

Sunspot Data Denoising using Wavelet

M. Bindusri, S. Koteswara Rao

Abstract: In data analysis, signal processing plays a prominent role since the received sunspot data continuously fluctuates. Sunspot number data is corrupted with Gaussian noise and for statistical analysis; the noise needs to be filtered using wavelet transform. Traditional methods, Fourier transform and Kalman filter has limitations when analyzing the sunspot number data. A Wavelet transform is a promising tool that provides the time-frequency representation of the data. Daily sunspot number data from 2001 to 2018 is analyzed using Daubechies wavelet transform. Daubechies wavelet transform provides flexibility and is used for wide ranges of data using different denoising techniques such as Rigrsure, Sqtwolog, Heursure, Minimaxi thresholding methods. Results showed Sqtwolog (Universal (or) global threshold) and Heursure gave the better- denoised results compared with the other two denoising threshold methods for the sunspot number data.

Index Terms: Denoising methods- Heursure, Minimaxi, Rigrsure, Sqtwolog, Sunspot number, wavelets.

I. INTRODUCTION

A. Sunspot

Study of sunspots began after the invention of telescope in the 17th century, but the sunspot data collection started in middle of 18th century. Sunspots are the violent disturbances on sun's surface which affect atmospheric phenomena on the earth's surface, influenced by the galactic rays. Surrounding areas are lighter when the sunspot storm happens. Sunspots contraction or expansion depends as they move across the sun's surface. When Sunspots are more, delivers more energy to the earth's atmosphere so globally the temperature should rise [1]. They are the one with regions of reduced surface temperature caused by the magnetic flux concentrations. Sunspots appear in pairs of opposite magnetic polarity. If the sunspot size is large and it is as big as the earth then the magnetic field is thousand times stronger than the surface of the earth. Sunspot consists of two parts- the dark area inside the region is called Umbra and the surrounding area is less dark called penumbra [2]. The difference between the temperatures of umbra and penumbra makes the sunspot appear brighter on the sun's surface [3]. Surrounding area of the sunspot consists of gas pressure which depends on temperature. Sunspots appear in the low latitudes near the solar equator and don't appear below 5 degrees or above 40 degrees towards north and south latitudes. Sunspots are usually represented with a sunspot number [4].

B. Sunspot number

Sunspot number is used to determine the number of sunspots (or) sunspots group present on the sun's surface. Sunspot number (S) is also called Wolf number (or) Zurich number (or) International sunspot number.

$$S = K (10g + s)$$

Where **K** depends on the location, **g** is the sunspots group and **s** is the sunspot individual number present on the sun's surface. Sunspot number data has fluctuated as it is mixed with the true data and the noise. For data analysis, signal processing plays an important role because the data gathered is corrupted with noise. To remove the noise for further analysis of data different methods exists, Fourier transform, Kalman filter and wavelet transform. Fourier transform, provides only frequency information of the signal and cannot provide frequency information along with the time axis. Kalman filter removes disruptions in the data by initialization and propagation of error covariance. Implementation of Kalman filter is impracticable in large scale models [5]. In this paper for sunspot data denoising, Daubechies wavelet transform is used.

II. WAVELET TRANSFORM

A Wavelet transform is represented by a family of wavelets. Wavelets are daughter wavelets which are scaled and translated (or) mother wavelets of a finite length decaying oscillatory waveform. Mathematically wavelet transform is represented in

$$X(a, b) = \int_{-\infty}^{\infty} x(t) \varphi_{c,d} * (t) dt$$

Where '**d**' is the factor for translating and '**c**' is the scaling one. Wavelet transforms are frequently used in present day communications for analysis of data in both frequency and time domain to overcome the Fourier transform drawback. Wavelet decomposes the data into various components with time varying functions called wavelets. Wavelet transforms require less number of wavelet coefficients to represent the data which is complicated and it is more efficient [6]. Wavelet transform is a mathematical function in which the data is analyzed with the mother wavelet. Wavelet transforms are used for real time non stationary applications [7]. Wavelet transforms are suited for data denoising, lack of discontinuities in the data [8], original data reconstruction using different wavelets thresholding methods. For Denoising sunspot number data Daubechies wavelet transform is represented in this paper [9].

