On πgβ–Closed Sets and Mappings in Neutrosophic Topological Spaces

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Abstract— *The real life situations always include indeterminacy. The Mathematical tool which is well known to deal with indeterminacy is Neutrosophy. The notion of Neutrosophic set is generally referred as the generalization of Intuitionistic fuzzy set. The Purpose of this article is to define the new class of sets called πgβ-closed sets in Neutrosophic topological spaces. The properties and characterizations of πgβ-closed sets are discussed and its relationships with other Neutrosophic sets are studied. Further we define πgβ –closed mappings and πgβ –open sets and some of its properties are touched upon.*

Index Terms—Neutrosophic πgβ--Closed Set, Neutrosophic πgβ--Open Set, Neutrosophic πgβ—Closed mappings.

I. INTRODUCTION

The idea of fuzzy sets was put forth by L.A. Zadeh [16] which deals with membership. The intuitionistic fuzzy sets introduced by K.Atanassova [2] deals with membership and non-membership. Samrandache[3] extended these ideas and introduced a new concept called Neutrosophic set that studies membership, non-membership and indeterminacy. The concept of Neutrosophic topological spaces was introduced by A.A Salama and S.A. Albowi [9]. In the literature, numerous authors studied about β-open sets . S.Tahiliani[14] introduced and studied πgβ-closed sets in topological spaces. The concept of πgβ-closed sets was studied under intuitionistic fuzzy topological spaces by T.Jenitha Premalatha and S.Jothimani[6] The concepts of πgα-closed mappings were studied by N.Semivasagan, O.Ravi and S. Satheesh Kanna[12].in Intuitionistic fuzzy topological spaces. In this article we define Neutrosophic π-generalized beta closed set and Neutrosophic π-generalized beta closed mappings and investigate their properties.

II. PRELIMINARIES

Definition 2.1[9] :

Let X be a nonempty fixed set. A Neutrosophic set (NS)A in X is an object having the form A ={<x,μ_A(x), $\sigma_A(x)$, γ_A(x) >:xϵ X}, where μ_A(x) denotes the degree of membership function, σ_A(x) denotes the degree of indeterminacy and γ_A(x) denotes the degree of non-membership respectively of each element x ϵ X to the set A.

Remark 2.1[9]:

A NS A={<x,μ_A(x) $\sigma_A(x)$, γ_A(x)>: x ϵ X} can be identified by an ordered triple<μ_A, σ_A, γ_A> in $]^-0,1^+[$ on X.

Definition 2.2 [9]

0_N ={<x,0,0,0>: xϵ X}, 1_N ={<x,1,1,1>: xϵ X} defines the NS 0_N and 1_N respectively.

Definition 2.3 [9] :

Let A=<μ_A, σ_A, γ_A> be an NS on X, the complement of the set A (C(A)) is given by :

C(A)= ={<x ,γ_A(x), $1-\sigma(x)$, μ_A(x)>:xϵ X}

Definition 2.4 [9] :

Let X be a non empty set, and NSs A and B be given by A={<x,μ_A(x), $\sigma_A(x)$, γ_A(x)>:xϵX},B=<x,μ_B(x),σ_B(x),γ_B(x) > : xϵ X},then (A⊆ B). ⇔ μ_A(x) ≤ μ_B(x) and γ_A(x) ≥γ_B(x) and σ_A(x) ≤ σ_B(x) for every x ϵ X.

Definition 2.5 [9] :

Let x be a non empty set, and A={<x,μ_A(x), $\sigma_A(x)$, γ_A(x)>:xϵX},B=<x,μ_B(x),σ_B(x), γ_B(x)> : xϵ X} be NSs. Then

(1) A ∩ B may be defined as A∩B=<x, μ_A(x)∧ μ_B(x), $\sigma_A(x)$∧$\sigma_B(x)$, γ_A(x) ∨ γ_B(x)>

(2) A∪ B may be defined as A∪B =<x, μ_A(x)∨μ_B(x), $\sigma_A(x)$∨$\sigma_B(x)$, γ_A(x)∧γ_B(x)>

Definition 2.6 [9] :

Let A and B be Neutrosophic sets then

A| B may be defined as A|B =<x, μ_A∧γ_B, $\sigma_{A.}\sigma_{B,}\gamma_A$∨$\mu_A$>

Definition 2.7[9] :

A Neutrosophic topology (NT) in a non-empty set X is a family τ of Neutrosophic subsets in X satisfying the following axioms

(NT_1) 0_N, 1_N ∈ τ,

(NT_2) G_1∩ G_2 ∈ τ for any $G_{1,}$ G_2 ϵ τ

(NT_3) ∪ G_i ∈ τ. ∀{ G_i: $i \in J$} ⊆ τ.

Here the pair(X,τ) is called a Neutrosophic topological space (NTS) and any NS in τ is known as a Neutrosophic open set (NOS in X. The complement $A^C$of a NOS A in NTS is called a Neutrosophic closed set (NCS) in X.

