Natural Convection in a Square Enclosure Filled with Micropolar Fluid: Effect of Thermal Radiation

C Sivarami Reddy, V Ramachandra Prasad, K Jayalakshmi

Abstract: The present examination manages the investigation of shaky laminar common convective stream in a square walled in area loaded up with micropolar liquid. The vertical dividers of the nook are kept up at various uniform temperatures and best and base dividers are thermally protected. Nonlinear overseeing conditions defined in dimensionless frame and unraveled by numerically with Marker and Cell Method (MAC). Calculations have been done to break down with the impacts of Rayleigh number (Ra), Prandtl number (Pr) and vortex consistency parameter (k) both for feeble and solid fixation cases. Acquired processed outcomes have been exhibited as streamlines, isotherms and vorticity profiles and talked about through graphically. The impact of vortex thickness parameter (k) on stream rate just as rate of warmth exchange is analysed.

Keywords: Natural Convection, Micropolar Fluid, Cavity, Numerical results, MAC.

I. INTRODUCTION

Micropolar liquid is a subject of microphoric liquid hypothesis. The displaying of micropolar liquids and its detail depiction is presented by Eringen [1-3]. The micropolarliquids speaks to the liquids have the bar-like components, for example, fluid gems. The embodiment of micropolar liquid hypothesis of liquid streams lies in augmentation of constitutive conditions for Newtonian liquid streams so progressively complex liquid, for example, creature blood, fluid precious stones, colloidal suspension, grease, molecule suspension, and fierce shear streams, and so on can be portrayed by this hypothesis.

Common Convective warmth move in shut geometries has pulled in light of a legitimate concern for some inquires about because of the coupled warmth condition with liquid stream. A few scientists examined on normal convection in walled in areas loaded up with Newtonian liquid (k=0). An incredible audit article on Natural Convection in nooks with Newtonian liquid is given by Vahl Davis and Jones [4]. Their examination abridges and talks about the principle commitment and gives quantitative correlation between them.

The basic Rayleigh numbers for characteristic convection in air filled pit with differentially warmed have been dissected by Vahl Davis [5]. Common convection of micropolar liquid in a triangular nook is contemplated with time-subordinate by Sheremet et al. [6]. They are broke down warmth exchange rate of micropolar liquid in a correct edge wavy pit by numerically. The impact of a Local Heat Source on common convection in Trapezoidal Cavity Filled with a Micropolar Fluid is analyzed by Sheremet et al. [7]. They examined normal Nusselt number and liquid stream rate in trapezoidal depression with different estimations of warm parameter and Local Heat Source of an inclination divider. Characteristic convection of micropolar liquid in two vertical dividers under the impact of attractive field is broke down by Harshad et al. [8]. They contemplated hypothetically the limit layer stream of an incompressible micropolar liquid under uniform attractive field and movement happens because of the lightness drive between vertical dividers. Normal convection in micropolar liquid is analyzed with limit component strategy is displayed by MatejZadravec et al. [9].

Common convection is a physical wonder, where because of the nearness of temperature contrast between the body surfaces lightness powers showed up. The greater part of the liquids close to the uniform hot divider will have their thickness diminished, and an upward close divider movement will be actuated. Characteristic convection in a rectangular fenced in area loaded up with a micropolar liquid was exhibited in crafted by Hsu and Chen [10], where they displayed a parametric investigation of the impact of microstructure on the stream and warmth exchange. In their examination they utilized cubic splines collocation technique. They demonstrated that warm exchange rate and in this way likewise Nusselt number of a micropolar liquid is diminished contrasted with the Newtonian liquid. Chamkha et al. [11] led a numerical investigation of completely created normal convective of a micropolar liquid in a vertical channel. Characteristic convection of micropolar liquid with slip limit conditions under the vertical channel is analyzed by Ashmawy [12]. They saw that the expansion in the slip limit parameter expands the speed and just as declines the microrotation. Broad surveys of the hypothesis and uses of micropolar liquid streams can be seen in the audit articles [13, 14] and in the ongoing books [12, 15].