III. DAUBECHIES WAVELET

Daubechies wavelet transform is one of the discrete wavelet transform where the wavelet samples are in discrete manner [10]. Daubechies wavelet is represented with (dbN), N indicates the Daubechies wavelet index number.

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Db6 wavelet level 5 is used which consists of detail coefficients also called wavelet coefficients and approximation coefficients also called scaling coefficients [11]. Detail coefficients obtained from the high pass filter and approximation coefficients results from low pass filters.

$$\text{Scaling coefficients } (a_{i,j}) = (y(n), \Phi_{i,j}(n))$$

$$= \sum_{i=0}^n y(n) * \Phi_{i,j}(n)$$

$$\text{Wavelet coefficients } (c) = (y(n), \Psi_{i,j}(n))$$

$$= \sum_{i=0}^n y(n) * \Psi_{i,j}(n)$$

Where $y(n)$ is the sunspot data, $\Phi(n)$ is the scaling basis, $\Psi_{i,j}(n)$ is the wavelet basis

Daubechies wavelet handles the signals accurately, which are more complicated because db6 wavelet uses original values from the data. Daubechies wavelet properties are:

1. Daubechies wavelet transform is asymmetric in both scaling and wavelet function.
2. Daubechies wavelets are orthogonal.
3. Daubechies biorthogonal wavelets.

Daubechies wavelet asymmetry property: By using low pass filters and high pass filters of Daubechies wavelet transform, the resulting frequencies shows different time intervals. Therefore Daubechies wavelet represents that no filters are symmetry in nature.

Orthogonal Daubechies wavelet: Daubechies wavelets are orthogonal with 'k' vanishing moments and minimum length of '2k'. Wavelet function (Ψ) consists 'k' vanishing moments and scaling parameter (Φ) generate lesser than or equal to 'k' polynomials of degree. Vanishing moments in wavelet decay the data towards lower frequencies [12].

Daubechies Biorthogonal wavelets: Biorthogonal wavelets are of compactly supported Daubechies wavelets. Daubechies Biorthogonal wavelets results in two wavelet functions and two scaling functions.

IV. DENOISING OF SUNSPOT DATA USING DAUBECHIES WAVELET THRESHOLDING METHODS

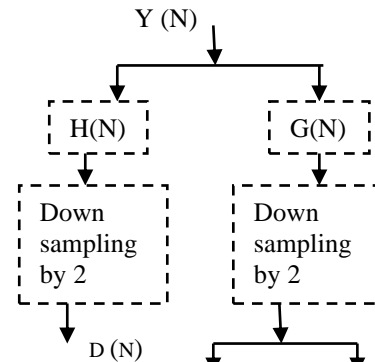
Denoising of sunspot data is the scaling down of noise from the original data, whether the noise may be random (or) white which has mean zero and the variance one [13]. Denoising algorithm main aim is to protect the information of the data and to achieve noise cut down from the data [14]. Wavelet denoising aim is to retrieve the noiseless data from the original by using Daubechies wavelet thresholding methods. By using denoising methods, the atmospheric noises and solar storms doesn't knock out the communications. Denoising the sunspot data is based on denoising threshold and threshold selection parameter [15].

$$y_{i,j} = w_{i,j} + \sigma \epsilon_{i,j}$$

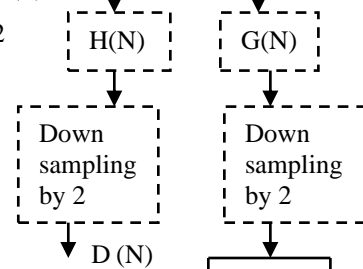
Where i indicates the decomposition level and j denotes the index of the coefficient, $w_{i,j}$ represents the daubechies wavelet coefficients of the corrupted sunspot data. Wavelet coefficients magnitude is of lesser in value it is considered as pure form of noise and should be equal to zero [16]. Threshold value is compared with the each level of the daubechies wavelet coefficient to posses whether it exists in the data or not [17].

