Analysis of Seismic Signal using Maximum Entropy Method

K. Himaja, K. S. Ramesh, S.Koteswara Rao

Abstract: ***Seismogenic disturbances are unpredictable hard knocks and are inevitable in nature. Earthquakes are one of the major seismic disturbances that are generated due to the sudden movement of the tectonic plates resulting in great loss to humanity. During the earthquake, abnormal energy is suddenly emanated into the earth's lithosphere thereby generating the seismic waves. Seismic signals thus generated travel through the earth layers and are highly combined with locally generated noise. The noise thus associated with seismic signals can be eliminated using FIR based band pass filter. In this paper an attempt is made to apply Maximum Entropy Method for deriving the frequency components of the seismic signals, for which the power spectrum of the seismic signals is analyzed.***

Index Terms: Adaptive signal processing, Applied statistics, Maximum Entropy Method, Seismology, Stochastic Signal Processing.

I. INTRODUCTION

A. Seismology

Earthquakes are defined as the shaking or trembling of the ground, which are by the unexpected discharge of energy in the inner part of the earth. The frequent cause of occurrence of an earthquake is sliding of tectonic plates. The point at which seismic waves start travelling is called as focus. There are other types of earthquakes which are caused by volcanic activity called as volcanic earthquakes. In accordance with these there are other types of earthquakes which spread energy in all directions. They are collision earthquakes and explosion earthquakes which are like nuclear blasts. From the ancient time onwards these earthquakes are regarded as the face or expression of fury [1]. So, for studying about earthquakes the department of science introduced a new branch called as seismology, which is defined as the study of earthquakes and the person who studies about earthquakes is called as seismologist. For measuring of the earthquakes they designed a device known as seismometer. The seismic waves are formed because of energy released during earthquakes and these waves travel in all directions through the earth's surface by reflecting and refracting at each interface. The seismometer shows readings of the seismic waves and how they travel on the surface of the earth. The Seismic waves are two types. One of them is Body Waves and other is Surface Waves [2]. Body waves majorly consist of two types: S-waves called secondary waves and P waves called primary waves. The primary waves are longitudinal waves that expand and compress, these are the first waves to appear during earthquake. Thus on the seismogram, as these are the quick moving waves through the solid surfaces. The secondary waves are the transverse waves which can shift perpendicular to the direction of propagation and these are slower than the primary waves. The fluids cannot support the perpendicular motion, so these travel in solids only. The other type of seismic waves is surface waves. Where the surface waves travel over the surface of the earth and these are most damaging waves with largest amplitudes and surface waves are of two types which are: Rayleigh waves and love waves. The Love waves are titled after the Augustus Edward Hough Love who was a British mathematician. Love waves are the fastest surface waves which moves on the surface side-to-side but it is slower than the s-waves and occurs after the primary and secondary waves [3]. The Rayleigh waves named after Rayleigh. It rolls like a wave rolls along the ground through a ocean or lake and it moves ground down and up and side to side in the same way of the wave propagation and mostly ground shakes due to these Rayleigh waves which are the largest waves.

B. Discrete-Time Random Process

Digital signal processing is concerned with design and analysis of deterministic discrete-time signals. A deterministic signal is represented as a mathematical model, which can be repeatedly reproduced [4]. The deterministic signal may include different types of signals like unit sample, a complex exponential etc. But in many applications only random process is considered. This process is determined as a group of various signals in terms of statistical possessions or properties. Thus, a random process is used in seismic signals. A random process commonly consists of unwanted signals and the noise is eliminated by filters.

C. Seismic Signal Processing

Seismic data processing can be measured as a system of cascaded operations or actions that attenuate unwanted signal associated with seismic data. Geometrical corrections are removed, such that the final image will truly show a map (seismic image) of the subsurface. Every seismic wave includes necessary geometrical features which specify its nature [4]. These features include transmitter position, receiver position and the position of subsurface reflection point. The last feature is most critical. Before processing, the location is unknown; hence estimation is done by assuming the point of reflection. The point is assumed to be under the location on the surface mid-way between the transmitter and receiver of that specific trace. The point is denoted as mid-common point or depth common point. Traces reflected from the same mid common point are termed as mid-common point gather. The following sections deals with maximum entropy algorithm, mathematical modeling of maximum entropy method, the Simulation and results and the paper is concluded in the last section.

