

# Experimental Evaluation and Empirical Formulation of Hydraulic Jump Characteristics in Sloping Prismatic Channel

Sanjeev Kumar Gupta, R. C. Mehta, Piyush Singhal

**Abstract:** Hydraulic jump is frequently used for dissipation excess energy downstream of hydraulic structure. This abundance energy, whenever left unchecked, will have unfavorable impact on the banks and downstream of the channel bed. In this paper hydraulic jump characteristics are experimentally evaluated and empirical correlations for depth ratio and relative height are produced in sloping channel by adopting the impact of both strategy Froude number and approaching Reynolds number and neglecting the frictional effect. The developed empirical correlations are validated using Gandhi (2014) data. The present correlation of jump characteristics gives better agreement with experimental data and can be used for preliminary design.

**Index Terms:** hydraulic jump, Froude number, Reynolds number, energy dissipation etc

## I. INTRODUCTION

Phenomenon of hydraulic jump is observed downstream of hydraulic structure such dams, barrage, over flow spillway, sluice gate etc. In this phenomenon flow regime changes from super critical to sub critical flow with rise in depth of flow and converting kinetic energy of rapidly flowing stream in turbulent energy and ultimately into heat energy which is dissipated into the atmosphere [1]. Froude number is an important dimensionless parameter in the study of hydraulic jump. The jump becomes more turbulent and more energy is dissipated as Froude number increases. It is defined as  $F_r = V/\sqrt{gY}$  [2]. The first sketch and description of this phenomenon was provided almost five century ago by Leonardo Da Vinci (1508-1513). The original work has to be done by Belanger (1849) by developing a correlation for depth ratio as [3].

$$\frac{Y_2}{Y_1} = \frac{1}{2} \left( \sqrt{1 + 8F_r^2} - 1 \right) \quad (1)$$

Bakhmeteff and Matzke (1932) gave an analytical equation for relative energy loss in terms of initial Froude number for rectangular horizontal channel as [4]

$$\frac{E_L}{E_1} = \frac{8F_r^4 + 20F_r^2 - \sqrt{8F_r^2 + 1} - 1}{8F_r^2(2 + F_r^2)} \quad (2)$$

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Bradley and Peterka (1957) developed an equation for distance of the rapidly varied flow and relative energy depletion in rectangular horizontal channel in terms of initial Froude number as [5]

$$\frac{L_J}{Y_2} = 220 \tanh \left( \frac{F_{r1} - 1}{22} \right) \quad (3)$$

$$\frac{E_L}{E_1} = 1 - \frac{\frac{Y_2}{Y_1} + \frac{F_{r1}^2}{2 \left( \frac{Y_2}{Y_1} \right)^2}}{1 + \frac{F_{r1}^2}{2}} \quad (4)$$

Chow (1959) developed an analytical equation for efficiency of the jump, relative energy loss and dissipation index for hydraulic jump in rectangular horizontal channel in terms of initial Froude number as [6]

$$\eta_j = \frac{E_2}{E_1} = \frac{(8F_r^2 + 1)^3 - 4F_r^2 + 1}{8F_r^2(2 + F_r^2)} \quad (5)$$

$$\frac{E_L}{E_1} = 1 - \frac{E_2}{E} \quad (6)$$

$$\frac{E_L}{Y_1} = \frac{(\phi - 1)^3}{4\phi} \quad (7)$$

Elevatroski (1959) developed mathematical model for relative length of the jump for horizontal channel from his experimental data as [2]

$$\frac{L_j}{Y_1} = 6.9 \left( \frac{Y_2}{Y_1} - 1 \right) \quad (8)$$

Silvester (1964) gave an analytical equation using momentum principle for energy loss in rectangular horizontal channel as [7]

$$\frac{E_L}{E_1} = \frac{2 - 2\phi + F_{r1}^2(\phi^2 - 1)/\phi^2}{2 + F_{r1}^2} \quad (9)$$

Rajaratnam and Subramanya (1968) collapsed the water surface profile data collected by Bakhmeteff and Matzke (1935) and Rajaratnam (1965) with the intention of creating one general non dimensional profile for any hydraulic jump.