Decomposition of Daubechies wavelet coefficients:

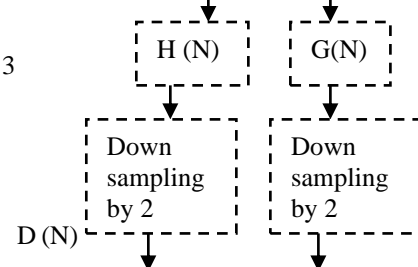
LEVEL 1



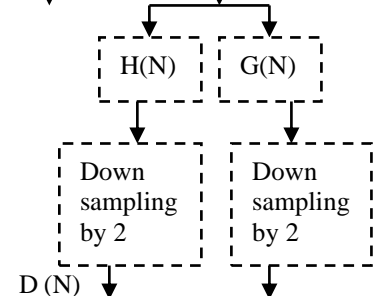
LEVEL 2



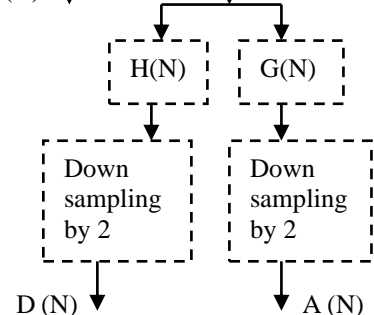
LEVEL 3



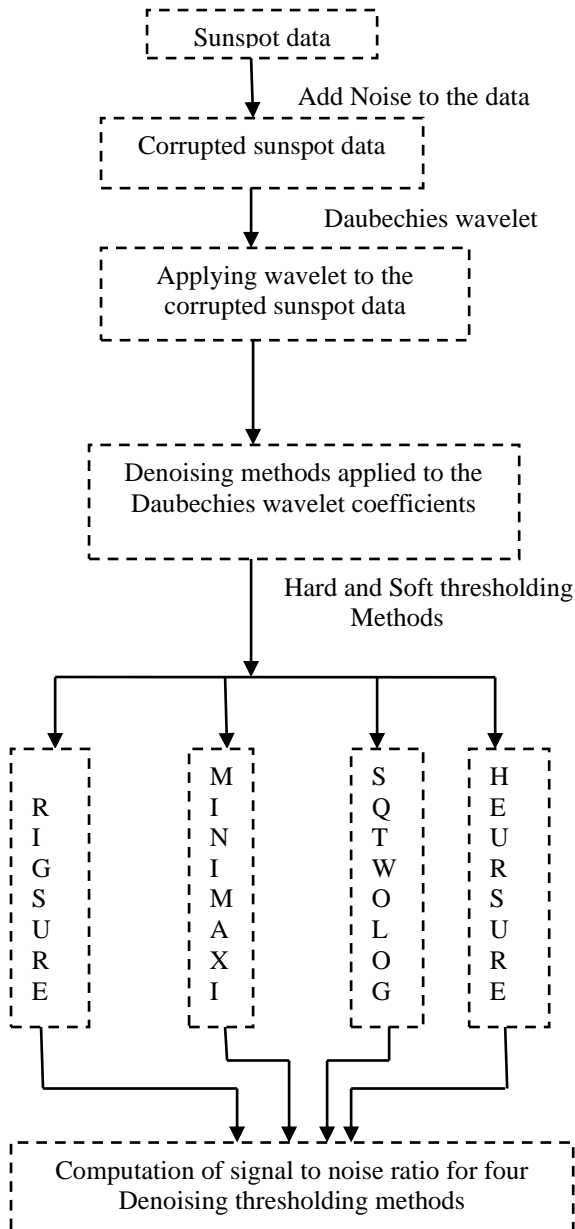
LEVEL 4



LEVEL 5



Flow chart for proposed model:



Denoising using wavelet consists of three steps:

1. Decomposition of the data using Daubechies wavelet: Decomposes the data in a discrete manner using mother wavelet (Daubechies wavelet). Eventually, the decomposition takes up to the level 5 using the high pass and low pass filters. High pass filters are denoted by H_0 while low pass filters are denoted by G_0 . At each level HPF produces the detail coefficients $D(N)$ and LPF results the approximation coefficients $A(N)$ [18].
2. Applying threshold methods to each level of the wavelet coefficients: Selecting of threshold methods and apply to each level of the wavelet decomposition coefficients for obtaining the modified coefficients [19].
3. Reconstruction of the data using thresholding methods: By holding the modified coefficients, denoised data is obtained [20].

A. Daubechies wavelet thresholding methods

Wavelet thresholding methods are applied to high frequencies that are the detail coefficients which are more affected by noise. Daubechies wavelet thresholding is categorized into hard thresholding (or) soft thresholding [21].

