An Efficient ECG Approximation using Chebyshev Polynomial Interpolation

Om Prakash Yadav, Shashwati Ray

***Abstract:** An ECG (Electrocardiography) is a simple, noninvasive method plotting potentials generated due to cardiac activity. The recorded signal is also called as ECG (Electrocardiogram) consist of waves viz., P, QRS, T and U are variable shape and timing characteristics. These signals are often contaminated with noises of variable frequency and amplitude during acquisition and transmission. These noises must be reduced for better clinical evaluation. Volume of data generated through ECG recorders is also very large. Lagrange-Chebyshev interpolation technique along with total variation approach has been presented for approximation of ECG signals of MIT-BIH database. The standard ECG assessment tools has been utilized to measure the performance the proposed method. The results obtained are found to be better than exiting techniques.*

***Index Terms:** ECG signal; Total Variation Denoising; First Difference; Second Differences; Majorization-Minorization optimization; Bottom-Up Algorithms; Chebyshev Nodes; Lagrange Interpolation.*

I. INTRODUCTION

An ECG is an important tool for cardiologist and helps in determination of diseases due to malfunctioning of heart. The signal consist of waves viz P, QRS, T and U of different shapes, amplitudes and frequencies which is captured by electrodes placed over skin [1-2]. The morphology and timing pattern as shown in Figure 1 of these signals are of utmost important.

The magnitude of ECG signals are very small (upto 1mV) and these signals are always contaminated with noises (variable frequency and amplitude) during acquisition and transmission [3].

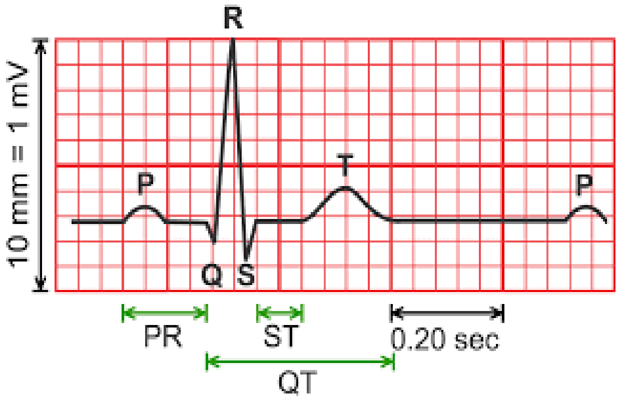

Figure 1. A STANDARD ECG Signal with Components Waves and Intervals on ECG grid.

The noises change the characteristic of ECG signals even upto an extent that original signal loses its significance. Thus, it is necessary to reduce noises from ECG signals upto the extent where ECG retains diagnostic features [4].

Since, ECG signals are quasi-stationery signals; significant noise removal cannot be achieved using a single filter. Conventional filters were designed to filter out noises of 50 Hz frequency. Infinite impulse response (IIR) and finite impulse response (FIR) filters with unacceptably long transient time were widely used to reduce PLI noises [5–7]. Also determination of cut-off frequency for these filters was not so easy. Adaptive filters minimize the error between noisy ECG and a reference ECG with high transient time especially on the QRS complex [8]. Adaptive filters like least mean square (LMS), time varying least mean square (TVLMS), normalized least mean square (NLMS), and recursive least mean square (RLS), transform domain least mean square (TDLMS) which suffered from numerical instability were also found in literature [9]. Baseline wandering is reduced to significant level using linear and polynomial filtering. Low frequency noises were reduced using median filters in [10]. Wavelet transform which allows analysis of signal in both time and frequency scales were also find application in ECG denoising. Various filters using wavelet transform can be found in [11–14]. Selection of threshold and decomposition level is still a challenge. Neural networks and genetic algorithms were also applied to reduce noise from ECG signals [15]. An effective ECG enhancement technique using total variation was proposed in [16]. In this paper, noises present in MIT-BIH ECG signals are reduced through total variation concept.