They proposed empirical relation based on their experiments as [8]

$$\frac{Y_2}{Y_1} = 1.3F_{r_1} - 0.355 \quad (10)$$

$$\frac{H_j}{Y_1} = 1.3F_{r_1} - 1.355 \quad (11)$$

$$\frac{L_j}{Y_1} = 5.08F_{r_1} - 7.82 \quad (12)$$

Herbrand (1973) proposed expression for sequent depth ratio, relative height of the jump and dissipation index by neglecting frictional effects and assuming that the channel bottom is horizontal [9]

$$\frac{Y_2}{Y_1} = \sqrt{2F_{r_1}} - \frac{1}{2} \quad (13)$$

$$\frac{H_j}{Y_1} = \sqrt{2F_{r_1}} - \frac{3}{2} \quad (14)$$

$$\frac{E_L}{Y_1} = \frac{1}{4\left(\frac{Y_2}{Y_1}\right)} \left[ \left(\frac{Y_2}{Y_1} - 1\right)^3 \right] \quad (15)$$

Ohtsu and Yasuda (1994) saw that the limit layer influencing the bounce after the conduit entryway moving the hop area to different tail water level in the even cook's garment. The aftereffect of this examination demonstrated that the drag constrain on the vertical ledge of a constrained hop for the undeveloped supercritical stream is bigger than that for completely created supercritical stream [10]. H. Chanson and T. Brattesberg (2000) examined the impact of air water shear stream in a pressure driven bounce. Conveyance of air focus, mean air water speed and bubbly recurrence recorded and introduced in this paper. The speed profile has a comparable shape as divider fly stream [11]. Bushra and Noor Afzal (2002) proposed the condition overseeing the structure of tempestuous water powered hop in a channel of subjective cross-segment where inactivity, hydrostatic weight angle, Reynolds ordinary pressure and bed rubbing powers were considered. They proposed their connection for relative length of the hop as [12]

$$\frac{L_j}{Y_1} = 6.9 \left( \frac{Y_2}{Y_1} - 1 \right) \quad (16)$$

Kucukali S. what's more, Chanson H. (2008) measure choppiness in the bubbly stream area of water driven hop [13]. M. Naseri and F. Othman (2012) decide the length of water driven hop utilizing counterfeit neural systems [14]. Sanjeev Kumar Gupta (2013) developed empirical correlations for relative length and relative energy loss for free hydraulic in horizontal & sloping channel. [15, 16] N. Y. Saad and E. M. Fattouh (2016) develop correlation using ANN model for different characteristics of flow over weir [17]. After literature survey it is found that most of the researcher developed correlation only considering the effect of Froude number and also very little information is available for sloped channel jump. In this paper correlation are developed for sloped channel considering the effect of both approach Froude number and incoming Reynolds number.

## II. EXPERIMENTAL PROCEDURE

### A. Experimental Set-up

A steady head (over head) supply tank of volume 3.65 x 3.65 x 3 m<sup>3</sup> is accessible to limit the mistake because of progress in stream amid experimentation. Water is come to the bay tank of volume 0.48 x 0.35 x 0.85 m<sup>3</sup> through the associating funnel of distance across 10 cm gave managing valve by a divergent siphon.

A constant channel of length 2.40 m, width 0.25 m and tallness 0.35 m is given at the upstream end of the working segment to guarantee uniform stream dispersion. The test were performed in 2.3 m long channel having width of 0.23 m and tallness 0.32 m. Base surface is made of concrete for smooth stream to limit the arrangement of vortexes and rollers. Side dividers are made of Perspex sheet to envision the area of beginning and end position of the hop. Parallel rails are mounted at the highest point of the side dividers for sliding of guide checks toward measure the profundity at various positions along the length and along the width of the channel. A flexible sharp edged vertical managing door is utilized at the upstream and downstream of the working segment to control the profundity of stream into the working area. A release tank with rectangular sharp peaked weir of length 0.32 m is utilized to quantify the release. For the experimentation in the sloping channel the rectangular bed is placed at different angles from the horizontal axis.

### B. Experimental Procedure

In the present study experiments were performed on a sloping prismatic channel of dimension 2.3 m x 0.23m x 0.32m in Channel Flow Laboratory of Applied Mechanics Department of MNNIT Allahabad. A movement of keeps running at different estimation estimations of release was tried tentatively and hop was produced by working the back door and conduit gateway. For each run starting profundity, profundity after bounce and length of water powered hop were assessed.