Hard thresholding:

In Hard thresholding, the coefficients are set to zero; if the absolute value is less than the threshold otherwise the coefficients are not modified. Hard thresholding satisfies

$$H(n) = \begin{cases} w(n), & \text{if } w(n) \geq T \\ 0, & \text{otherwise} \end{cases}$$

Where ' $w(n)$ ' indicates the daubechies wavelet coefficients and ' T ' denotes the threshold value foreach wavelet coefficient.

Soft thresholding:

Soft thresholding settles the coefficients to zero if the wavelet coefficient values are less than the threshold, otherwise the thresholding value equal to the sign function which multiplies the coefficient and the subtraction value of the coefficient from the threshold [22].

$$S(n) = \begin{cases} \text{sgn}((w(n))(w(n) - T)), & w(n) \geq T \\ 0, & \text{otherwise} \end{cases}$$

Where $\text{sgn} = -1, 1, 0$

If the coefficient $w(n)$ is equal to zero it returns 0, if the coefficient is greater than zero it returns one, coefficient is lesser than zero it returns -1.

B. Thresholding methods

Thresholding methods are based on mathematical calculations; methods like Sqtwolog, Rigrsure, Heursure, Minimaxi are used for suppressing the wavelet coefficients and to retain the data without noise.

Rigrsure threshold method:

SURE is stein unbiased risk estimator, it is an unbiased estimator of the mean square error of a nonlinear estimator which provides an indication of the accuracy of the estimator.

Let $\mu \in \mathbb{R}^d$ is a white Gaussian noise parameter and $y \in \mathbb{R}^d$ is known data and Gaussian noise is distributed with zero mean [23].

$f(y)$ is an estimator of μ from y

$$f(y) = y + g(y)$$

Where $g(y)$ is weakly differentiated functions not assumed differentiable but only integrable.

$$\begin{aligned} SURE(f) &= d\sigma^2 + \|g(y)\|^2 + 2\sigma^2 \sum_{i=1}^d \frac{\partial}{\partial y_i} (g(y_i)) \\ &= -d\sigma^2 + \|g(y)\|^2 + 2\sigma^2 \sum_{i=1}^d \frac{\partial}{\partial y_i} (f(y_i)) \\ E\mu\{SURE(f)\} &= MSE(f) \\ E\mu\|f(y) - \mu\|^2 &= E\mu(d\sigma^2 + \|g(y)\|^2 \\ &\quad + 2\sigma^2 \sum_{i=1}^d \frac{\partial}{\partial y_i} (g(y_i))) \end{aligned}$$

Rigrsure threshold selection is in form

$$thr = \sigma r + wn$$

σ is the noisy data standard deviation, w_n is the wavelet coefficient from the vector $(w_1, w_2, w_3, \dots, w_n)$, thr indicates the rigrsure threshold value yields by minimizing the risks.

Sqtwolog threshold method:

Threshold values are calculated by the square root log (or) universal threshold which uses fixed form of threshold value.

$$ths = \sigma_s \sqrt{2 \log(N_j)}$$

$$\sigma_s = MAD_s / 0.6745 = \text{Median}(ws) / 0.6745$$

Where ' w ' is the wavelet coefficient to scale (s), σ_s is the mean absolute deviation value and N_j is the length of noisy data.

Minimaxi threshold method:

Minimaxi threshold estimator value is best when it is in the worst case. It satisfies the Baye's estimator criterion that minimizes the loss function.

$$thm = \begin{cases} \sigma(0.3936 + 0.10829 \log N), & N > 32 \\ 0, & N < 32 \end{cases}$$

Where N is the length of the noisy data and σ is the standard deviation of the noisy data.

An estimator $\delta^M: x \rightarrow \Theta$ is called Minimax with respect to a risk function $R(\Theta, \delta)$ it achieves the smallest maximum risk among all estimators [23].

Heursure threshold method:

Heursure is a fusion of Sqtwolog and Rigrsure [23]. If the Rigrsure threshold is small it impacts on the Heursure, which means that the signal to noise ratio is small in that situation Sqtwolog threshold gives the best threshold.

