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Spectrum Sensing using Single Ring Law

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***Abstract**: The concept of cognitive radio is becoming increasing popular as it is a prominent solution for spectrum scarcity problem. A smart and wise usage of available spectral resources is an interesting feature of cognitive radio. In the terminology of cognitive radio, a primary (licensed) user and a secondary (unlicensed) user are usually heard. A secondary user transmits data only if the primary user does not use the alloted spectral resources. For sensing the presence or absence of the primary user, spectrum sensing is necessary. Conventionally many techniques such as energy detection (ED), eigenvalue based approaches have been designed for spectrum sensing. Recently large random matrix theory based analytics has shown that Single Ring Law (SRL) can be an effective solution for binary hypothesis testing problems. Hence, in this paper spectrum sensing for multiple antenna system is investigated using SRL based parameters. In Rayleigh and Nakagami fading channels, the SRL based detection is employed and has been found to be a consistent tool for spectrum sensing.*

***Index Terms**: spectrum sensing, single ring law, detection probability, opportunistic spectrum access, secondary user, energy detection*

I. INTRODUCTION

With the dawn of LTE and consecutive 5G systems, multiple input multiple output (MIMO) and massive MIMO systems have started to gain momentum [1]. Parallel studies show that the radio spectrum that is allotted across many communication services has been underutilized [2]. Some of the frequency bands are unutilized, some are less utilized and few others are over utilized. A cognitive radio facilitates a new communication system where in the user can transmit data on the same resources as that of the primary user, but the primary user does not suffer from interference due to the presence of secondary user. Such a radio helps in efficient resource utilization without demanding new resources. This is also termed as opportunistic spectrum access. The process of detecting if the primary user is present or absent is known as spectrum sensing.

Spectrum sensing can be accomplished by either blind or non-blind means. In non-blind techniques, the secondary user needs a prior knowledge of the waveforms and modulations used by the primary user. However blind techniques do not need such information. Hence cognitive radios with blind spectrum sensing have been popular. The simplest among all of these is the ED [3] and later eigenvalue based spectrum sensing techniques have also been popular [4, 5]. A sensing technique is generally chosen as a tradeoff between detection probability and computational complexity.

Recently, in a massive MIMO system the large data acquired at the base station has been modeled as a large random matrix [6, 7]. Corresponding asymptotic eigen spectral density was found to satisfy Marchenko Pastur (MP) law and SRL [8]. Both the MP law and SRL have been demonstrated to have powerful detection capabilities [9, 10]. Exploring this fact further, in this paper we investigate the ability of SRL for spectrum sensing.

We first develop a detection framework applicable to multiple antenna sensing systems and there by develop an SRL based spectrum sensing algorithm. Accordingly, using a parameter called the mean spectral radius (MSR) of the received data, detection of the primary user transmissions has been performed in both Rayleigh and Nakagami fading channel environments. It is found that the SRL based detection can render better probability of detection compared to not only ED but also eigenvalue based strategies like Akaike Information Criterion (AIC) and Minimum Description Length (MDL) based sensing approaches.

The rest of the paper is organized as follows. Section 2 illustrates the detection framework along with the SRL based sensing algorithm. Section 3 provides a comparison of the detection performance of the simulated system for ED, AIC, MDL and SRL based detection. The paper is concluded in section 4.

II. SYSTEM MODEL – SPECTRUM SENSING

Consider a multiple antenna sensing system with N_r sensing antennas grouped over an observation window of length M . The received data from all the N_r antennas at n[th] time instant can be represented as a column vector

$$X(n)=\left[x_1(n) \quad x_2(n) \quad \cdots \quad x_{N_r}(n)\right]^T \tag{1}$$

Using the convolutional relation between transmit data form the primary user and the channel behavior and the addition of an Additive White Gaussian Noise (AWGN), the received data $X(n)$ can be shown as

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$$X(n) = \mathbf{H}_{\mathbf{ch}} S(n) + W(n) \tag{2}$$

where, $W(n) = [w_1(n) \quad w_2(n) \quad \cdots \quad w_{N_r}(n)]^T$, $S(n) = [s(n) \quad s(n-1) \quad \cdots \quad s(n-L+1)]^T$ are the noise vector and the primary signal vector respectively and $\mathbf{H}_{\mathbf{ch}}$ is the channel matrix, given as

$$\mathbf{H}_{\mathbf{ch}} = \begin{bmatrix} h_1(0) & h_1(1) & \cdots & h_1(L-1) \\ h_2(0) & h_2(1) & \cdots & h_2(L-1) \\ \vdots & & \vdots & \vdots \\ h_{N_r}(0) & h_{N_r}(1) & \cdots & h_{N_r}(L-1) \end{bmatrix} \tag{3}$$