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II. MAXIMUM ENTROPY METHOD

This is an eccentric method of assessing the power spectral density of fixed time series by applying known facts about the autocorrelation functions. But in assessing of density of a power spectra through conventional methods by directly taking autocorrelation functions and by taking the correlation functions as zero at all lags, where it does not contain an estimation and it takes little treatment from the assessing the lags to decrease the effect of auto correlation functions [5]. The assessing principle used for estimation in which the spectrum should be more casual (random) or it should have the maximum entropy of any spectrum power which contains reliable data to be measured. At last the output of this method gives a high resolution when compared to the normal conventional methods with very little amount of growth in time [6]. This method is a main application of an uncertainty which is known as entropy, it is the evaluation of possible casual (random) functions of a screen topic to numerous constraints, with the help of entropy maximum method. The entropy maximum method or maximum entropy method (MEM) defines that undetermined probabilities should be taken for increasing or maximizing the entropy of the topic to available constraints. It is the finest method used for problems where the past information of varieties is available.

Maximum Entropy Method Principle

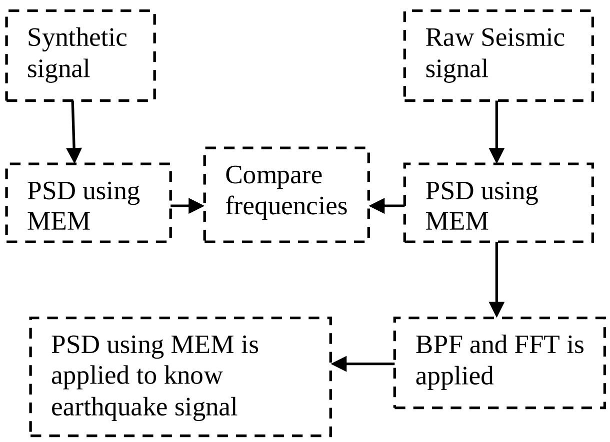
The discrete Fourier transform of the autocorrelation function is power spectrum of a stationary process is given by

$$p(f) = T\sum_{k=-\infty}^{\infty} R(KT)\, exp\,(-2\pi jkTf) \qquad (1)$$

By looking at the implied assumptions about the autocorrelation function it is useful to compare various spectral estimates. In the correlogram method the infinite sum in Eq.(1) is truncated[7]. To smooth some of the effects of this the q estimates of the autocorrelation function are multiplied by the window function w(k). The new estimation is given by

$$S(f) = T\sum_{k=-q}^{q} R(KT)\, w(k)\, exp\,(-2\pi jkTf) \qquad (2)$$

Flow chart for proposed model used in this paper:



III. MATHEMATICAL MODELLING

The auto correlation is represented by $x_r(k)$ for a process for $log|k| \leq p$, expressing the extrapolated values by $e_r(k)$, it must be clear that some of constrains must be placed on $e_r(k)$. The equation is given by

$$x_p(e^{jw}) = \sum_{k=-x}^{x} x_r(k)e^{-jwk} + \sum_{|k|>x} e_r(k)e^{-jkw} \qquad (3)$$

Then $x_p(e^{jw})$ must be related to particular spectrum i.e., $x_p(e^{jw})$ must be real-valued and positive for all values of ω.[8] In common however only constraining $x_p(e^{jw})$ to be original and positive is not enough to assure a distinctive extrapolation. Therefore, approximately additional constraints should be enforced on group of acceptable extrapolations [9]. One of the constraints is suggested by burg, to make it in such a way to increase the uncertainty of the method. The entropy is used for knowing the uncertainty, a entropy maximum extrapolation corresponds to finding the sequence of auto correlations, $e_r(k)$ that makes x(n) as casual [10]. In positions to spectrum power this links to the constraints given by $x_p(e^{jw})$ which is to be flat.