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In the present paper, roused the creator to research the issue of completely created two dimensional model of regular convective incompressible micropolar liquid stream in a nook. In addition, the more practical warm limit conditions and warm radiation are connected additionally its impact is considered.

II. BASIC EQUATIONS

The physical model considers a two dimensional unsteady flow within a closed enclosure filled with micropolar fluid of length L is depicted in Figure. 1. The vertical walls of the cavity are quantified at two fixed dissimilar temperatures $\left(T_h, T_c\right)$.

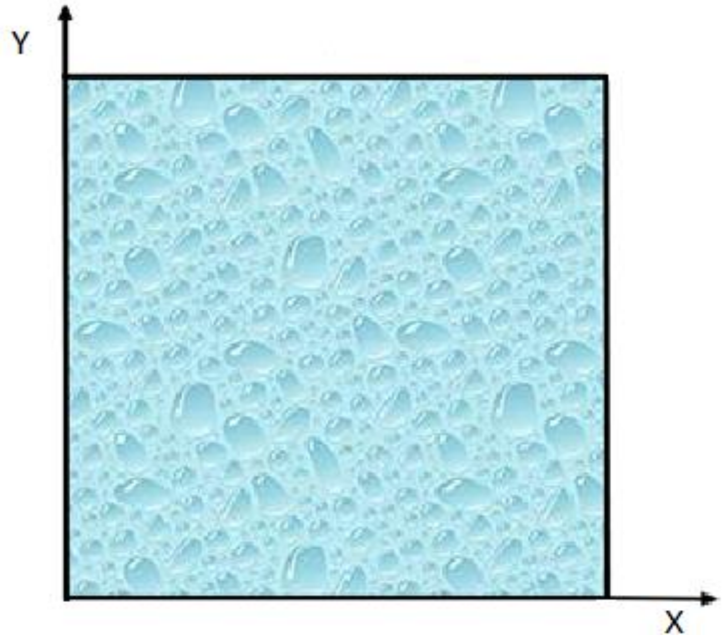

Fig. 1 Physical model

The left and right dividers are kept up at uniform temperature where and the rest of the dividers are thermally protected. The stream field is thought to be incompressible, Newtonian, and laminar with insignificant thick dispersal. There are no warmth age and synthetic responses, and furthermore warm radiation has been contemplated. The liquid properties are thought to be consistent, with the exception of thickness. Under these suspicions, the overseeing conditions for the two dimensional gooey incompressible liquid stream for characteristic convection in a square cavity loaded up with micropolar liquid can be spoken to as,

Continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu+k}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{k}{\rho}\frac{\partial \eta}{\partial y} \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\mu+k}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{k}{\rho}\frac{\partial \eta}{\partial x} \tag{3}$$

Angular momentum:

$$\frac{\partial \eta}{\partial t}+u\frac{\partial \eta}{\partial x}+v\frac{\partial \eta}{\partial y}=\frac{\gamma}{\rho j}\left(\frac{\partial^2 \eta}{\partial x^2}+\frac{\partial^2 \eta}{\partial y^2}\right)-\frac{2k}{\rho j}\eta+\frac{k}{\rho}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right) \tag{4}$$

Energy:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-\frac{1}{\rho c_p}\left(\frac{\partial q_{rx}}{\partial x}+\frac{\partial q_{ry}}{\partial y}\right) \tag{5}$$

By using the Rosseland diffusion approximation, the flux inside the enclosure is calculated as

$$q_{rx} = -\frac{4\sigma^*}{3a_R}\frac{\partial T^4}{\partial X} \qquad q_{ry} = -\frac{4\sigma^*}{3a_R}\frac{\partial T^4}{\partial Y}$$

Where $\gamma = \left(\mu + \frac{k}{2}\right)j$, it is expected that the temperature contrasts inside the stream are to such an extent that the term T^4 can be considered as a direct capacity of temperature. This is cultivated by growing in a T^4 Taylor arrangement about T^4_c and dismissing higher request terms.