In (3), L is the length of the channel impulse response, which is assumed to be slowly varying in time and assumed constant in the window of length M. The hypothesis testing problem can be framed for the detection as follows

$$\begin{aligned} H_0 &: \overline{\mathbf{X}} = \overline{\mathbf{W}} \\ H_1 &: \overline{\mathbf{X}} = \mathbf{H}_{\mathbf{ch}} \overline{\mathbf{S}} + \overline{\mathbf{W}} \end{aligned} \tag{4}$$

where $\overline{\mathbf{X}} = [X(1) \quad X(2) \quad \cdots \quad X(M)]$ is the matrix of the received data whose dimension is $N_r \times M$ and $\overline{\mathbf{W}} = [W(1) \quad W(2) \quad \cdots \quad W(M)]$ is the noise matrix. IN accordance with (4) the primary user data is arranged as

$$\overline{\mathbf{S}} = \begin{bmatrix} s(1) & s(2) & \cdots & s(M) \\ s(0) & s(1) & \cdots & s(M-1) \\ 0 & 0 & \cdots & \vdots \\ & \vdots & \cdots & s(M-L+1) \end{bmatrix} \tag{5}$$

A sample covariance matrix $\overline{\mathbf{R}}_x = \mathbf{E}\{\overline{\mathbf{X}}\overline{\mathbf{X}}^{\mathbf{H}}\}$ can be calculated using $\overline{\mathbf{X}}$. Using this covariance matrix, the SRL for large dimensional matrix $\overline{\mathbf{X}}$ can be devised [20, 25]. Asymptotically, as N_r tends to infinity, the entries of $\frac{1}{\sqrt{M}}\overline{\mathbf{X}}$ are independent identically distributed (i.i.d) with zero mean and σ^2variance. Consider the spectral density of eigenvalues of $\frac{1}{M}\overline{\mathbf{R}}_x$ that follow MP law. By a unitary Haar transformation $\mathbf{U}$ on $\frac{1}{M}\overline{\mathbf{R}}_x$, consider the resultant matrix $\mathbf{Z} = \sqrt{\frac{1}{M}\overline{\mathbf{R}}_x}\mathbf{U}$. The eigen spectral density of the absolute eigenvalues (r) of $\mathbf{Z}$ follow a distribution given by

$$f_Z(r) = \begin{cases} \frac{2}{c} r & \sqrt{1-c} \le r \le 1 \\ 0 & otherwise \end{cases} \tag{6}$$

Distribution in (19) implies that the r of matrix $\mathbf{Z}$ fall within two circles, an inner circle with radius $\sqrt{1-c}$, where $c = \frac{N_r}{M}$ and the outer circle with radius 1. Also if entries of $\frac{1}{M}\overline{\mathbf{R}}_x$ are correlated, then the inner ring radius decreases compared to $\sqrt{1-c}$ [10]. Exploring this fact for application to signal detection in a multiple antenna system, a signal detection or a spectrum sensing can be performed. Note that $\overline{\mathbf{R}}_x$ has the dimension $N_r \times N_r$, which leads to only N_r eigenvalues.

The principle of sensing using SRL can be justified as follows. If the received data matrix has uncorrelated entries, it corresponds to a case that the data matrix is purely noise. If the matrix corresponds to the signal case, the received data must possess corrleated entries because presence of the primary user signal gets convolved with channel response that leads to correlated entries in the received data matrix. Hence spectrum sensing problem reduces to finding if the data matrix has correlated or uncorrelated entries.

According to SRL, if the entries are correlated, the ring radius is lesser than it should be for an uncorrelated cased. Hence by theoretically obtaining the ring radius for an uncorrelated case, and checking the ring radius from the received data with the theoretical value, we can identify the presence or absence of the primary user. This technique is illustrated in Table I.