For a Gaussian casual (random) process with spectrum power$x_p(e^{jw})$, the uncertainty is

$$H(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} ln\, x_p(e^{jw})d\omega \qquad (4)$$

Therefore, for Gaussian process with a taken fractional autocorrelation arrangement $x_r(k)$ for $|k| \leq p$, the MEM of spectrum is one which increases issue to the constraint that for the inverse direct-time Fourier transform of $x_p(e^{jw})$ matches to group of correlations to $|k| \leq p$,

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} x_p(e^{jw})e^{jkw}\, d\omega = x_r(k)\,;|k| \leq p \qquad (5)$$

The $e_r(k)$ values which maximize the uncertainty can be identified by using the H(x) with reference to $e_r^*(k)$ equates to null (zero) as

$$\frac{\partial H(x)}{\partial\, e_r^*(k)} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{x_p(e^{jw})}\frac{\partial x_p(e^{jw})}{\partial\, e_r^*(k)}d\omega = 0 \qquad ; \quad |k| > p \quad (6)$$

By the equation mentioned above

$$\frac{\partial x_p(e^{jw})}{\partial\, e_r^*(k)} = e^{jkw} \qquad (7)$$

When this Eq. (7) is substituted into Eq. (6) we get

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} x_p(e^{jw})e^{jkw}\, d\omega = 0 \qquad ; |k| > p \qquad (8)$$

For $X_q(e^{j\omega}) = 1/\, x_p(e^{jw})$, the equation tells about the inverse discrete-time fourier transform of $X_q(e^{j\omega})$ is a finite-length sequence that is equates to null for $|k| > p$,

$$X_q(k) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_q(e^{j\omega})(e^{j\omega})d\omega = 0 \qquad ; |k| > p \qquad (9)$$

Therefore,

$$X_q(e^{j\omega}) = \frac{1}{x_p(e^{jw})} = \sum_{k=-p}^{p} X_q(k)e^{j\omega} \qquad (10)$$

Which takes the entropy maximum spectrum power for Gaussian process which is denoted by $\hat{k}_{mem}(e^{j\omega})$, is an all pole power spectrum,

$$\hat{k}_{mem}(e^{j\omega}) = \frac{1}{\sum_{k=-p}^{p} X_q(k)e^{j\omega}} \qquad (11)$$

By implementing theorem of factorization the equation can be given as

$$\hat{k}_{mem}(e^{j\omega}) = \frac{|b(0)|^2}{A_p(e^{j\omega})A_p^*(e^{j\omega})} = \frac{|b(0)|^2}{\left|1+\sum_{k=1}^{P} A_p(K)e^{-jk\omega}\right|^2} \qquad (12)$$

It also can be denoted in terms of vectors $p_a = [1, p_a(1), \ldots\ldots, p_a(a)]^T$ and $e = [1, e$^jω, ….. , e^jpω]^T, the MEM spectrum can be given as

$$\hat{k}_{mem}(e^{j\omega}) = \frac{|b(0)|^2}{|e^H P_a|^2} \qquad (13)$$

By defining the MEM spectrum which residues to know the coefficients $P_a(k)$ and b(0) [11,12]. By this the given constraint in the Eq(15), coefficient should be elected in such a way that the inverse discrete time Fourier transform of $\hat{k}_{mem}(e^{j\omega})$ given by autocorrelation sequence that matches given values of x_r(k) for $|k| \leq p$ [13,14]. The coefficients $P_a(k)$ are the results of an autocorrelation then equations are

$$\begin{bmatrix} x_r(0) & x_r^*(1) & \dots\dots & x_r^*(p) \\ x_r(1) & x_r(0) & \dots\dots & x_r^*(p-1) \\ \vdots & \vdots & & \vdots \\ x_r(p) & x_r(p-1) & & x_r(0) \end{bmatrix} \begin{bmatrix} 1 \\ p_a(1) \\ \vdots \\ p_a(a) \end{bmatrix} = \epsilon_p \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (14)$$

And if, $|b(0)|^2 = x_r(0) + \sum_{k=1}^{a} p_a(k) x_r^*(k) = \epsilon_p$ (15)

By this entropy maximum method spectrum is given as

$$\hat{k}_{mem}(e^{j\omega}) = \frac{\varepsilon_p}{|e^H P_a|^2} \quad (16)$$

Where P_a is a solution to equation. $P_a(k)$ and ε_p are all-pole coefficients. Note that since $k_{mem}(e^{j\omega})$ is an all-pole power spectrum, x_r(k) satisfies the yule-walker equations

$$x_r(l) = \sum_{k=1}^{p} P_a(k) x_r(k-l) \quad \text{for } l > 0 \quad (17)$$

The properties of entropy maximum method have been extensively understood and also for spectrum estimation tool, the entropy maximum method subjects to various interpretations [12, 15]. The properties of maximum entropy method have been studied extensively and as a spectrum analysis tool, the maximum entropy method is subject to different interpretations [16]. On the other hand it can be claimed that since the entropy maximum extrapolation imposes an all-pole model on data, unless the process is known to be constraint with this model, then the estimated spectrum may not be very accurate [17].