The remaining article is arranged as: the theoretical concept of total variation for denoising is presented in section 2. In section 3, we provide the theory related to Lagrange-Chebyshev interpolation. In section 4, implementation and the results obtained are discussed. The conclusion is presented in the last section.

II. DENOISING USING TOTAL VARIATION

The first difference total variation (TV), i.e., sum of error between consecutive sample points, of an N point discrete signal $x(n)$ can be derived as [17]

$$TV(x) = \sum_{n=1}^{N} |x(n) - x(n-1)| \qquad (1)$$

It is found that signals with spurious detail show high TV [18]. Thus, reducing the TV of the signal removes unwanted detail and preserves important diagnostic features TV denoising (TVD) observes contaminated signal y(n) as

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$$y(n) = x(n) + w(n) \qquad n = 0, \ldots, N \qquad (2)$$

where x(n) is signal to be cleaned and w(n) is random noisy signal of same length. Thus, the cost function (3) can be optimized through TVD by reducing noise level and preserving significant features [19].

$$\underbrace{\arg min}_{x} = \left\{ f(x) = \frac{1}{2}\sum_{n=0}^{N-1} |y(n) - x(n)|^2 + \lambda TV(x) \right\} \quad (3)$$

The λ here refers to the regularization parameter which regularizes extent of denoising and it must be chosen intelligently [20].

Since the ECG signals are quasi-stationary signals, first difference TVD may introduce staircase effect leading to small flat regions in the denoised signal [21]. So, higher-order differences can be a better option for ECG signals. So, a variant of TVD is proposed in this paper which would reduce the noise and staircase effect of first difference TVD.

Let $x(n)$ be the discrete signal to be denoised is of form

$$x(n) = [x(0), \ldots, x(N-1)] \qquad (4)$$

Expressing the TV of the signal $x(n)$, in terms of second order differences as

$$TV(x) = \sum_{n=2}^{N-1} |\{x(n) - x(n-1)\} - \{x(n-1) - x(n-2)\} \qquad (5)$$

Therefore, the optimization problem reduces to

$$\underbrace{\arg min}_{x} = \left\{ f(x) = \frac{1}{2}\sum_{n=0}^{N-1} |y(n) - x(n)|^2 + \lambda \sum_{n=2}^{N-1} |\{x(n) - xn-1-\{xn-1-x(n-2)\} \right. \qquad (6)$$

Now, $x(n)$ can be derived from $y(n)$ by minimizing (6). Since l1 norm is not differentiable, we minimize the objective function using Majorization-Minimization (MM) algorithm.

The MM method solves lengthy and complicated optimization function by dividing them into a series of simple problems and convergence of the solution is also guaranteed [22]. If $x, y \in R$, f and g be real valued functions on R^n, then the function g majorizes the function f at y if:

(a) $g(x) \geq f(x)$ for all x

(b) $g(y) = f(y)$

While minimizing the objective function f iteratively, let $x(k)$ be the current best minimizer at the k^{th} iteration. A majorizing function g is constructed that majorizes f at $x(k)$. If $x(k)$ minimizes g, the procedure is terminated otherwise a new solution $x(k+1)$ is found by minimizing g,

$$f(x^{k+1}) \leq g(x^{k+1}) \leq g(x^k) \qquad (7)$$

A new majorizing function is constructed at $x\,(k+1)$, and the steps are repeated to produce a decreasing sequence of function values. In order to construct a majorizer for the objective function given by (6), the property of quadratic majorizers has been exploited. The function $f(x) = |x|$ has a quadratic majorizer at each x_k except at $x_k = 0$. If $x(k) \neq 0$ then the majorizer for $f(x)$ is given by [22]

$$|x| = \frac{1}{2|x_k|}x^2 + \frac{1}{2}|x_k| \qquad (8)$$

Therefore, the cost function $f(x)$ is solved indirectly by $G_k(x)$, $k = 0,1,2, \ldots$, where each function $G_k(x)$, is a majorizer of $f(x)$. Using (8) we can construct the majorizer for (6) as

$$G_k(x) = \frac{1}{2}\sum_n |y(n) - x(n)|^2 + \frac{1}{2}\lambda \sum_n \left[\frac{(x(n)-x(n-1))^2}{|x_k|} + xk \qquad (9)\right.$$