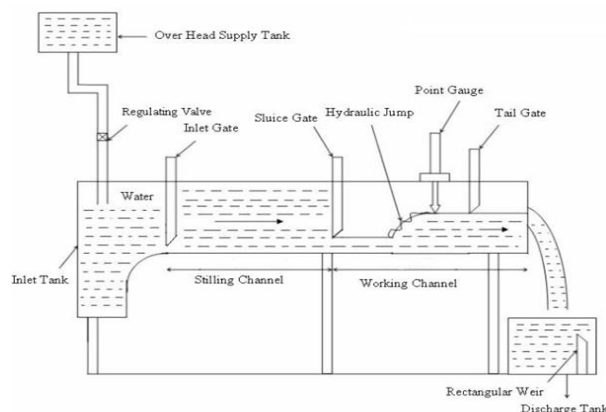


Fig 1 Experimental Setup

The above trials were performed continuously at different valve opening. The volume stream rate in the channel is evaluated with the help of sharp topped rectangular weir. The pre hop profundity, profundity after the hop and bounce stature of water streaming over the upper part of sharp peaked weir are estimated by point measure.



III. DIMENSIONAL ANALYSIS

In view of premise of water powered hop wonder, the essential variable which influence the hop example and vitality dispersal are  $Y_1, Y_2, V_1, V_2, L_j, H_j, E_1, E_2, E_L, E_{RL}, \rho, g, \mu, \epsilon$  and  $\eta$  which can be explored as

$$f(Y_1, Y_2, V_1, V_2, L_j, H_j, E_1, E_2, E_L, E_{RL}, \rho, g, \mu, \epsilon, \eta) = 0$$

Utilizing Buckingham's  $\pi$ -hypothesis and treating  $Y_1, g$  and  $\rho$  as rehashing factors, the accompanying dimensionless gatherings are produced

$$f\left(\frac{Y_2}{Y_1}, \frac{H_j}{Y_1}, \frac{L_j}{Y_1}, \frac{E_L}{Y_1}, \frac{E_1}{Y_1}, \frac{E_2}{Y_1}, \frac{E_2}{E_1}, \frac{E_L}{E_1}, \frac{V_2}{V_1}, \frac{V_1^2}{gY_1}, \frac{\rho V_1 Y_1}{\mu}, \frac{\epsilon}{Y_1}\right) = 0$$

Similarly for sloping prismatic channels hydraulic jump  $Y_1 = d_1 \cos\theta, Y_2 = d_2 \cos\theta$

The following dimensionless groups are developed for sloping prismatic channels

$$f\left(\frac{d_2}{d_1}, \frac{H_j}{d_1}, \frac{L_j}{d_1 \cos\theta}, \frac{E_L}{d_1 \cos\theta}, \frac{E_1}{d_1 \cos\theta}, \frac{E_2}{d_1 \cos\theta}, \frac{E_2}{E_1}, \frac{E_L}{E_1}, \frac{V_2}{V_1}, \frac{V_1^2}{gd_1 \cos\theta}, \frac{\rho V_1 d_1 \cos\theta}{\mu}, \frac{\epsilon}{d_1 \cos\theta}\right)$$

From the dimensional investigation it is discovered that all the pressure driven bounce attributes are the capacity of methodology Froude number, approaching Reynolds number and incline of the channel. It is noticed that the impact of surface unpleasantness couldn't be contemplated as it couldn't be conceivable to change the harshness of the

$$\frac{d_2}{d_1} = f(Fr_1, Re_1, \theta) \quad \frac{H_j}{d_1} = f(Fr_1, Re_1, \theta)$$

IV. RESULTS AND DISCUSSION

A. Variation of Sequent Depth Ratio with methodology Froude number and Slope of the Channel

Figure 2 demonstrates direct variety of sequent profundity proportion with methodology Froude number which shifted from 2.5 to 7.57, 2.58 to 7.61, 2.56 to 7.62 and 2.73 to 7.58 for channel incline 00, 20, 40 and 60 individually. From this figure it is seen that sequent profundity proportion increments with increment in methodology Froude number and slant of the channel bed. It is clear from the assume that roughly 97%, 100%, 98% and 100% of test information are existing in  $\pm 10\%$  of best fit bend with R2-estimation of 0.9496, 0.9779, 0.966 and 0.9739 for 00, 20, 40 and 60 channel bed inclines separately. Hardly any information focuses are veered off from best fit bend which might be because of error in estimation of profundity of stream and release.