Definition 2.8 [9] :

Let (X, τ) be NTS and A ={<x,μ_A(x), $\sigma_A(x)$, γ_A(x)>:xϵ X}, be NS in X. Then Neutrosophic interior and Neutrosophic closer are defined by:

Nint(A)=∪{U,U is a NOS in X & U ⊆ A}

Ncl (A)= ∩{V,V is a NCS in X & A⊆ V}

Definition 2.9 [4] :

Let A ={<x,μ_A(x), $\sigma_A(x)$, γ_A(x)>:xϵ X} be a NS in a NTS (X, τ) , then it is called a Neutrosophic semi closed set (NSCS), if Nint(Ncl(A)) ⊆A. and A is said to be a Neutrosophic semi open set (NSOS) if A⊆Ncl(Nint(A)).

Definition 2.10 [15] :

Consider a NS A of a NTS (X, τ) is an Neutrosophic pre closed set (NPCS) if Ncl(Nint(A)) ⊆ A, (resp. Neutrosophic

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(pre open set (NPOS), β-open set (NβOS), β-closed set (NβCS) if (A⊆Nint(Ncl(A)), A⊆Nint(Ncl(A)), Nint(Ncl(Nint(A))) ⊆A resp.)

Definition 2.11 [1] :

Let A be a NS in NTS, then A is a

(i) Neutrosophic α-closed set (NαCS) if Ncl (Nint(Ncl (A))) ⊆ A.

(ii) Neutrosophic regular open set (NROS) if A=Nint (Ncl(A)).

Definition 2.12[7]:

Consider a NS A of a NTS, then A is a Neutrosophic generalized closed set (NGCS) if Ncl(A) ⊆ U wherein A ⊆U & U is a NOS in NTS. The set of all NβCSs (resp. NβCSs) of a NTS (X, τ) is denoted by NβC(x) (resp. NβO(x)).

Definition 2.13 [10]:

Consider a NS A in NTS .Then it is said to be a Neutrosophic *w* – closed (NWCS) if Ncl (A) ⊆ U wherein A ⊆ U & U is a NSO and a NS A of a NTS is said to be a Neutrosophic *w* – open (NWOS) if A^c = NWCS.

Definition 2.14 [4]:

Consider a NS A in NTS. Then Neutrosophic semi closure of A (scl(A) (resp. Neutrosophic semi interior of A (sint(A))is defined as Nscl(A) = ∩{K/K is a NSCS in X and A ⊆K. (resp. Nsint(A) = ∪{K / K is a NSOS and K ⊆ A.)

Definition 2.15 [11]

Consider a NS A in NTS. Then A is a Neutrosophic generalized semi closed set (NGSCS) if Nscl(A) ⊆ U whenever A ⊆ U and U is a NOs in (X, τ).

Definition 2.16 [8]

Consider a NS A in NTS. Then A is a Neutrosophic semi pre open set (NSPOS) if we can find a NPOS B with B⊆A⊆cl(B) and A is a Neutrosophic semi pre closed set (NSPCS) if we can find a NPCS B with int (B)⊆A⊆B.

III. NEUTROSOPHIC Π - GENERALIZED BETA CLOSED SETS

In this chapter the notion of NπgβCS is putforth and their properties are investigated.

Definition 3.1

Consider a NS A in NTS. The Neutrosophic beta interior & Neutrosophic beta closure of A are defined as

Nβint(A)= ∪ {G,G is a NβOS in X and G ⊆ A}

Nβcl(A)=∩{K,K is a NβOS in X and A ⊆ K}

Remark 3.1

Consider a NS A in NTS, then

(1) Nβcl(A)=A∪Nint(Ncl(Nint(A))),

(2) Nβint(A)=A∩Ncl(Nint(Ncl(A))).

Definition 3. 2

Consider a NS A in NTS. Then it is a Neutrosophic generalized beta closed set (NGβCS) if Nβcl(A)⊆U whenever A ⊆ U and U is a NOS.

Definition 3.3:

Consider a NS A in NTS. Then it is a Neutrosophic π-open if A =∪ {G,G is a NROS in NTS}

Remark 3.2

Each NOS is NSOS in NTS.

Remark 3.3 .

Let A and B be NROSs, then A∪B is NOS in NTS.

Remark 3. 4.

Each NπOS is NOS in NTS.

Definition 3.4:

Let A be a NS. Then A is aNeutrosophic π generalized beta closed sets (NπgβCS) in NTS if Nβcl(A)⊆U wherein A ⊆ U & U is a NπOS in NTS . The set of all NπgβCSs of a NTS (X, τ) is given as NπgβC.

Example 3.1:

Let X = {x1, x2}, τ = {0, G, 1} be a NT on X , G = < x ,(0.4, 0.5, 0.7), (0.8, 0.4, 0.6) > Then the NS A = < x , (0.3, 0.2, 0.7), (0.6, 0.2, 0.8) > is a NπgβCS in X .

Theorem 3.1:

Each NCS is a NπgβCS.

Proof. Consider a NCS A and assume A ⊆ U & U is a NπOS in NTS. Obviously Nβcl(A)⊆Nscl(A)⊆Ncl(A) and A is a NCS ,Nβcl(A)⊆Ncl(A)=A⊆U. Thus A is a NπgβCS. However the reverse implication is not true.

Example 3.2:

Let X = {x_1 , x_2 } , τ = {0 , G , 1 } be a NT on X , G = < x ,(0.2, 0.4, 0.6), (0.3, 0.6, 0.7) > Then the NS A = <x, (0.1, 0.3, 0.8), (0.2, 0.5, 0.7)> is a NπgβCS but not NCS.

Theorem 3.2:

Each NSCS is a NπgβCS .