IV. SIMULATION AND RESULTS

The data used in this paper for the analysis is obtained from Wail. A. Mousa et al [4]. A synthetic signal is generated and is considered as input signal. This is compared with the power spectrum of raw seismic data and is then subjected to the Maximum Entropy Method. Fig.1. shows the generated synthetic signal.

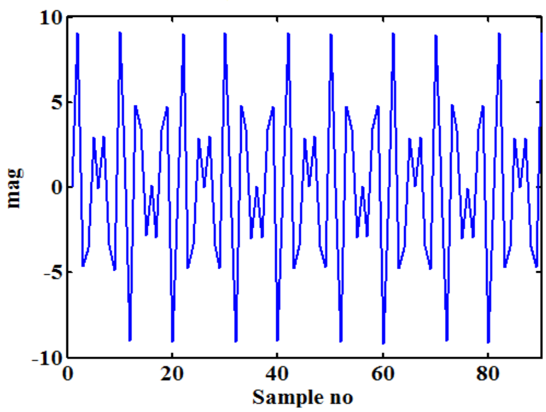


Fig.1.Synthetic signal

The Power Spectral Density of the synthetic signal is given in Fig.2. From the figure it is observed that peaks at the normalized frequency components are at 0.49 and 0.69, by applying Maximum Entropy Method and comparing the results we can see that results are very well correlated [18]. For which the Power Spectral Density of the synthetic signal is plotted with the normalized frequency and the corresponding Power Spectral density are shown in Fig.2.

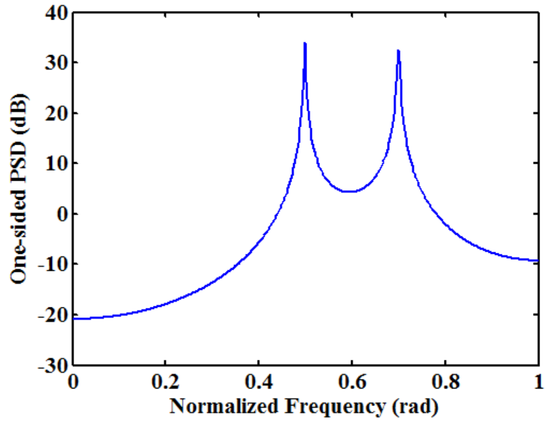


Fig.2.PSD using MEM for synthetic signal

The original seismic data is plotted in the Fig.3. The raw data that is plotted for the events like direct waves, head waves or reflected waves, surface waves and reflections that are observed in the seismic records.

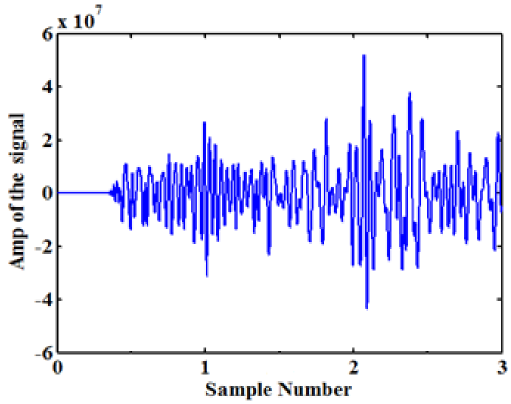


Fig.3.Raw seismic signal

The bias or shift present in the seismic records are removed by considering the mean and are then de-trended. The de-trended signal is plotted which is detached from the mean in Fig.4.

