B_1}) \subseteq \text{swg-cl}(F_{B_2}) \subseteq \text{swg-cl}(F_{B_1}) \subseteq \text{swg-cl}(F_{B_2}) \subseteq \text{swg-cl}(F_{B_1}) \subseteq \text{swg-cl}(F_{B_2}) \subseteq \text{swg-cl}(F_{B_1}) \subseteq \text{swg-cl}(F_{B_2}) \subseteq \text{swg-cl}(F_{B_1}) \subseteq \text{swg-cl}(F_{B_2}) \subseteq \text{swg-cl}(F_{B_1}) \subseteq \text{swg-cl}(F_{B_2}) \).
every soft swg-open set $F_D$ containing $f_A$ is such that $F_D \not\subseteq F_B$. This implies by definition 4.8., $f_A \notin \text{swg-int}(F_B)$ implies $f_A \in (\text{swg-int}(F_B))^c$ and $\text{swg-cl}(F_B)^c \subseteq (\text{swg-int}(F_B))^c$. Thus $(\text{swg-int}(F_B))^c = \text{swg-cl}(F_B)^c$. (ii) and (iii) follows from (i).

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