Proof. Consider a NSCS A in NTS. Suppose A⊆U & U is a NπOS in (X,τ). By assumption, Nβcl(A)⊆Nscl(A)⊆A ⊆U. Thus Nβcl(A)⊆U. Hence A is a NπgβCS. However the reverse implication is not true.

Example 3.3:

Let X = {x1, x2 } ,τ = {0 , G , 1 }, G =< x ,(0.2, 0.4, 0.6), (0.3, 0.6, 0.7) >.Then the NS A =< x ,(0.1, 0.3, 0.8), (0.2, 0.5, 0.7) > is a NπgβCS but not NSCS in X , since Nint(Ncl(A)) = (0.2, 0.4, 0.2), (0.3, 0, 0.7) ⊄ A.

Theorem 3.3:

Each NαCS is a NπgβCS.

Proof. Consider a NαCS A in NTS. assume A ⊆ U & U is a NπOS in NTS. By assumption, Ncl(Nint(Ncl(A)))⊆A Thus, Nint(Ncl(A))⊆A. Also,Nint(A)⊆A,Ncl(Nint(A)⊆ Ncl(A),hence Nint(Ncl(Nint(A)))⊆Nint(Ncl(A))⊆A, which implies Nβcl(A)⊆A⊆U.Hence A is a NπgβCS. However the reverse implication is not true.

Example 3.4:

Let X = {x1 , x2 } , τ = {0 , G1 , G2 , 1 } be a NT on X , where G1 =< x ,(0.3, 0.5, 0.7), (0.2, 0.4, 0.6) > and G2 =< x ,(0.2, 0.4, 0.8), (0.1, 0.3, 0.7) > then NS A =< x ,(0.3, 0.4,0.7), (0.2, 0.3, 0.8) >, is a Nπgβ CS but not NαCS.

Theorem 3.4:

Each NPCS is a NπgβCS.

Proof. Consider NPCS A in X .suppose A ⊆ U and U is a NπOS. By given condition, Ncl(Nint(A))⊆A. Therefore Nint(Ncl(Nint(A)))⊆Nint(A)⊆A.which implies Nβcl(A)⊆A⊆U. Therefore A is an NπgβCS. However the reverse implication is not true.

Example 3.5:

Let X = {x1 , x2 } & τ = {0 , G1 , G2 , 1 } be a NT , G1 =< x ,(0.2, 0.7, 0.3), (0.4, 0.2, 0.8) > and G2 =< x ,(0.3, 0.7, 0.1), (0.8, 0.2, 0.8) > Then the NS A = G_1 ,is a Nπgβ CS but not NPCS since Ncl(Nint(A)) =< x ,(0.3, 1 0.3), (0.8, 0.8 0.4) > ⊄ G1 or A.

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Theorem 3.5:

Each NβCS is a NπgβCS .

Proof. Consider a NβCS A in NTS. By assumption ,Nβcl(A)⊆ A wherein A ⊆ U & U is NπOS. By(Remark 3.3) Nβcl(A)⊆U wherein A ⊆ U & U is NOS.

Hence A is a Nπgβ. However the is not true.

Example 3.6:

Let X = {x} , τ = {0, G , 1 } be a NT , G =< x ,(0.5, 0.2, 0.6) > then A =< x ,(0.5, 0.1, 0.7), > is a NπgβCS but not NβCS. Since Nint(Ncl(Nint(A)) ⊄ A.

Theorem 3.6:

Each NRCS is a NπgβCS.

Proof. Let A be an NRCS. By defn. A=Ncl(Nint(A)).This implies Ncl(A)=Ncl(Nint(A)). Thus Ncl(A= A. Hence A is an NCS in NTS . By Thm 3.1, A is a NπgβCS in X. The reverse implication need not hold.

Example 3.7:

Let X = {x1 , x2 }, τ = {0 ,G ,1 } be a NT on X , where G =< x ,(0.5, 0.4, 0.3), (0.6, 0.2, 0.7) > then A =< x ,(0.3, 0.2, 0.3), (0.5, 0.1, 0.7) > is a NπgβCS but not NRCS as Ncl(Nint(A)) = < x, (0.3, 0.6, 0), (0.7, 0.8, 0.6) >≠ A.

Theorem 3.7:

Each NWCS is a NπgβCS.

Proof. Consider a NWCS A . Suppose A ⊆ U and U is a NπOS. By our assumption Ncl(A)⊆U wherein A⊆U , since Nβcl(A)⊆Ncl(A) & A is a NWCS, Nβcl(A)⊆Ncl(A)⊆U, wherein A⊆U & U is NSO. Hence A is a NπgβC. The reverse implication need not hold.

Example 3.8:

Let X = {x1 , x2 } , τ = {0 , G , 1 } be a NT , G =< x ,(0.6, 0.2, 0.4), (0.3, 0.1, 0.6) > then A =< x ,(0.5, 0.2, 0.6), (0.2, 0.1, 0.7) > is a NπgβCS but not NWCS since Ncl(A)= <x, (1, 0.8, 0.6), (0.6, 0.9, 0.3) ⊄ G .

Theorem 3.8:

Each NGCS is a NπgβCS.