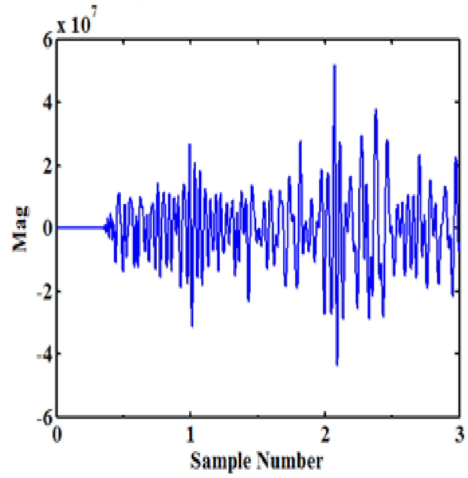


Fig.4.Detrended Raw seismic signal

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As shown in the Fig.5 the de-trended raw seismic signals are subjected to the maximum entropy method for obtaining the Power Spectral Density (PSD). It is observed that the maximum peak is obtained at 0.0958 π normalized frequency and the calculation is as follows.

$$w = \frac{2\pi f}{fs} = 0.0958\pi$$

$$= \frac{2\pi f}{500}$$

$$= \frac{2\pi}{fs} f = 0.0958\pi$$

Tonal Frequency $f = \frac{500}{2} * 0.0958$

$= 23.950$ Hz

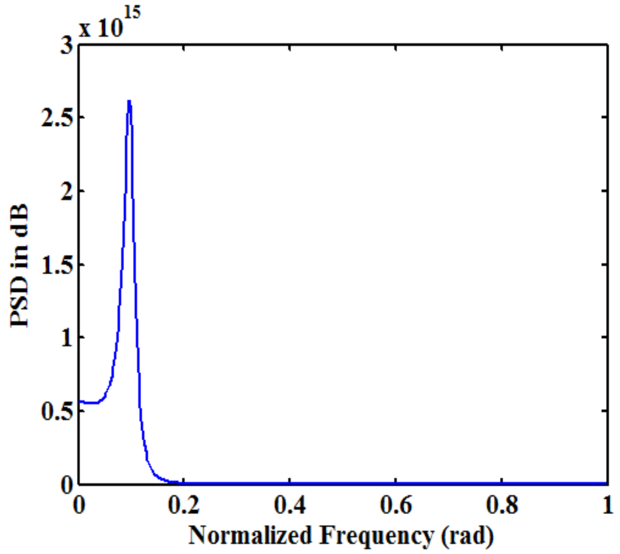

Fig.5.Raw signal spectrum analysis using MEM

It is shown from the data[4] by applying band pass filter the range of frequency components of seismic data falls between 15Hz to 60Hz. Conforming this, by using Band Pass Filter (BPF) with the Finite Impulse Response (FIR) order 8 is accomplished. And the corresponding frequency spectrum of the band pass filter is given in the Fig.6.

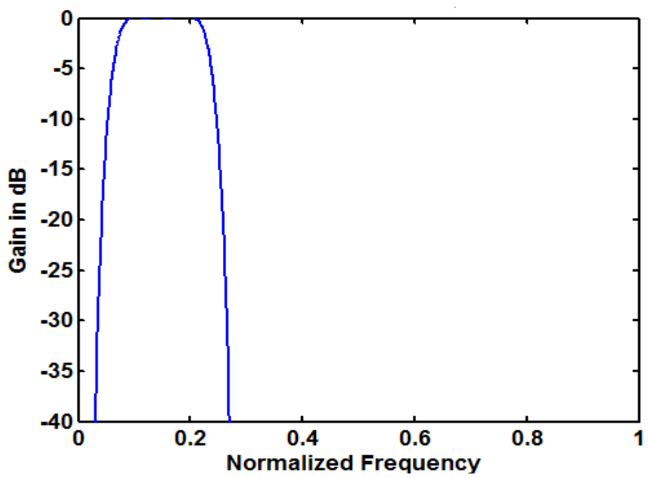

Fig.6.FIR BPF frequency spectrum

By performing the convolution process between the de-trended signal and Finite Impulse Response band pass filter signal. In which the band pass filter extracts the noise in a particular frequency. The Finite Impulse Response signal of 8^{th} order contains 8 filters used for the detrended seismic signal. And the results are shown in Fig.7. It is observed that Finite Impulse Response band pass filter is used to convert into an ideal filter in the frequency domain. Then it is translated to the discrete time domain. And the corresponding results of the infinite impulse response are shown in the Fig.7. To compensate this, a window function is multiplied with the ideal impulse response. Some measurements which are very hard in the time domain are very easy in the frequency domain for one object. Considering the measurement, the harmonic distortion is hard to evaluate the distortion of a sine wave by observing at the signal. The amplitudes and the harmonic frequencies are visible with the amazing clarity when the matching signal is displayed on a spectrum analyzer [20]. Another example is noise analysis for those frequencies. The total noise amplitude which is measured by an oscilloscope is looking at an amplifier's output noise. To determine the signal strength in certain frequency bands the signal power which proceeded through the filter was measured. Repeating the measurements and by tuning the filters, a spectrum could be obtained as shown in plot of the Fig.7.