Table I: SRL based spectrum sensing algorithm

SRL based Spectrum Sensing
Input: Received data matrix $\overline{\mathbf{X}}$, estimate of inner ring radius $\hat{r}$
1. Calculate $\overline{\mathbf{R}}_x = \mathbf{E}\{\overline{\mathbf{X}}\overline{\mathbf{X}}^H\}$
2. Compute the absolute eigenvalues of $\mathbf{Z} = \sqrt{\frac{1}{M}\overline{\mathbf{R}}_x}\mathbf{U}$
3. Calculate $MSR = \frac{1}{N_r}\sum_{i=1}^{N_r}\lvert\lambda_i\rvert$, where λ_i are the eigen values of $\mathbf{Z}$
4. Perform the test criterion: $MSR \underset{H_1}{\overset{H_0}{\gtrless}} \hat{r}$
Output: H_0 or H_1

According to Table I, $\hat{r}$ is the threshold used in the test criterion. It is the estimate of inner ring radius $r = \sqrt{1-c}$, which can be either estimated or calculated based on the noise power estimation or the number of antennas in the sensing system respectively. If N_r is very large, asymptotically the eigen spectral density satisfies SRL. In that case $\hat{r} = \sqrt{1-c}$ can be used. Otherwise, it can be estimated by simulating a noise alone condition which imlpies an SNR of -60 dB to -90 dB. Subsequently the MSR of the absolute eigenvalues can be estimated as $\hat{r}$.

III. SIMULATION RESULTS AND DISCUSSION

The system set up considered for simulation is as follows. The baseband data of a QPSK modulated wave is transmitted through a static channel with Brazil-A type power delay profile and the channel is assumed constant over a time window $M = 400$.

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The number of receive antennas is considered as $N_r = 4$ or $N_r = 8$. First we verify the validity of SRL for the case of $N_r = 8$.

Let the received data be obtained for $N_r = 8$ at an SNR of -80 dB which represents the no noise case. The absolute eigenvalues in accordance with (6) are distributed as shown in Figure 1. Similarly for an SNR of 5 dB, the eiegnvalues are shown in Figure 2. For generating Figures 1 and 2, $c = 0.16$ is used, which implies $N_r = 8, M = 50$. From Figures 1 and 2, we can obseve that ring radius decreases with increasing SNR.