Proof. Consider a NGCS A in NTS. Suppose A ⊆ U & U is a NπOS. By given, Ncl(A)⊆ U, wherein A ⊆ U, since Nβcl(A)⊆ Nscl(A)⊆Ncl(A)⊆ U , wherein A ⊆ U. Then A is a NπgβCS in NTS. However the converse need not hold.

Example 3.9:

Let X = {x1, x2 } , τ = {0 , G , 1 } be a NT,

G =< x ,(0.3, 0.6, 0.4), (0.2, 0.5, 0.3) > then A =< x ,(0.2, 0.5, 0.6), (0.2, 0.4, 0.4) > is a NπgβCS but not NGCS since Ncl(A)= <x, (0.4, 1, 0.3), (0.3, 0.5, 0.2) ⊄ G.

Theorem 3.10:

Each NαGCS is a Nπgβ CS.

Proof. Consider a NαGCS A in NTS. Suppose A ⊆ U and U is a NπOS. By given, Ncl(Nint(Ncl (A)))⊆U. Therefore Nint(Ncl(A))⊆U. Also NintA⊆A,Ncl(Nint (A))⊆Ncl(A)thusNint(Ncl(Nint(A)))⊆Nint(Ncl(A))⊆U which shows Nβcl(A)⊆U, wherein A ⊆ U. Thus A is a NπgβCS. However the converse need not hold.

Example 3.10 : The NS set A as defined in above Example 3.9 is NπgβCS but not NαGCS

Theorem 3.11:

Each NGPCS is a NπgβCS.

Proof. Consider a NGPCS A in X .Suppose A ⊆ U , U ϵ NπOS. By given and (Remark 3.3) Ncl(Nint(A))⊆U. Thus Nint(Ncl(Nint(A)))⊆Nint(U)⊆ U .Which shows Nβcl(A)⊆U, wherein A⊆U & U is NOS. Thus A is a NπgβCS in NTS. However the converse need not hold.

Example 3.11:

Let X = {x1 , x2 },τ = {0 , G , 1 } be a NT, where G =< x ,(0.2, 0.5, 0.4), (0.4, 0.6, 0.5) > then A =< x ,(0.2, 0.4, 0.8), (0.2, 0.4, 0.6)> is a NπgβCS but not NGPCS as Ncl(Nint(A)= <x, (1, 0.5, 0.2), (0.5, 0.4, 0.4) ⊄ G .

Theorem 3.12:

Each NGβCS is a NπgβCS.

Proof. Consider a NGβCS A. By assumption,

Nβcl(A)⊆U, wherein A ⊆ U & U is NπOS. By assumption and (Remark 3.3) Nβcl(A)⊆ Nscl(A) ⊆ U, wherein A ⊆ U & U is NOS. Then A is a Nπgβ CS. However the reverse implications does not hold.

Example 3.12:

Let X = {x },τ = {0 , G1 , G2, 1 } be a NT, where G1=< x ,(0.5, 0.2, 0.6) > G2==< x ,(0.3, 0.1, 0.6) > then A =< x ,(0.2, 0.1, 0.7)> is a NπgβCS but not NGβCS

Remark 3.5:

In general the intersection of any two NπgβCS is not a NπgβCS.

Example 3.13:

Let X = {x1 , x2 } , τ = {0 , G1 , G2, G3 , G4 G5 , 1 } be a NT on X , G1 =< x ,(0.1, 0.6, 0.3), (0.2, 0.5, 0.3) > and G2 =< x ,(0.1, 0.7, 0.2), (0.1, 0.6, 0.3) > G3 =< x ,(0.1, 0.7, 0.2), (0.2, 05, 0.3) >, G4 =< x ,(0.1, 0.6, 0.3), (0.1, 0.6, 0.3) > and G5 =< x ,(0.3, 0.5, 0.2), (0.3, 0.4, 0.3) > then NS =< x ,(0.1, 0.6,0.3), (0.2, 05, 0.3) >, B=<x, (0.3, 0.5, 0.2), (0.1, 0.6, 0.3)> are NπgβCS A∩B is not a NπgβCS.

Theorem 3.13:

Consider a NTS X. For each A ∈NπgβC(X) and for each B ∈NS(X), A⊆B⊆Nβcl(A) => B ∈Nπgβ C(X).

Proof. Consider the set B contained in U .suppose U is a NπOS. Given A ⊆ B , A ⊆ U and A is a NπgCS, Nβcl(A)⊆ U , wherein A ⊆ U , By assumption, B ⊆Nβcl(A),Nβcl(B)⊆Nβcl(A) ⊆ U . Therefore Nβcl(B) ⊆ U.Hence B is an NπgβCS.

Theorem 3.14:

If A is a NπOS, Nπgβ CS in NTS, then A is a NβCS in NTS.

Proof. Consider a NπOS A. as we know A ⊆ A, by given Nβcl(A)⊆ A. since A ⊆β Ncl(A). Thus Nβcl(A) = A. Therefore A is a NβCS.

Theorem 3.15:

Consider a NTS X. If NS A is NπOS and NCS of NTS, then the following are equivalent:

(a) A is NGCS in NTS

A is NπgβCS in NTS.

Proof. (a) ⇒ (b): Consider a NGCS A in X . By Thm 3.8, A is NπgβCS.

(b)⇒ (a): Consider a NπgβCS A. Then Nβcl(A)⊆U wherein A⊆U & U is NπOS in X, => Nβcl(A)⊆Ncl(A)⊆U, wherein A⊆U , because A is NπOS & NCS, A is NGCS.