III. LAGRANGE-CHEBYSHEV INTERPOLATION

The n[th] degree Chebyshev polynomials are obtained [23] as:

$$T_n(x) = \cos\left(n\cos^{-1}(x)\right) \text{ for } n \geq 1 \qquad (10)$$

The Chebyshev polynomial has $n+1$ zeros (nodes or points) in the interval [-1, 1], which can be calculated as:

$$x_j = \cos\left(\frac{2j+1}{2(n+1)}\pi\right) \text{ for } 0 \leq j \leq n \qquad (11)$$

Chebyshev interpolation produces a sequence of polynomials $p(x)$ that converge uniformly to $f(x)$ over $[-1,1]$ [24][25]. If $f(x)$ is a continuous function on $[-1, 1]$, the polynomial interpolation of degree n can be obtained by interpolating between the values of $f(x)$ at $n+1$ significant points in the interval. Let $f(x_j)$, $0 \leq j \leq n$ be a set of $N+1$ numbers representing the samples of ECG sequence vector of length N in $[-1, 1]$, then there exists a unique polynomial p of order $n \leq N$ that interpolates these data, i.e., $p(x_j) \approx f(x_j)$ for each j.

If the interpolating polynomial is

$$p_n(x) = c_0 + c_1 x + c_2 x^2 + \cdots + c_{n-1}x^{n-1} + c_n x^n \qquad (12)$$

We require that

$$c_0 + c_1 x_j + c_2 x_j^2 + \cdots + c_{n-1}x_j^{n-1} + c_n x_j^n = f(x_j) \qquad (13)$$

The matrix representation of (13) is arranged as

$$\begin{bmatrix} 1 & x_0 & \cdots & x_0^{n-1} & x_0^n \\ 1 & x_1 & \cdots & x_1^{n-1} & x_1^n \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 1 & x_n & \cdots & x_n^{n-1} & x_n^n \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} f(x_0) \\ f(x_1) \\ \vdots \\ f(x_n) \end{bmatrix} \qquad (14)$$

For a unique solution of (14), the Vandermonde determinant on the extreme left should be non singular [26]. The Vandermonde determinant equals the product of the terms $(x_i - x_j)$ for i > j, therefore the points $x_0, \ldots x_n$ should be distinct for the determinant to be non zero.

Setting the coefficients as the interpolated values $f(x_j)$; $0 \le j \le n$, the Lagrange's polynomial [27, 28] $p_n(x)$ can be expressed as

$$p_n(x) = \sum_{j=0}^{n} l_j^n(x) f(x_i) \quad (15)$$

where l_j^n are (n + 1) Lagrange polynomials of degree $\le n$.

$$l_j^n(x) = \prod_{j=0, j\neq i}^{N} \frac{x - x_j}{x_i - x_j} \quad (16)$$

The interpolation error $E_n(x)$ is then calculated as [29]

$$E_n(x) = \mathrm{f}(\mathrm{x}) - \mathrm{p_n}(\mathrm{x}) = \frac{1}{\mathrm{n}+1} \mathrm{f}^{(\mathrm{n}+1)}(\xi) \prod_{\mathrm{j}=0}^{\mathrm{n}} (\mathrm{x} - \mathrm{x_j}),$$

$$\xi \in [-1, 1] \quad (17)$$

Therefore,

$$|E_n(x)| \le \frac{1}{(n+1)!} \left| \prod_{j=0}^{n} (x - x_j) \right| max_{-1 \le \xi \le 1} \left| f^{(n+1)}(\xi) \right| \quad (18)$$

To minimize the upper bound for $|E_n(x)|$, we can minimize the product $\prod_{j=0}^{n} (x - x_j)$

A better choice of interpolating points $x_0, \dots, x_n$ to ensure uniform convergence is the set of zeros of the Chebyshev polynomial $\mathrm{T}_{n+1}(\mathrm{x})$, instead of equally spaced nodes [27].