V. SIMULATION AND RESULTS

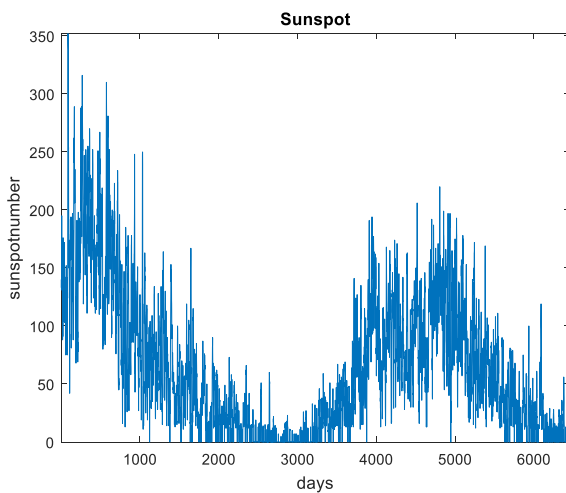


Fig1: Sunspot data from the year 2001-2018

Fig1 shows the Sunspot data taken from the SILSO (sunspot index large scale solar observatories) from the year 2001 to 2018. Data collection was usually mixed between the true data and noise.

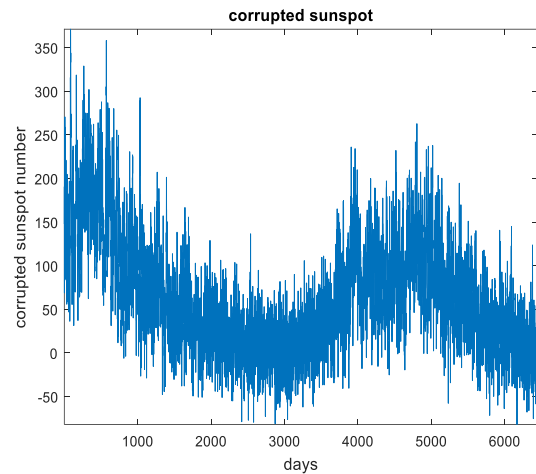
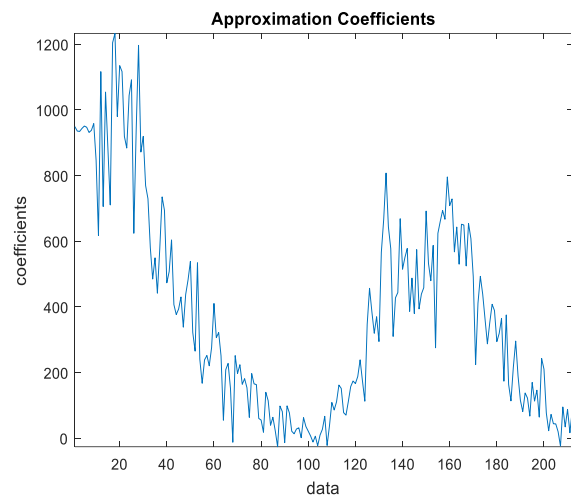
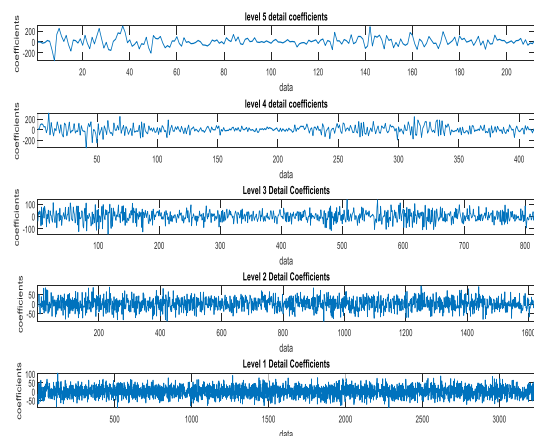


Fig2: Sunspot data corrupted with Gaussian noise

Fig2 corresponds to the sunspot data corrupted with white Gaussian noise with zero mean and variance one. White Gaussian noise values are not correlated with time and the powers at all frequencies are equal.