$$T^4 = 4TT_c^3 - 3T_c^4$$

Using the following non-dimensional variables

$$\tau = \frac{t\alpha}{L^2}, (X,Y) = \frac{(x,y)}{L}, U = \frac{uL}{\alpha}, V = \frac{vL}{\alpha}, P = \frac{pL^2}{\rho\alpha^2}, \theta = \frac{T - T_c}{T_h - T_c}, N = \frac{\eta L^2}{\alpha}$$

$$Pr = \frac{\upsilon}{\alpha}, Ra = \frac{g\beta(T - T_c)L^3 Pr}{\upsilon^2}$$

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{6}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + (1+K)Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) + K.Pr\frac{\partial N}{\partial Y} \tag{7}$$

$$\frac{\partial V}{\partial \tau} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + (1+K)Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) - KPr\frac{\partial N}{\partial X} + Ra.Pr\theta \tag{8}$$

$$\frac{\partial N}{\partial \tau} + U\frac{\partial N}{\partial X} + V\frac{\partial N}{\partial Y} = \left(1 + \frac{K}{2}\right)Pr\left(\frac{\partial^2 N}{\partial X^2} + \frac{\partial^2 N}{\partial Y^2}\right) - 2KPrN + KPr\left(\frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y}\right) \tag{9}$$

$$\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \left(1 + \frac{4Nr}{3}\right)\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{10}$$

Where the radiation parameter is $Nr = \frac{4\sigma^* T_c^3}{a_R \alpha}$, σ^* is Stephan Boltzmann constant and a_R is the Rosseland mean spectral absorption coefficient.

With the following boundary conditions are

$\tau = 0\ U = V = \theta = N = 0$

For $0 \le Y \le 1\ \ 0 \le X \le 1$

$\tau > 0 \quad U = V = 0,\ \frac{\partial \theta}{\partial Y} = 0\ N = -n\frac{\partial U}{\partial Y}$ at Y=0, 1.

$U = V = 0, \theta = 1\ N = -n\frac{\partial V}{\partial X}$ at X=0

$U = V = 0, \theta = 0\ N = -n\frac{\partial V}{\partial X}$ at X=1

The fluid motion is represented using the stream function ψ evaluated with dimensionless velocity components U and V. The worthy relationship between the velocity components and stream function ψ are

$U = \frac{\partial \psi}{\partial Y}$ and $V = -\frac{\partial \psi}{\partial X}$

The Local Nusselt numbers are computed with the following relation

$$Nu = -\left.\frac{\partial \theta}{\partial X}\right|_{X=0,1}$$

III. METHOD OF SOLUTION

Non-dimensional administering halfway differential conditions (7)- (10) were comprehended numerically. The discretization of the time subordinate is performed by certain plan utilizing amazed matrix, its outstanding as MAC cell strategy. The velocity– weight inclination solid coupling present in the progression and the projection strategy [16] executing on energy conditions. The exceptional limited contrast plan of V. Ambethkar et al. [17] and Harlow and Welch [18] is actualized to limit the numerical dispersion for the shift in weather conditions terms. The Poisson condition of weight angle which is settled utilizing a quickened full multigrid technique, while the focal limited distinction discretized conditions are understood utilizing the SOR (progressive over-unwinding) strategy [19] with the decision of unwinding factors. At long last, the intermingling of the numerically processed outcomes is set up at each time venture as per the accompanying basis:

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$$\sum_{i,j}\left|\Omega^{k+1}_{i,j}-\Omega^{k}_{i,j}\right|<10^{-6}$$

The conventional variable Ω stands for U,V,P,θ,N and k represents, and k signifies the cycle time levels. In the above imbalance the subscripts i,j speaks to the space facilitates X and Y individually.

Code validation

The exactness of the MAC numerical strategy utilized for the arrangement of the issue under thought, it was approved (in the wake of making the essential changes) with the issue of the impact of lightness proportion on twofold diffusive normal convection in a cover driven cavity revealed before by Tapas Ray Mahapatra et al. [20]. The examinations for the streamlines, isotherms, Iso-focus shapes are exhibited in Figure 2 for case consistently warmed and consistently thought with N=50, . Tapas Ray Mahapatra et al. [20] utilizing Control-volume based limited contrast. These examination gives great understanding. This loans certainty into the commendable precision of the present numerical code.