Definition 3.5:

The Neutrosophic π-kernel (Nπ-ker (A)) =∩{O,O is NπOS in X and A⊆ O}

Remark 3.6:

Let A⊆ X ,then A is Nπgβ-Closed if Nβcl(A)⊆Nπ-ker(A).

Theorem 3.16:

Let A ⊆X. Then it is NπgβCS iff Nβcl(A)⊆ Nπ−ker(A).

Proof. By given condition A is NπgβCS, Nβcl(A)⊆ A for arbitrary NπOS U such that A⊆U.Thus Nβcl(A)⊆ Nπ−ker(A). Conversely, let U be any Nπos with A ⊆ U . By assumption, Nβcl(A)⊆ Nπ−ker(A) ⊆ U .Then A is NπgβCS.

Theorem 3.17:

If NS P is NπOS & NπgβCS, then it is β-closed.

Proof. Since P is NπOS and NπgβCS, Nβcl(P)⊂P, but P ⊂ Nβcl(P) Hence, P is β -closed.

Theorem 3.18:

Consider a NπgβCS A in NTS. Then Nβcl(A)\A does not contain any nonempty NπCS.

Proof. Consider a non-empty NπCS B of Ncl(A)\A. hence A ⊂ X \B , such that A is NπgβCS & X \A is NπOS. Therefore Nβcl(A) ⊂ X \A, or, B⊂ X \Nβcl(A). Because B ⊂ Ncl(A), this contradicts the given condition.

Corollary 3.1:

Consider a NπgβCS P in NTS. Then P is NβCS iff

Nβ Ncl(P)|P is Nπ-Closed.

Proof. Necessity: Consider a NπgβCS P. By given condition Nβcl(P)=P. we have Nβcl(P)\P = φ which is π-Closed.

Sufficiency: Assume that Nβcl(P)\P is NπCS. Then by Theorem 3.18, Nβcl(P)\P=Nπ, i.e., βNcl(P)=P. Thus, P is βCS.

IV. NEUTROSOPHIC Π - GENERALIZED BETA OPEN SETS

In this chapter Neutrosophic π generalized beta open sets are defined and its properties are analysed.

Definition 4.1:

A Neutrosophic π- generalized beta open sets (Nπgβ OS) in (X, τ) if its complement Ac is a NπgβCS in NTS. The set of all NπgβOSs of a NTS is denoted by NπgβO(X).

Example 4.1:

Let X = {x1 , x2 }, τ = {0 , G , 1 } is a NT , G =< x ,(0.3, 0.5, 0.2), (0.4, 0.6, 0.5) > then A =< x ,(0.6, 0.7, 0.1), (0.6, 0.6, 0.3) > is a N πgβOS.

Theorem 4.1 :

For arbitrary NTS , we have : Every NOS, NSOS, NαOS, NGOS, NPOS, NβOS is a NπgβOS. But the converses are not true in general.

Proof. The proof is obvious

Example 4.2:

Let X = {x1 , x2 }, τ = {0 , G , 1 } be a NT , where G =< x ,(0.3, 0.5, 0.2), (0.4, 0.6, 0.5) > then

A =< x ,(0.6,0.7, 0.1), (0.6, 0.6, 0.3) > is a NπgβOS, but not NOS, since Nint(A)= <x, (0.3, 0.5, 0.2), (0.4, 0.6, 0.5)> ≠ A

Example 4.3:

Let X = {x1 , x2 }, τ = {0 , G ,1 } be NT , G =< x ,(0.3, 0.5, 0.2), (0.4, 0.6, 0.5) > then A =< x ,(0.6, 0.7, 0.1), (0.6, 0.6, 0.3) > is a NπgβOS but not NSOS since Ncl(Nint(A)) = <x, (1, 0.5, 0.3), (0.5, 1, 0,4)> ⊉ A.

Example 4.4:

Let X = {x1 , x2 }, τ = {0 , G , 1 } be a NT , G =< x ,(0.3, 0.5, 0.2), (0.4, 0.6, 0.5) > then A =< x ,(0.6, 0.7, 0.1), (0.6, 0.6, 0.3) > is a N πgβOS but not N αOS, since Ncl(Nint(A)) = <x, (0.3, 0.5, 1), (0.4, 0.6, 0.5)> ⊉ A.

Example 4.5 :

Let X = {x1 , x2 }, τ = {0 , G , 1 }be a NT on X , where G =< x ,(0.3, 0.5, 0.2), (0.4, 0.6, 0.5) > then A =< x ,(0.6, 0.7, 0.1), (0.6, 0.6, 0.3) > is N πgβOS but not NPOS since Nint(Ncl(A)) = <x, (0.3, 0.5, 0.2), (0.4, 0.6, 0.5)> ⊉ A.

Example 4.6:

Let X = {x1 , x2 }, τ = {0 , G , 1 } be a NT on X , where G =< x ,(0.3, 0.5, 0.2), (0.4, 0.6, 0.5) > then A =< x ,(0.6, 0.7, 0.1), (0.6, 0.6, 0.3) > is N πgβOS but not NβOS since Ncl(Nint(Ncl(A)) = <x, (1, 0.5, 0), (0.5, 1, 0)> ⊉ A.