a) Approximation coefficients

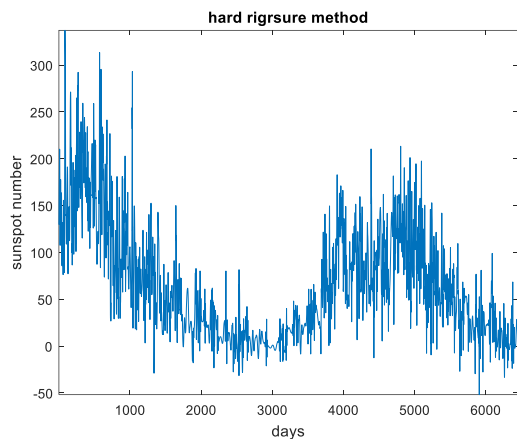


b) Detail coefficients

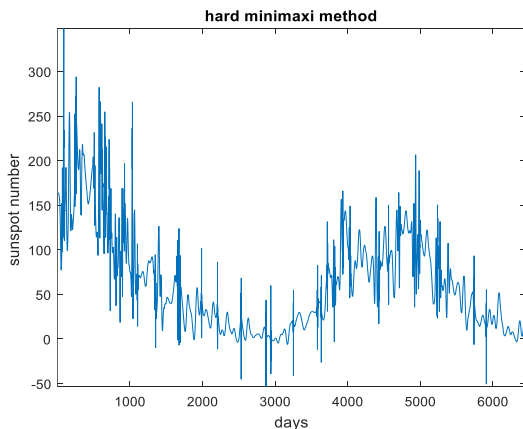
fig3: Daubechies wavelet approximation and detail coefficients

Fig3; By using Daubechies wavelet decomposition of the noisy sunspot data, approximation and detail coefficients are obtained using high pass and low pass filters. At each level of the filters produces the data i.e., half the frequency band with no loss of information.

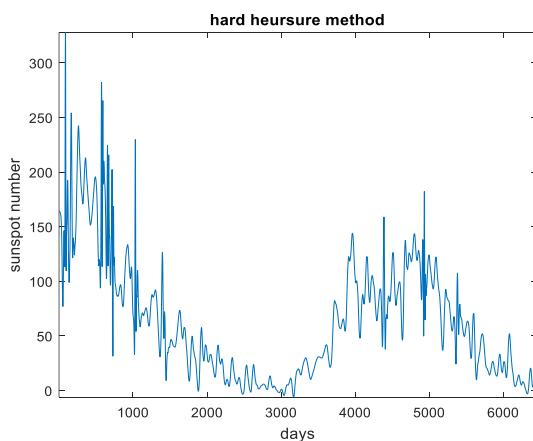
Decimation by 2 represents the half the number of samples as the entire data is halved by the time resolution. By applying Daubechies wavelet, at low frequencies the frequency resolution approach is used while at high frequencies time resolution is used arbitrarily. Approximation coefficients and detail coefficients at level 5 in figure3 are in the same fashion. Therefore Denoising of level 5 Daubechies wavelet detail coefficients is done by using Hard and Soft thresholding methods such as Rigrsure, Minimaxi, Sqtwolog and Heursure.