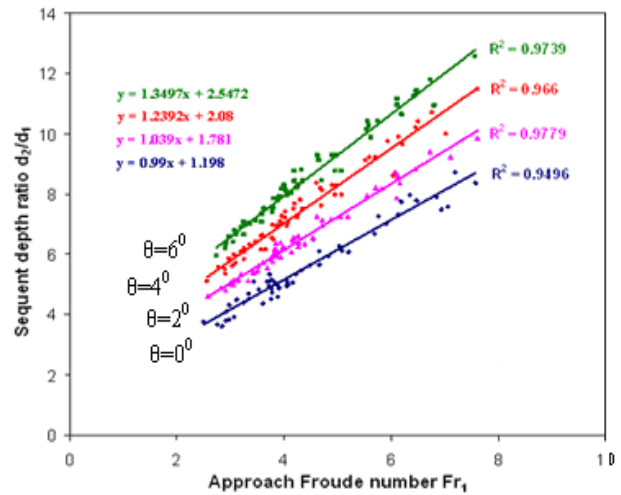


Fig 2 Variety of Sequent Depth Ratio with methodology Froude number and Slope of the Channel

A. Variation of Relative Height of Jump with methodology Froude number and Slope of the Channel

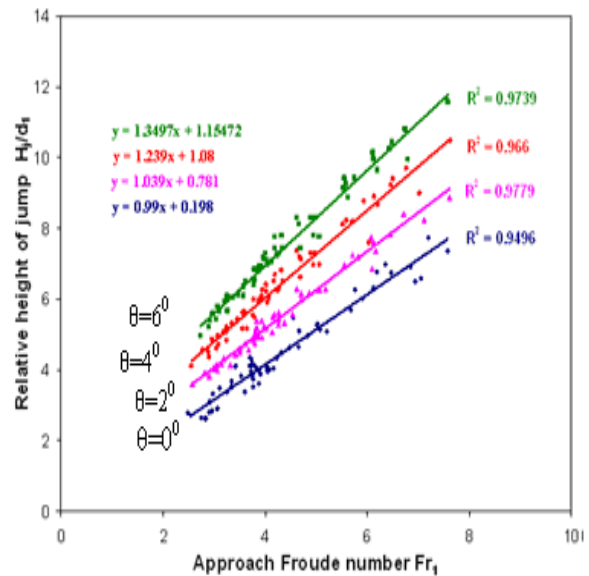


Fig 3 Variety of Relative Height of Jump with methodology Froude number and Slope of the Channel

C. Variation of Sequent Depth Ratio and Relative Height of the Jump with Incoming Reynolds number

Fig. 4 and 5 indicates non - direct variety (polynomial) variety of pressure driven hop attributes with approaching Reynolds number. It is seen from the figures 4 and 5 that sequent profundity proportion, Relative stature of the hop diminishes with increment in approaching Reynolds number individually.

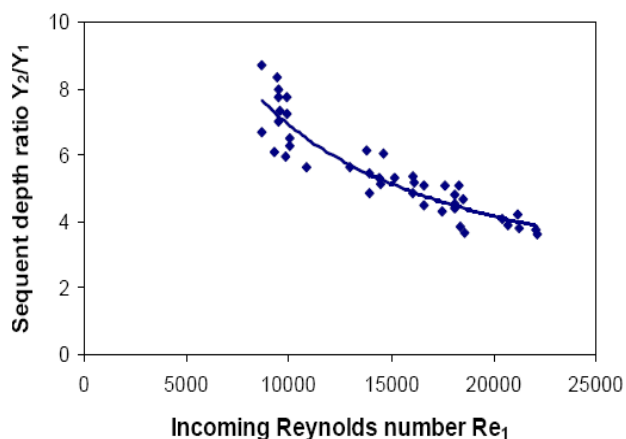


Fig 4 Variation of Sequent Depth Ratio with Incoming Reynolds number

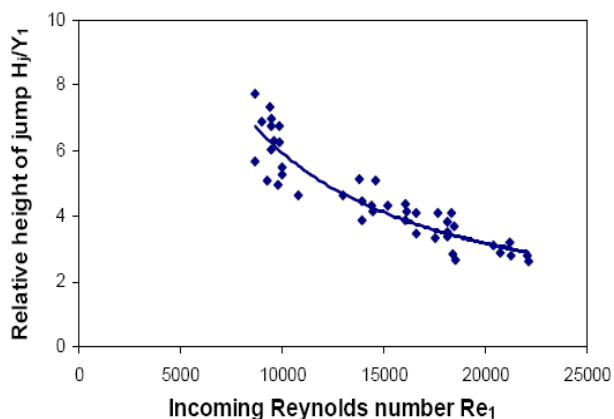


Fig 5 Variation of Relative Height of Jump with Incoming Reynolds number

### V. EMPIRICAL MODELS

Buckingham’s  $\pi$ -theorem is used to develop the computational empirical correlations with the help of relapse investigation of exploratory information. Based on straight fitting between various hop attributes and dimensionless group developed by Buckingham’s  $\pi$ -theorem, the following computational empirical correlations were developed.