Theorem 4.2:

Consider a NTS X. Suppose A ∈ NπGO. then V ⊆ Ncl(Nint(Ncl(A))) whenever V ⊆ A and V is NCS of NTS.

Proof. By given we have A ∈ NπGO(X). Then Ac is an NπgβCS . Then Nβcl(Ac)⊆ U , wherein Ac ⊆ U and U is a NπOS => Nint(Ncl(Nint(Ac)))⊆U Thus U c ⊆ Ncl(Nint(Ncl(A))) wherein U c ⊆ A, and U c is NCS in X . Replacing U c by V impiles V ⊆ Ncl(Nint(Ncl(A))) wherein V ⊆ A & V is NCS .

Theorem 4.3:

Consider a NTS X. then for all P ∈ Nπgβ O(X) and for all Q ∈ NS(X),Nβint(P) ⊆ Q ⊆ P=> Q ∈ NπgβO(X).

Proof. By given Pc ⊆ Qc⊆ (Nβint(P))c . Let Q c ⊆ U & U be a NπOS. Because Pc ⊆ Q c Pc ⊆ U. But Pc is a Nπgβ CS, β Ncl(Pc) ⊆ U . Further Q c ⊆ (Nβint(P))c =Nβcl(Pc), Thus βN cl(Qc) ⊆ Nspcl(Pc) ⊆ U . Therefore Q c is a NπgβCS=> Q is an NπgβOS .

Remark 4.1:

The Union of two NπgβOS need not be a NπgβOS.

Example 4.7

Define NTS as in example 3.13.Then A= <x, (03, 0.4, 0.1),(0.3,05,0.2) and B=<x,(0.2,0.5,0.3), (0.3,0.4,0.1) are NπgβOS but A U B is not a NπgβOS.

Theorem 4.4 :

A NS A ∈ NTS is a NπgβOS iff G ⊆Nβint(A) whenever G is an NπCS and C ⊆ A.

Proof. Necessity: Assumption that A is a NπgβO. Suppose G is a NπCS , G ⊆ A. Then G c is a NπOS. with A c ⊆ G c. Because G c is a NπGSPCS, Nβcl(A c)⊆ G c Therefore (Nβint(A))c ⊆ G c . Thus G ⊆Nβint(A).

Sufficiency: Assume that A is a NS. Suppose G ⊆Nβint(A) wherein G is a NCS, G ⊆A. Then Ac ⊆ G c and G c is a NπOS. By given condition, (Nβint(A))c ⊆ G c , which implies Nβcl(A c)⊆ G c.

Thus Ac is a NπgβCS. which implies A is an NπgβOS.

Theorem 4.5:

Consider a NTS X. Let P, Q ⊂ X , If Q is NπGO and Nβint(Q)⊂ P then P ∩ Q is NπgβO

Proof. By given condition Q is NπgβO & Nβint(Q)⊂P, Nβint(Q) ⊂P ∩ O ⊂Q, by Theorem 4.3, P ∩ Q is NπgβOS.

Theorem 4.6:

Consider a πgβOS A in a NTS, then S = X Whenever S is Nπ-open and Nβint(A)∪Ac ⊂ S .

Proof. Let S be a NπOS and β Nint(A) ∪ Ac ⊂ S . Now S c ⊂ Nβcl(A c \Ac). Since S c is NπCS and A c is NπgβCS by Theorem 3.18, S c = ϕ .which implies S = X .

Theorem 4.7:

Consider a πgβOS P in NTS , Q be a NαOS. Then P intersection Q is a πgβOS in (X ,τ).

Proof. Consider a arbitrary πCS R of X with

R ⊂ P ∩ Q. Then R ⊂ P and by Thm 4.4, R ⊂ Nβint (P) ={U : U is βOS & U ⊂ P}. trivialy, R ⊂ (U ∩ Q), U is a open set in X contained in P. Because U intersects Q ,Q is a βOS⊂ P ∩ Q for every open set U ⊂ P, R ⊂ Nβint(P ∩ Q), and by Thm 4.5, (P∩ Q) is a πgβ OS.

Theorem 4.8:

Consider a NTS (X , τ). Let P be a subset of X. such that P is a πgβCS, then Nβ cl(P)\P is πgβOS.

Proof. Consider a NπgβCS P. let Q be a πCS. such that Q ⊂Nβcl(P)\P. Then, Q = ϕ. So, Q ⊂ Nβint(Nβcl(P)\P). By Thm 4.4 Nβ cl(P)\P is πgβOS.

Lemma 4.1:

For a arbitrary subset A of a NTS, Nβint(Nβcl(A)\A = ϕ.

Theorem 4.9:

Suppose P⊂ Q ⊂ X, Nβcl(P)\P is πgβOS. Then Nβcl(P)\Q is also πgβOS.

Proof. Assume that Nβcl(P)\P is πgβOS .

let R be a πCS. such that R⊂ β Ncl(P)\Q . Then R⊂Nβcl(P)\P. By Theorem 4.4 and Lemma 4.1, R⊂Nβcl(P)\P= ϕ.Thus, R = ϕ. Therefore R⊂Nβcl(P)\Q.

Theorem 4.10 :

Consider a NTS X. Let P ⊂ X , Nβint(Nβcl(P) − P) = ϕ.