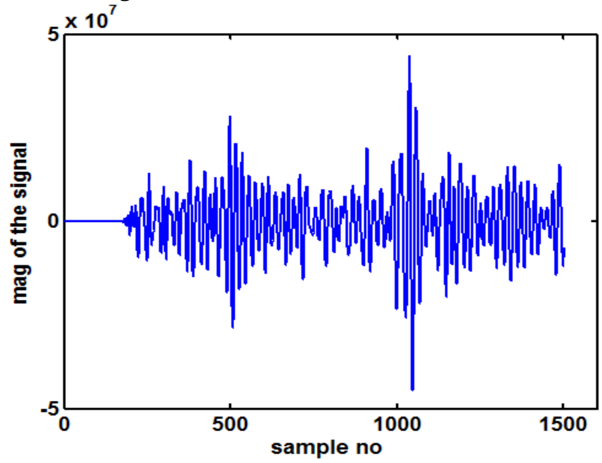

Fig.7.FIR Band Pass filtered signal

The seismic signal's Fast Fourier Transform spectrum after BPF in the normal frequency vs magnitude in decibel is shown in the Fig.8.

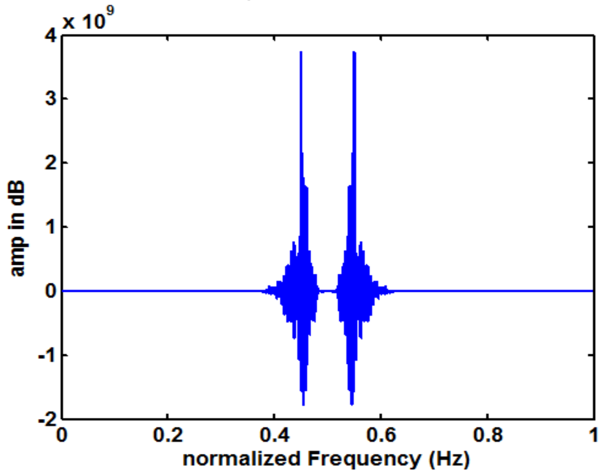

Fig.8.FFT spectrum seismic after BPF in norm freq vs mag in db representation

The Band Pass Filter is applied to the above plot and the analysis is made by using Maximum Entropy Method as shown in Fig.9. The corresponding frequency components are calculated as follows.

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$$w = \frac{2\pi f}{fs} = 0.1056\pi$$
$$= \frac{2\pi f}{500}$$
$$= \frac{2\pi}{fs} f \ = 0.\,1056\pi$$
$$\text{Tonal Frequency } f \ = \frac{500}{2} * 0.1056$$
$$= 26.40 \text{ Hz}$$

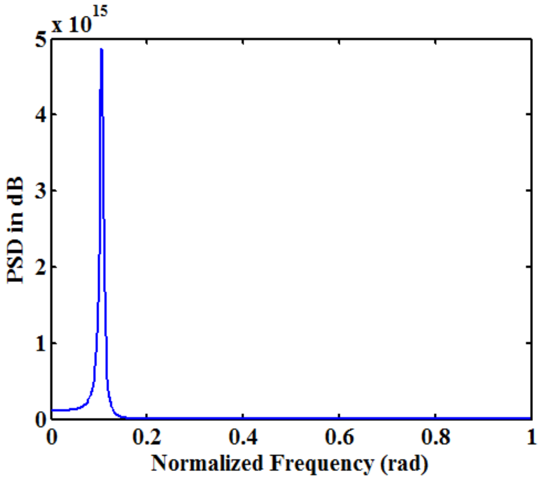

Fig.9. Signal spectrum analysis using Maximum Entropy Method after BPF

V. CONCLUSION

The seismic signal power spectrum analyzed in this paper using Maximum Entropy Method gives the better estimation of power spectrum and frequency components present in the seismic signals, when compared to the conventional methods. It is concluded that the results obtained using Maximum Entropy Method are very well correlated with the synthetic signal for the analysis of seismic signals.

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