If transformation of interval is also required and can be derived (for $x \in [a, b]$) as $y \in [-1,1]$ by using

$$x = \frac{(b-a)y + (a+b)}{2} \quad (19)$$

And the roots of Chebyshev polynomial $T_n(y)$ in $y \in [-1,1]$ can be obtained as

$$y_j = cos\left(\frac{2j+1}{2(n+1)}\pi\right) \quad 0 \le j \le n \quad (20)$$

Thus, x_j in $[a, b]$ is now calculated using (21)

$$x_j = \frac{(b-a)y + (a+b)}{2} \quad 0 \le j \le n \quad (21)$$

The interpolation error now is given by

$$|f(x) - p_n(x)| = \frac{1}{2^n (n+1)!} \left| \frac{b-a}{2} \right|^{n+1} max_{-1 \le \xi \le 1} \left| f^{(n+1)}(\xi) \right| \quad (22)$$

where $p_n(x)$ is the Lagrange interpolating polynomial based on Chebyshev nodes. Delving deeper into the advantages of using Chebyshev interpolating nodes, we observe that Runge phenomenon does not occur with the effect that the error tends to decrease with the increasing degree of the Lagrange Chebyshev interpolating polynomial, whereas the same may not be true for equally spaced nodes. Alternatively, we can express the nth degree interpolating polynomial $p_n(x)$ as a sum of Chebyshev polynomials $T_k(x_j)$ [30].

$$p_n(x) = \sum_{k=0}^{n} c_k T_k(x), \quad x \in [-1,1] \quad (23)$$

where the coefficients c_k are defined as

$$c_k = \frac{2}{n+1} \sum_{j=0}^{n} f(x_j) T_k(x_j), \quad k = 0, \dots, n \quad (24)$$

where

$$x_j = cos\left(\frac{(2j+1)\pi}{2(n+1)}\right) \quad \text{j=0,…,n} \quad (25)$$

Let

$$x = \theta, \theta \in [-\pi, \pi] \quad (26)$$

then,

$$c_k = \frac{2}{n+1} \sum_{j=0}^{n} g(\theta_j) \cos(k\theta_j) \quad k = 1, \dots, n \quad (27)$$

with

$$\theta_j = \frac{(2j+1)\pi}{2(n+1)} \quad (28)$$

Replacing $f(cos\ \theta)$ by a periodic function $g(\theta)$,

$$c_k = \frac{2}{n+1} \sum_{j=0}^{n} g(\theta_j) \cos(k\theta_j), \quad k = 0, \dots, n \quad (29)$$

Thus c_k is discrete approximation to the Fourier series coefficients

$$c_i^s \approx \frac{1}{\pi} \int_{-\pi}^{\pi} g(\theta) \cos(k\theta) d\theta \quad (30)$$

Applying the trapezoidal rule approximation

$$c_i^s = \frac{1}{\pi} \frac{\pi}{(n)} \sum_{k=0}^{n+1} g\left(\frac{(2j+1)\pi}{2(n+1)}\right) cos\left(\frac{(2j+1)\pi}{2(n+1)}\right) \quad (31)$$

which is same as (30) for c_k.
Thus (31) is the discrete Fourier transform of the transformed function $g(\theta) = f(cos\theta)$ [30]. Chebyshev interpolation can effectively be considered as the partial sum of the approximate Chebyshev series expansion obtained by replacing the Fourier transform c_k^s by discrete Fourier transform c_k.