Proof. Consider a NπgβCS P. let Q be a πCS. Q ⊂ Nβcl(P) − P. By Thm 3.18, Q = ϕ, by Remark 4.2,

β Nint(Nβcl(P)− P) = ϕ. Therefore Q⊂(Nβcl(P)−P). Thus Nβcl(P)−P is πgβOS.

V. NEUTROSOPHIC ΠGB CLOSED MAPPINGS & RESULTS

In this chapter we define NπgβC- mappings and discuss some of their properties.

Definition 5.1: Consider a map f : (X, τ) → (Y, σ) . Then it is called a Neutrosophic πgβC-mapping (NπgβC-map) if f(A) is a NπgβCS in (Y, σ) for each NCS A in NTS X .

Example 5.1:

Let X = {a, b} Y = {u, v}. $\tau 1$ = {0, G1 ,1 } and $\tau 2$ = {0, G2 ,1 } , G_1 = < x, (0.4, 0.5, 0.8), (0.2, 0.5, 0.4)> , G2 = < x, (0.3, 0.5, 0.7), (0.6, 0.5, 0.8)> , A = < x, (0.8, 0.5, 0.4),(0.4, 0.5, 0.2)> is a CS. Now the mapping f : $\tau 1 \rightarrow \tau 2$ is a NπgβC-mapping.

Definition 5.2: Consider a map f : X → Y. Then it is called a Neutrosophic πgβO- mapping (NπgβO- map) if f(A) is an NπgβOS in (Y, σ) for each NOS A in NTS X .

Example 5.2:

Let X = {a, b} Y = {u, v}. $\tau 1$ = {0, G_1,1 } and $\tau 2$ = {0, G2 ,1 } where , G1 =< x, (0.4, 0.5, 0.8), (0.3, 0.5, 0.7)> , G2 = < x, (0.6, 0.5, 0.1), (0.6, 0.5, 0.3)> , A = < x, (0.8, 0.5, 0.4), (0.7, 0.5, 0.3)> then f : $\tau 1 \rightarrow \tau 2$ is a NπgβO- mapping.

Theorem 5.1:

Each NC-mapping is a NπgβC-mapping, not the converse.

Proof. Consider a NC-map f : X → Y. Suppose A is a NCS in X. Given that f is a NC- map, which shows that f(A) is a NCS in (Y, σ). We know that every NCS is a NπgβCS, f(A) is a NπgβCS in (Y, σ). Thus the above mapping is a NπgβC- mapping.

Example 5.3:

Let X = {a, b} Y = {u, v}. $\tau 1$ = {0, G1 ,1 } and $\tau 2$ = {0, G2 ,1 } where , G1 =< x, (0.5, 0.5, 0.7), (0.1, 0.5, 0.4)> , G2 = < x, (0.1, 0.5, 0.8), (0.2, 0.5, 0.7)> , A = < x, (0.7, 0.5,0.5), (0.4, 0.5, 0.1)> is a CS in X , f : $\tau 1 \rightarrow \tau 2$ is a NπgβC-mapping, not NC-mapping.

Theorem 5.2:

Each NGC-mapping is a NπgβC-mapping

Proof: Consider a NGC- map f : X → Y. Let A be a NCS. By given condition f(A) is a NGCS in (Y, σ). Because every NGCS is a NπgβCS, therefore f(A) is a NπgβCS in (Y, σ). Therefore it is a NπgβC-mapping. However the converse need not hold.

Example 5.4:

Let X = {a, b} Y = {u, v}. $\tau 1$ = {0, G1 ,1 } and $\tau 2$ = {0, G2 ,1 } where , G1 =< x, (0.3, 0.5, 0.4), (0.2, 0.5, 0.3)> , G2 = < x, (0.2, 0.5, 0.6), (0.2, 0.5, 0.4)> , A = < x, (0.4, 0.5,0.3), (0.3, 0.5, 0.2)> is a Closed set in X now f : $\tau 1 \rightarrow \tau 2$ is a NπgβC- mapping, not NGC-mapping.

Theorem 5.3:

Each NβC- mapping is a NπgβC- mapping.

Proof: Consider a NβC-map f : X → Y . Let A be a NCS. By assumption f(A) is a NβCS in (Y, σ). Beacuse every NβCS is a NπgβCS, then f(A) is a NπgβCS in (Y, σ). Therefore f is a NπgβC-mapping.

Example 5.5:

Let X = {a, b} Y = {u, v}. $\tau 1$ = {0, G_1 ,1 } and $\tau 2$ = {0, G2 ,1 } where , G1 =< x, (0.3, 0.5, 0.7), (0.2, 0.5, 0.6)> , G2 = < x, (0.3, 0.5, 0.7), (0.2, 0.5, 0.8)> , A = < x, (0.7, 0.5,0.3), (0.6, 0.5, 0.2)> is a Closed set in X now f : $\tau 1 \rightarrow \tau 2$ is a NπgβC- mapping but not NβC-mapping.

Theorem 5.4:

Consider a map f : X→ Y . Let f(A) be a NRCS in (Y, σ) for each NCS A in NTS X. Then f is a NπgβC- mapping.

Proof. Consider a NCS A in NTS X. Then f(A) is a NRCS in (Y, σ). Because every NRCS is a NπgβCS, then image set of A under f is a NπgβCS in (Y, σ).Thus f is a NπgβC-mapping.