$$\frac{d_2}{d_1} = 912 \left( \frac{Fr_1^2}{Re_1} \right) + 25 \tan \theta + 3.456 \quad \dots\dots(17)$$

$$\frac{H_j}{d_1} = 912 \left( \frac{Fr_1^2}{Re_1} \right) + 25 \tan \theta + 2.456 \quad \dots\dots(18)$$

### VI. VALIDATION OF EMPIRICAL MODELS

#### A. Validation of Model Equation of Sequent Depth Ratio

The developed computational model equation (17) of sequent depth ratio is validated using Gandhi (2014) data and the figure of experimental value of  $Y_2/Y_1$  with predicted data of  $Y_2/Y_1$  is shown in fig 6. Every value are lying within  $\pm 10\%$  of the best fit curve and  $R^2=0.9761$ , unmistakably demonstrates the great fitting of present model condition.

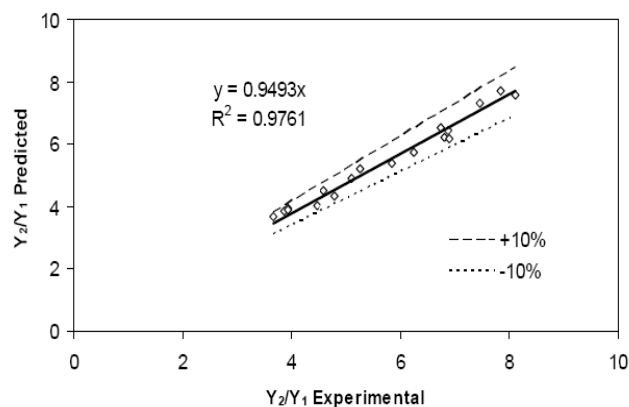


Fig 6 Validation of model equation 17 with Gandhi (2014) data

#### B. Validation of Model Equation of Sequent Depth Ratio

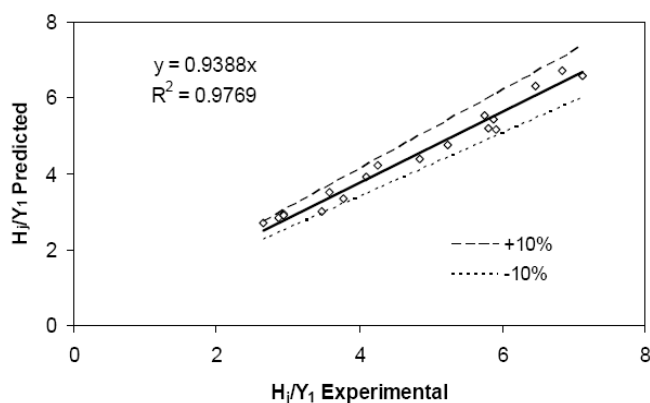


Fig 7 Validation of model equation 18 with Gandhi (2014) data

The developed computational model equation (18) of relative height of the jump is validated using Gandhi (2014) data and the figure of experimental data of  $H_j/Y_1$  with computational data of  $H_j/Y_1$  is shown in fig 7. Every value are lying within  $\pm 10\%$  of the best fit curve and  $R^2=0.9769$ , unmistakably demonstrates the great fitting of present model condition.

### VII. CONCLUSION

From the analysis it is found that the depth ratio of the hop and relative height of the hop increased with increment in methodology Froude number and slant of the channel however diminishes with increment in approaching Reynolds number.

Empirical correlations are developed that can be directly used in design of stilling channel of jump. The developed empirical models are applicable between approach Froude number 2.5 to 8.5 and approaching Reynolds number 5000 to 25000.



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NOMENCLATURE

$E_1$	Specific Energy before the Jump
$E_2$	Specific Energy after the Jump
$E_L$	Energy Loss
$E_{RL}$	Relative Energy Loss
$F_{r1}$	Approach Froude number
$Re_1$	Incoming Reynolds number
$H_j$	Jump Height
$L_j$	Jump Length
$R^2$	Coefficient of Determination
$V_1$	Flow velocity before the Jump
$V_2$	Flow velocity after the Jump
$Y_1$	Flow Depth before the Jump in Horizontal Channel
$Y_2$	Flow Depth after the Jump in Horizontal Channel
$d_1$	Depth of flow before jump in sloping channel
$d_2$	Depth of flow after jump in sloping channel
$\rho$	Density of the Water
$\mu$	Coefficient of Viscosity
$\eta$	Efficiency of the Jump

$\epsilon$  Roughness of the Surface  
 $\theta$  Slope angle

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