IV. METHODS AND RESULTS

The signals used are from MIT-BIH [31] database and are of 10 seconds with 11 bits per sample resolution. These records are sampled at 360 Hz. Following assessment tools are used to performance evaluation:

4.1. Computational Performance

Let y(n) and x(n) (of length N) be the true signal and reconstructed ECG signal respectively. The error between the signals is evaluated as:

$$e(n) = x(n) - y(n)$$

The performance of algorithms with respect to the signals is evaluated in terms of following parameters:

4.1.1. Mean Absolute Deviation

Mean absolute deviation (MAD) provides average of absolute error.

$$MAD = \frac{1}{N} \sum_{i=1}^{N} |e(n)|$$

4.1.2 Root Mean Square Difference

The root-mean-square difference (RMSD) is the square root of the mean of errors. The RMSD is more useful when large errors are particularly undesirable

$$MSD = \sqrt{\frac{\sum_{i=1}^{N} |e(n)|^2}{N}}$$

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4.1.3 Percentage Root-Mean-Square Difference

The percentage root mean square difference (PRD) is calculated by:

$$PRD = \sqrt{\frac{\sum_{i=1}^{N}|e(n)|^2}{\sum_{i=1}^{N}|y(n)-\bar{y}|^2}}$$

where $\bar{y}$ is the mean of the true signal. This parameter serves as best tool for ECG denoising.

4.1.4 Signal to Noise Ratio

Signal-to-noise ratio (SNR) represents signal strength over noise. A ratio higher than 1:1 indicates more signal than noise.

$$SNR(dB) = 20log_{10}\frac{\sum_{i=1}^{N}|y(n)-\bar{y}|^2}{\sum_{i=1}^{N}|e(n)|^2}$$

4.1.5 Cross Correlation Coefficients

A measure that determines the degree to which two variable's signals is associated. Value near to 1 indicates close resemblance between original and reconstructed signal.

$$CC = \frac{N(\sum xy)-(\sum x)(\sum y)}{\sqrt{[N\sum x^2-(\sum x)^2][N\sum y^2-(\sum y)^2]}}$$

SNR, CC is positive oriented parameters and are higher value are desirable whereas MAD, RMSD and PRD are negative oriented parameters and lesser value is expected.

4.2 ECG Signal Denoising using Total Variation

The ECG signal denoising by second difference TVD -MM to optimize $f(x)$ of [6] can be summarized as :

1. Store the ECG signal to be denoised.
2. Choose a suitable value for regularization parameter λ.
3. Fix the number of iterations as 30.
4. Initialize $x(0)$ as the original signal.
5. Choose $G_k(x)$ using (9) such that

(a) $G_k(x) > f(x)\ for\ all\ x$

(b) $G_k(x) = f(x_k)$

6. Set $x(k+1)$ as the minimizer of $G_k(x)$.

The above algorithm is to implemented for different values of λ and number of iterations till desired results are obtained. The quality assessment tools for different λ are tabulated in Table 1.

Table 1. Performance Parameters Different Values of Regularization Parameter.

λ	MAD	RMSD	PRD	SNR	CC
0.001	0.0040	0.1855	1.1403	45.5960	0.9999
0.005	0.0140	0.5094	3.1315	36.8216	0.9995
0.010	0.0222	0.7352	4.5193	33.6351	0.9990
0.050	0.0537	1.3295	8.1723	28.4897	0.9967
0.100	0.0222	0.7352	4.5193	33.6351	0.9990

From the Table 1, we can observe that best results are for $\lambda = 0.001$.. Result deteriorates for higher values of regularization parameter for both positive and negative oriented parameters.

Since better results are obtained for $\lambda = 0.001$, we fix this value and implemented the algorithm on 10 sets of ECG data. The result of second difference TVD is tabulated in Table 2.