Theorem 5.5:

Consider a NπgβC- map f : X → Y . Then f is a NC-mapping in (Y, σ). If Y is a Nπβa T1/2 space.

Proof: Consider a NCS A in NTS (X, τ). By given condition, f(A) is a NπgβCS in (Y, σ). Because Y is a Nπβa T1/2 space, f(A) is a NCS in Y. Thus f is a NC- mapping.

Theorem 5.6:

Consider the map f :X → Y . Let Y be a Nπβa$T_{1/2}$ space. Then the following are equivalent.

(a) f is a NπgβO-mapping,

(b) If A is a NOS in X then f(A) is a NπgβOS in Y,

(c) f(Nint(A)) ⊆ Ncl(Nint(Ncl(f(A)))) for every NS A in X.

Proof. (a) ⇒ (b) : The proof is trivial.

(b)⇒ (c) : Consider a NS A. which implies Nint(A) is a NOS in X. Thus f(Nint(A)) is a NπgβOS in Y. Since Y is an NπβaT1/2 space, f(Nint(A)) is a NOS in Y. Thus

f(Nint(A))=Nint(f(Ncl(A))⊆Nint(Ncl(Nint(f(A)))
⊆Nint(Ncl(f(A)) ⊆Ncl(Nint(Ncl(A))

(c)⇒ (a) : Consider a NOS A in X. By given condition, f(Nint(A))⊆Nint(Ncl(Nint(f(A)))).=>f(A)⊆Ncl(Nint(Ncl f(A)))). Thus f(A) is a NβOS in (Y, σ). Because NβOS is a NπgβOS, f(A) is a NπgβOS in (Y, σ). Therefore f is a NπgβO-mapping.

Theorem 5.7:

Consider a NπgβC- mapping f : X → Y. If Y is a $N\pi\beta bT_{1/2}$ space. Then f is a NGC- mapping

Proof: Consider a NCS A in (X, τ) . Then f(A) is a NπgβCS in (Y, σ) , by assumption. By given Y is a Nπβb $T_{1/2}$ space, then f(A) is a NGCS in Y. Therefore f is a NG Closed mapping.

Theorem 5.8:

Consider a NC- mapping and NπgβC- mapping f : X → Y and g : Y → Z respectively. Then g ∘ f : X → Z is a NπgβC- mapping.

Proof: Consider a NCS A. we have f(A) is a NCS in Y, by assumption. Since g is a Nπgβ Closed mapping, g(f(A)) is a NπgβCS in Z. Then g ∘ f is a NπgβC- mapping.

Theorem 5.9:

Consider the map f : X → Y. Let Y be a $N\pi\beta aT_{1/2}$ space. Then the following are equivalent.

f is a NπgβC- mapping,

(b) f(Nint(P)) ⊆ Nβint(f(A)) for each NCS P of X,

(c) Nint(f−1(Q)) ⊆f−1(Nβint(Q)) for every NS Q of Y.

Proof. (a) ⇒ (b) :Consider aNπgβC- map f . For if P is any NS in X. Then Nint(P) is a NOS. By assumption, f(Nint(P)) is a NπgβOS in Y. Since (Y, σ) is a $N\pi\beta aT_{1/2}$ space, f(Nint(P)) is a NβOS in (Y, σ). Thus Nβint(f(Nint(P)))=f(Nint(P)). Now f(Nint(P))= Nβint (f(Nint(P))) ⊆ Nβint(f(P)).

(b)⇒ (c) : Consider a NS Q in (Y, σ). Then f−1 (Q) is a NS . By assumption, f(Nint(f−1 (Q))) ⊆ NβNint(f(f−1(Q))) ⊆ Nβint(Q). Hence Nint(f−1 (Q)) ⊆ f−1 (Nβint(Q)).

(c)⇒ (a) : Consider a NOS P in X. Then Nint(P) = P and f(P) is a NS in (Y, σ). Then Nint(f−1(f(P))) ⊆f−1(Nβint(f(P))), Now P=Nint(P) ⊆Nint(f−1 (f(P)))⊆f−1(Nβint(f(P))).Thusf(P)⊆f(f−1(Nβint(f(P))))=Nβint(f(P)) ⊆ f(P). Hence Nβint(f(P)) = f(P) is an NβOS in Y. Therefore f(P) is a NπgβOS in (Y, σ). This implies f is a NπgβC-mapping.

Theorem 5.10:

Consider a NπgβC- map f : X, → Y.Y be a $N\pi\beta cT_{1/2}$ space, then f is a NGSC-mapping.

Proof. Let A be a NCS in X. By assumption, f(A) is a NπgβCS in (Y, σ). Given Y is a Nπβc $T_{1/2}$ space, then f(A) is a NGSCS in (Y, σ). This shows that f is a NGSC-mapping.

Theorem 5.11:

Consider a map f : X → Y. Then f is a Nπgβ open mapping if f(Nβint(A)) ⊆ Nβint(f(A))for every A ⊆X.

Proof. Consider a NOS A in X. Then Nint(A) = A. Now f(A) = f(Nint(A)) ⊆ f(Nβint(A)) ⊆ Nβint(f(A)), by assumption. Since Nβintf(A) ⊆ f(A). Thus Nβint(f(A)) = f(A). i.e., f(A) is a NβOS in X. which shows that f(A) is a NπgβOS in X. Therefore f is a Nπgβ open mapping.

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