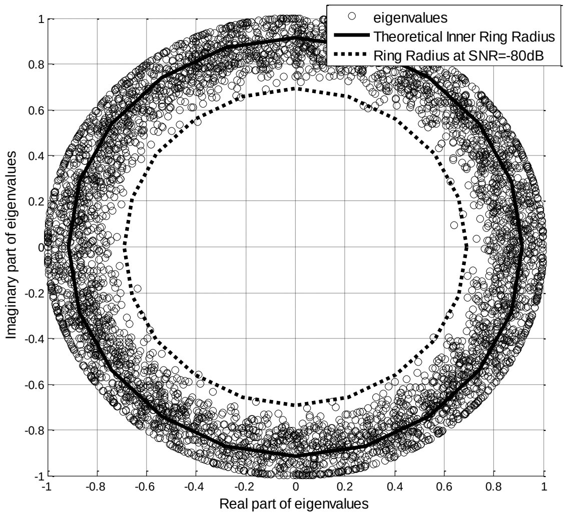

Figure 1: Eigen Spectral Density with SNR= -80 dB and $N_r = 8$

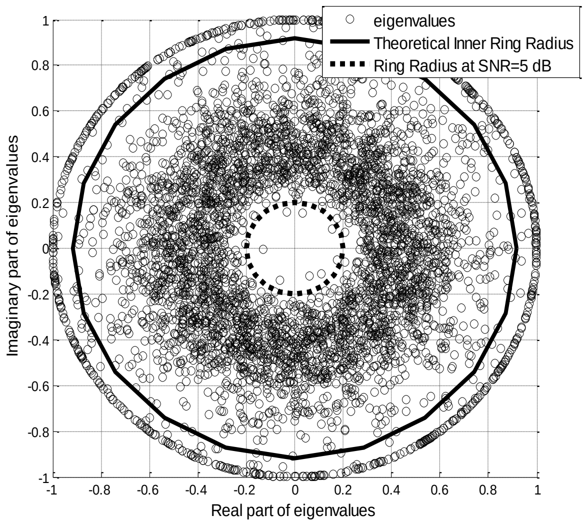

Figure 2: Eigen Spectral Density with SNR=5dB and $N_r = 8$

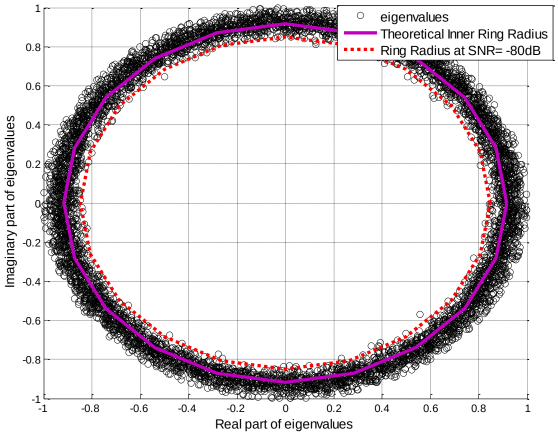

Figure 3: Eigen Spectral Density with SNR= -80 dB and $N_r = 64$

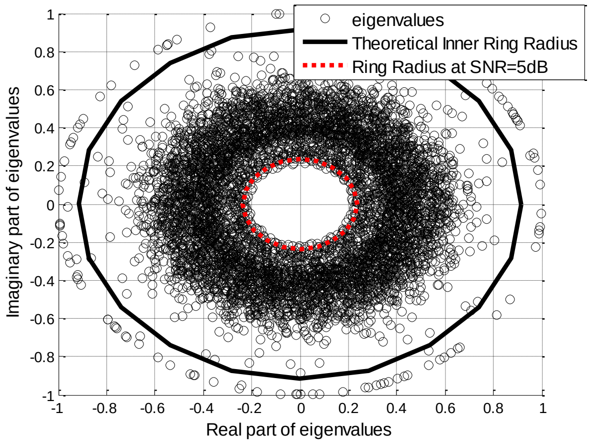

Figure 4: Eigen Spectral Density with SNR= 5 dB and $N_r = 64$

Now let $N_r = 64, M = 400$, which implies $c = 0.16$. The absolute eigenvalue distributions for SNR values of -80 dB and 5 dB as in Figure 3 and 4 respectively. Comparing both it can be seen that as SNR increases, the ring radius decreases. The radii obtained for both the SNR values for $N_r = 8$ and $N_r = 64$ are compared with their theoretical values in Table II.

Table II: Inner Ring Radius for different N_r and SNR values

N_r	SNR (dB)	Theoretical (Empirical) Inner Ring Radius	Inner Ring Radius from Simulations (Deviation)
$N_r = 8$	-80	0.9165 $\left(c = \frac{8}{50} = 0.16\right)$	0.6914 (25 %)
	-30		0.6904 (25 %)
	5		0.2049 (77 %)
$N_r = 64$	-80	0.9165 $\left(c = \frac{64}{400} = 0.16\right)$	0.8287 (10 %)
	-30		0.8243 (10 %)
	5		0.3355 (64 %)

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Highlight a section that you The following are the observations from Table II. At low SNR, the theoretical and simulated radii are more deviated for $N_r = 8$ compared to $N_r = 64$ case. For high SNR, in both the SNR cases the deviation is more because the SRL fails due t o correlated data entries. Note that the estimate of radius is empirically valid only if the entries are uncorrelated.

Hence for $N_r = 8$, the SRL fails at both low and high SNR. However the relative difference between the ring radius at both high and low SNR can itself serve to detect the presence or absence of a primary user, proving the ability of algorithm in Table I for spectrum sensing.

The inner ring radius be assessed practically speaking either under roughly no flag conditions, for example, SNR of - 80 dB or low flag conditions, for example, SNR of - 30 dB. In both these cases, the gauge is equivalent to seen from Table II.

For , eight distinctive Rayleigh blurring channels with Brazil A sort PDP are mimicked with multipaths at same estimations of postponement in all the four channels. Correspondingly the discovery probabilities are appeared as an element of SNR in Figure 5. The execution bends got for Nakagami channels with Brazil-A PDP are appeared in Figure 6. The outcomes in Figures 5 and 6 demonstrate that SRL based location is especially useful when the SNR is low. Anyway as SNR increments, however the enhancement offered by the SRL based discovery is superior to those offered by ED and AIC based location.