Table 2. Quality assessment matrix of TVD 2D at $\lambda = 0.001$

Signal	MAD	RMSD	PRD	SNR(dB)	CC
100	0.0207	0,6755	3.9683	34.5911	0.9992
104	0.0211	0.6985	2.5274	34.3528	0.9997
108	0.0222	0.7352	4.5193	33.6351	0.9990
112	0.0171	0.4616	2.2735	45.9685	0.9997
115	0.0167	0.4879	1.6410	41.2991	0.9999
117	0.0202	0.5522	2.3786	44.0140	0.9997
122	0.0248	0.6920	1.9812	42.1940	0.9998
201	0.0175	0.6027	3.0501	33.0086	0.9995
205	0.0181	0.4245	2.3008	39.7792	0.9997
207	0,0233	0.7586	2.5033	33.7068	0.9997
214	0.0212	0.7645	1.5988	36.1302	0.9999
220	0.0198	0.4778	1.5334	42.9549	0.9999

It can be observed from the Table 2. that RMSD and MAD are upto one and two decimal places. PRD is also less than 10 which indicates that the denoised signal is medically acceptable. Lowest SNR reported is 33 .00 dB can be considered to suppress noise to significant levels. Shape retentivity is also preserved as CC is almost near to 1. Hence, we can conclude that the method is medically suitable for ECG denoising and henceforth the denoised signals will be utilized for further processing.

4.3. ECG Approximation Through Lagrange-Chebyshev Interpolation

Here we need to construct an interpolating polynomial $p_n(x)$ using (23) with the N ECG samples using the Chebyshev nodes. The Lagrange-Chebyshev interpolation method can be summarized as:

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1. Fix the order n and the tolerance $\xi = 10^{-4}$ for the Lagrange-Chebyshev polynomial approximation.
2. Transform the domain of ECG signals from [a, b] to [-1,1] using (21) and calculate the x_j using (25).
3. Find $f(x_j)$ using the two adjacent samples around x_j.
4. Construct interpolating polynomial $p_n(x)$ using (23).
5. Calculate error$E_n(x) = max\,|f(x) - p_n(x)|$.
6. If $E_n(x) > \xi$ then $n = n + 1$ and go to step 2.

In order to approximate ECG signals of large intervals, high order polynomials were required which further increases computation time. Also errors for this approach were very high. So, to reduce the computation time and error, we segmented the whole signal into suitable (till optimum results are achieved) number of segments using Bottom up segmentation technique. Details and implementation procedure about the technique can be obtained from [32]. The approximation model can be made more flexible if segmentation is done before compression and after denoising. The signal reconstruction stage consists of sequentially appending the segments to obtain the complete reconstructed signal which does not require any selection of significant coefficients. The denoised signals thus obtained are then divided into 100 segments and then the individual segments are interpolated by 50th order Lagrange-Chebyshev interpolants. Table 3. Shows the result of Lagrange-Chebyshev interpolation for same set of signals.

Table. 3. Quality Assessment Matrics for Lagrange-Chebyshev Interpolation

Signal	MAD	RMSD	PRD	SNR (dB)	CC
100	$1.8344\text{x}10^{-6}$	$6.1657\text{x}10^{-5}$	0.0019	44.8285	0.9977
104	$2.8941\ \text{x}10^{-6}$	$9.5539\text{x}10^{-5}$	0.0017	43.0592	0.9982
108	$1.5131\ \text{x}10^{-7}$	$1.1162\text{x}10^{-5}$	$1.7153\ \text{x}10^{-4}$	66.9131	1.0000
112	$7.9530\ \text{x}10^{-7}$	$2.8741\text{x}10^{-5}$	$4.6281\ \text{x}10^{-4}$	55.9675	0.9999
115	$1.8073\ \text{x}10^{-6}$	$5.2451\text{x}10^{-5}$	0.0030	50.3352	0.9945
117	$2.5893\ \text{x}10^{-6}$	$1.3054\text{x}10^{-4}$	0.0014	43.6540	0.9987
122	$3.9897\ \text{x}10^{-6}$	$1.1316\text{x}10^{-4}$	0.0023	47.6133	0.9968
201	$8.4447\ \text{x}10^{-7}$	$3.3991\text{x}10^{-5}$	$9.7624\ \text{x}10^{-4}$	44.8090	0.9994
205	$7.0070\ \text{x}10^{-7}$	$3.8266\text{x}10^{-5}$	$1.1696\ \text{x}10^{-4}$	53.0546	1.0000
207	$1.2015\ \text{x}10^{-5}$	$2.4682\text{x}10^{-4}$	0.0139	23.1663	0.8837
214	$5.7031\ \text{x}10^{-6}$	$1.6179\text{x}10^{-4}$	0.0090	32.2947	0.9535
220	$3.9318\ \text{x}10^{-6}$	$1.4484\text{x}10^{-4}$	0.0031	41.4559	0.9941