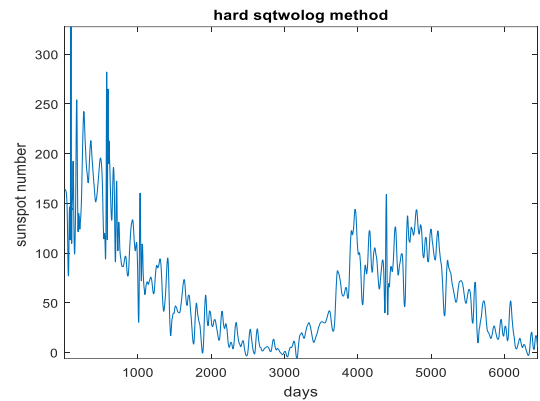
a) Hard thresholding Rigrsure method



b) Hard thresholding Minimaxi method



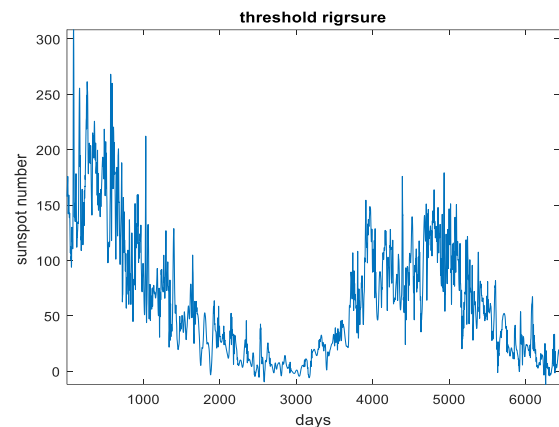
c) Hard thresholding Heursure method



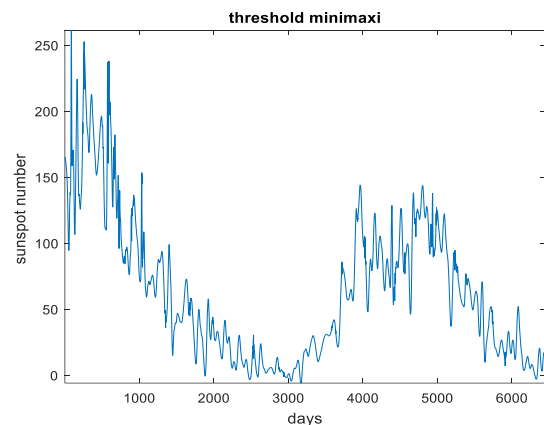
d) Hard thresholding Sqtwolog method

Fig4: Denoising using hard thresholding rigrsure, minimaxi, sqtwolog, heursure methods

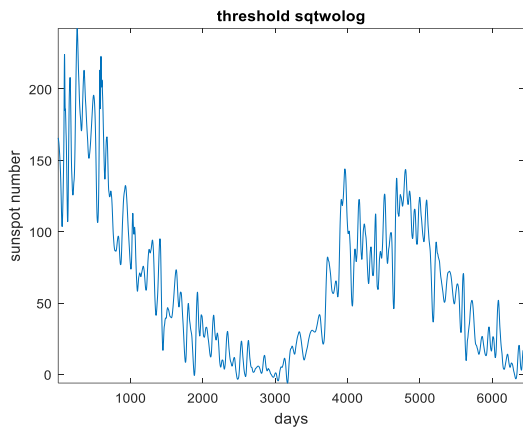
From fig4 Denoising using hard thresholding methods rigrsure, minimaxi, sqtwolog, heursure doesn't provide accuracy of the data, in hard thresholding methods there is a discontinuity in the data which leads to fluctuations. Compared to the hard thresholding, soft thresholding imparts the accuracy by smoothing the data using rigrsure, minimaxi, sqtwolog, heursure threshold methods.



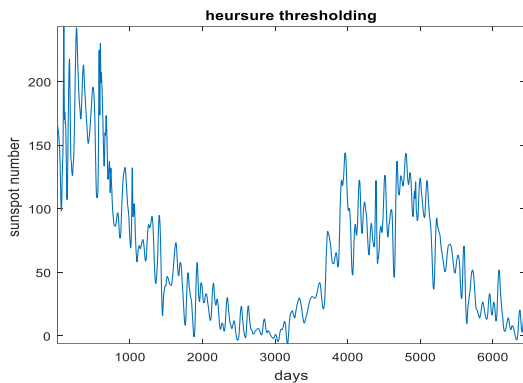
a) Soft thresholding Rigrsure method



b) Soft thresholding Minimaxi method



c) Soft thresholding Sqtwolog method



d) Soft thresholding Heursure method

Fig5: Denoising using Soft thresholding Rigrsure, Minimaxi, Sqtwolog, Heursure methods

Fig5; In Soft thresholding, Sqtwolog (or) universal (or) global threshold method and Heursure soft threshold method gives the best results compared with the rigrsure and minimaxi soft threshold denoising methods.

TABLE1: SNR results for Denoising methods

Thresholding methods	Soft thresholding	Hard thresholding
Sqtwolog	82.2876	82.2659
Minimaxi	82.2683	79.5377
Rigrsure	80.8150	76.6589
Heursure	82.29	82.2572

Table1 shows the signal to noise ratio for soft thresholding and hard thresholding denoising methods such as Rigrsure, Sqtwolog, Minimaxi, Heursure methods. Sqtwolog and Heursure gave the better signal to noise ratio results in hard thresholding and soft thresholding methods.