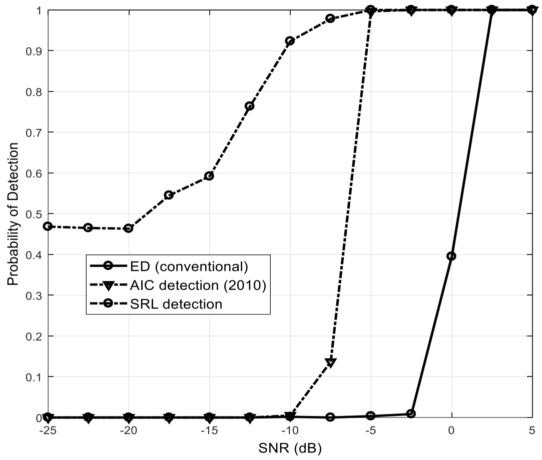

Figure 5: Detection Probability versus SNR (Rayleigh fading)

IV. CONCLUSION

In this paper, a detection framework based for massive sensing systems is considered. For detecting the presence of absence of the primary transmissions, a large random matrix theory based SRL has been employed. It is found that the SRL based sensing can be applied for both MIMO and massive MIMO systems. For simulated multiple antenna systems, with Rayleigh and Nakagami channel environments, SRL based detection has shown better detection perfomrance compared to conventional ED and the AIC based spectrum sensing technique. Hence it can be concluded that SRL based sensing can be an effective spectrum sensing system for multiple antenna sensing systems as well as massive sensing antenna systems, which can find place in either 4G or 5G systems.

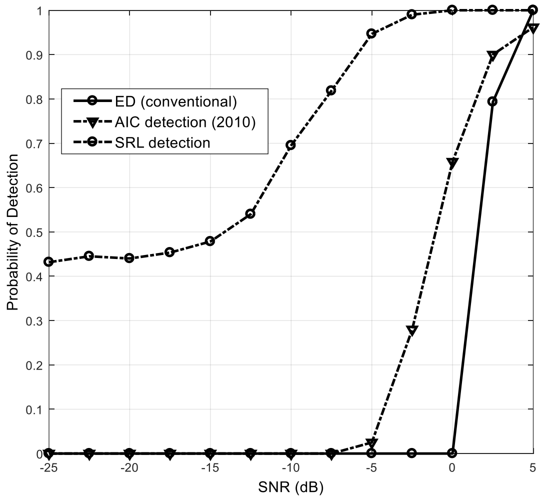

Figure 6: Detection Probability versus SNR (Nakagami fading)

REFERENCES

1. Ngo,Hien Quoc, Erik G. Larsson, and Thomas L. Marzetta. Energy and spectral efficiency of very large multiuser MIMO systems." IEEE Transactions on Communications 61 (4) (2013) 1436-1449.
2. Malik, Shahzad A.,Shah, M. A.,Dar,A. H., Haq, A., Khan, A. U., Javed, T., & Khan, S. A. "Comparative analysis of primary transmitter detection based spectrumsensing techniques in cognitive radio systems."Australianjournalofbasicandapplied sciences 4 (9) (2010) 4522-4531.
3. Urkowitz,Harry."Energydetectionofunknowndeterministic signals." Proceedingsof the IEEE 55.4 (1967) 523-531.
4. Zeng, Yonghong, andYing-Chang Liang."Spectrum-sensing algorithms for cognitive radio based on statistical covariances." IEEE transactions on Vehicular Techlogy 58 (4) (2009) 1804-1815.
5. Wang, Rui, and Meixia Tao. "Blindspectrumsensing byinformation theoreticcriteria." GlobalTelecommunicationsConference (GLOBECOM 2010), IEEE (2010) 1-5.
6. Ciuonzo, Domenico, Pierluigi Salvo Rossi,andSubhrakantiDey. "Massive MIMO channel-aware decision fusion." IEEE Transactions on Signal Processing 63 (3) (2015): 604-619.
7. Ding, Guoru, Xiqi Gao, Zhen Xue, Yongpeng Wu,andQingjiang Shi. "Massive MIMO for Distributed Detection with Transceiver Impairments." IEEE Transactions onVehicular Technology (2017).
8. Guionnet, Alice, Manjunath Krishnapur, and Ofer Zeitouni. "The single ring theorem." Annals of mathematics (2011): 1189-1217.
9. Pallaviram Sure,NarendraBabuC andChandraMohanBhuma, “Applicability of big data analytics to massive MIMO systems,” IEEE Annual India Conference (INDICON), IEEE (2016) 1-5.
10. Zhang, Changchun, and RobertC. Qiu. "Massive MIMOas abig data system: Random matrix models and testbed." IEEE Access 3 (2015) 837-851.

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