From the Table 3, we can observe that MAD and RMSD values are significantly reduced. PRD is also upto 2 decimal places. SNR is also improved to a much better value. CC is now more near to 1. Since all the parameters are within diagnostic limits, we can conclude the approach is suitable for ECG approximations.

4.4 Comparison of the Proposed Method with Existing Techniques

Various algorithms were implemented on the same set of signals and their performance is evaluated in terms of standard quality assessment tools. A comparative analysis is being tabulated for record 117 of MIT-BIH database in Table 4.

Table 1.4: Comparison of Exiting Methods with the Proposed Method

Method	MIT-BIH record	PRD
Truncated Singular Value Decomposition(SVD) Method [33]	117	1.180
JPEG2000 Compression Methods[34]	117	1.180
Beat Alignment Method in 2-D environment[35]	117	1.380
Dual Encoding Technique[36]	117	1.41
Support Vector Machines (SVD)[37]	117	3.9
Proposed	**117**	**0.0014**

As it be clearly inferred from the Table 1.4 that the proposed method is more suitable for ECG approximation when errors, i.e., PRD is considered for analysis.

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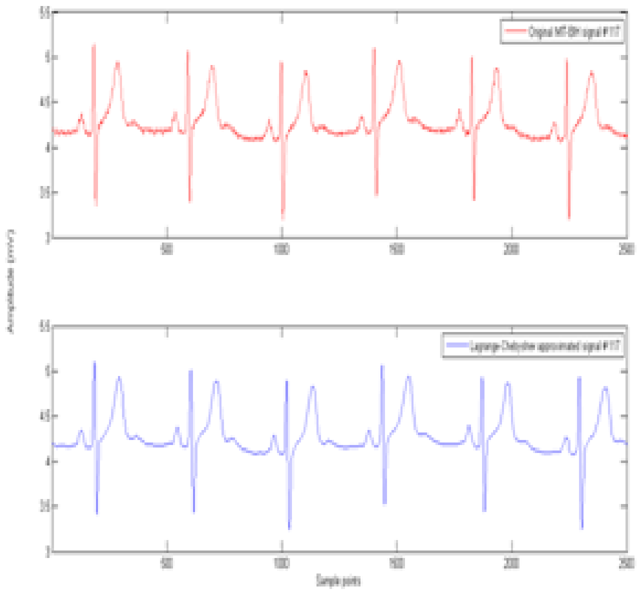

Figure 2. First 2500 Samples of Original and Lagrange – Chebyshev Reconstructed Record Number 117.

V. CONCLUSION

ECG signal records voltage potentials of cardiac activity. These signals are often contaminated with noises of variable amplitude and frequency. Also, the amount of ECG signals generated nowadays is voluminous in size. In this paper, an efficient polynomial approximation of ECG signals of MIT-BIH database has been presented. The proposed method approximates ECG signals through two cascaded stages, i.e., denoising, segmentation and then approximation. Second difference total variation method is implemented for ECG denoising and results obtained were quite convincing. Lagrange –Chebyshev interpolation technique has been utilized to approximate the denoised signals. Bottom-Up segmentation method has been incorporated to enhance the results. Performance of the method is measured in terms of standard parameters and is found to acceptable. The accuracy can be further improved segmenting ECG signal into large number of segments but that could be at the cost of computation time. A combination of TVD 1D-2D may provide better results.

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