VI. CONCLUSION

Wavelet transforms find a wide range of applications ranging from signal processing which applied to different signals and image compression techniques. In this paper, discrete wavelet transform carried through different filtering operations for localization of time-frequency analysis on the sunspot data. Most prominent information present in high amplitudes and the information which is less appear in low amplitudes. Denoising methods such as soft thresholding and hard thresholding are used for reconstruction of original data

using Rigrsure, Sqtwolog, Minimaxi, Heursure and the mathematical models for the four denoising methods are proposed. Thresholding methods Sqtwolog, Heursure reconstruct the data without the transitions and gave the better signal to noise ratio results.

REFERENCES

- HAN YANBEN, HAN YONGGANG (30-Aug 2013). Wavelet analysis of sunspot relative numbers.
- ASWATHY MARY PRINCE, Dr. SANISH THOMAS, Er. RAVI JOHN, Dr.D.P. JAYAPANDIAN (2013). A study on themed range periodicity of sunspot number during solar cycles 21, 22, 23and 24, International journal of scientific and research publications.
- Sunspots essay research paper.
- SATISH KUMAR KASDE, DEEPAK KUMAR SONDHIA, ASHOK KUMAR GWAL (September 2016), Volume 5. Analysis of sunspot time series during the ascending phase of solar cycle24 using the wavelet transform
- S. POSTALCLOGLU, K. ERKAN, E.D. BOLAT. Comparison of Kalman filter and wavelet filter for denoising.
- P.M. BENTLEY, J.T.E. MC DONNEL, Wavelet transforms an introduction, Volume 6.
- BABATUNDE S. EMMANUEL. Discrete wavelet mathematical transformation method for non-stationary heart sound signal analysis (August 2012), Vol: 7, No: 8.
- I.M. DREMIN, O.V. IVANOV, V.A.NECHITAILO LEBEDEV physical institute, Moscow117294, Russia. Wavelets and their use.
- BURHAN ERGEN, FIRAT University, TURKEY. Signal and Image denoising using wavelet transform.
- LEI LEI, CHAO WANG, XIN LIU (2013), Vol:7, No:9. Discrete wavelet transform decomposition level determination exploiting sparseness measurement.
- C EDRIC VONNESH, THIERRY BLU, MICHAEL UNSER (20-Aug-2007). Generalized Daubechies wavelet families, Volume 55.
- S.C SHIRALASHETTI (2014). An application of the Daubechies orthogonal wavelets in power system engineering, Recent advances in Information technology.
- LU JING-YI, LIN HONG, YE DONG, ZHANG YAN-SHENG (2016). A new wavelet threshold function and denoising application, <http://dx.doi.org/10.1155/2016/3195492>.
- BARTOSZ KOZLOWSKI, Journal of Telecommunications and Information technology, 2005. Time series denoising with wavelet transforms.
- E.HOSTALKOVA, A. PROCHAZKA. Wavelet signal and Image denoising.
- PIOTR LIPINSKI, MYKHAYLO YATSYMRISKYY. Efficient 1D and 2D Daubechies wavelet transforms with application to signal processing.
- M. PITCHAMMAL, N. RIGANA FATHIMA, S. SHAJUN NISHA (2016). Emprical evaluation of wavelet transforms using Shrinkage thresholding techniques with medical images., Vol:6.
- MARIO MASTRIANI. Denoising and compression in wavelet domain via projection onto approximation coefficients.
- YALI LIU (2015). Image denoising method based on threshold, wavelet transform and genetic algorithm, Vol: 8, No: 2.
- VAISHALI V. THORAT, ELECTRONICS and TELECOMMUNICATION ENGINEERING department, SAVITRIBHAI PHULE Pune University. Study of Denoising algorithms- Review paper.
- M. STNDAG, A. SENG, M. GKBULUT and F. ATA, "PRZEGLD ELEKTRO TECHNICZNY (2012), Vol: 89, No 5, pp 2047-2052. Performance comparison of wavelet thresholding techniques on weak ECG signals denoising.
- JEENA ROY, SALCE PETER, NEETHA JOHN (2013),Vol-2. Denoising using soft thresholding.
- DANIEL VALENCIA, Member IEEE, DAVID OREJUALA, JEFERSON SALAZAR, JOSE VALENCIA, Member IEEE. Comparison analysis between Rigrsure, Sqtwolog, Heursure and Minimaxi techniques using Hard and Soft thresholding methods.

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