IV. RESULTS AND DISCUSSION

The present MAC Computations are completed for the administering parameter both for the frail focus (n = 0) and for the solid fixation (n = 0.5) with Pr=0.71, Ra=103, 104, 105 and 106 with different estimations of K. Strikingly, no distinction is seen between the aftereffects of the feeble and solid fixation cases. Subsequently, we present here outcomes just for the instance of powerless fixation case (n = 0). We should see again that the instance of K = 0 speaks to the Newtonian liquid.

Present Results Mahapatra *et al*.[20]

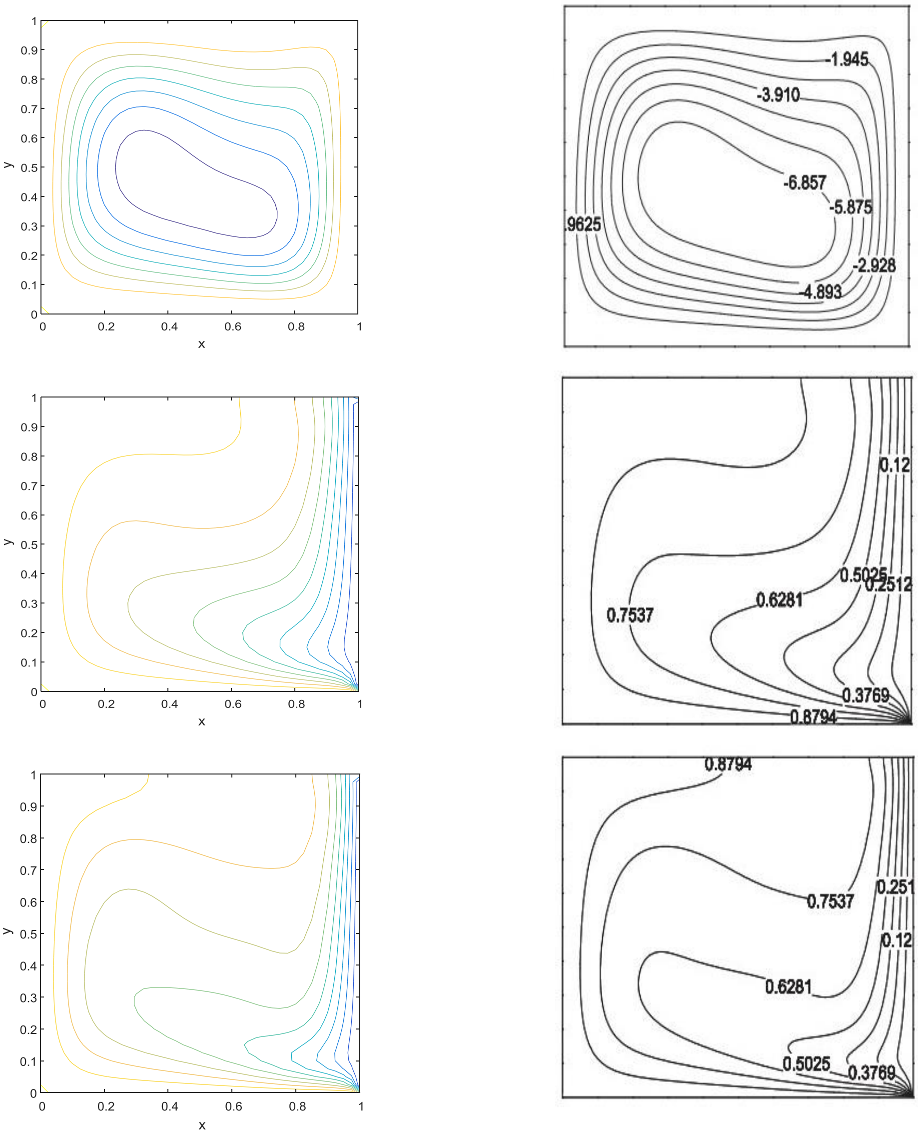

Fig. 2 Comparision of the Streamlines, Isotherms and Isoconcentrations for uniformly heated and uniformly concentrated left and bottom walls with N=50, Pr=0.7 and Ra=10^3

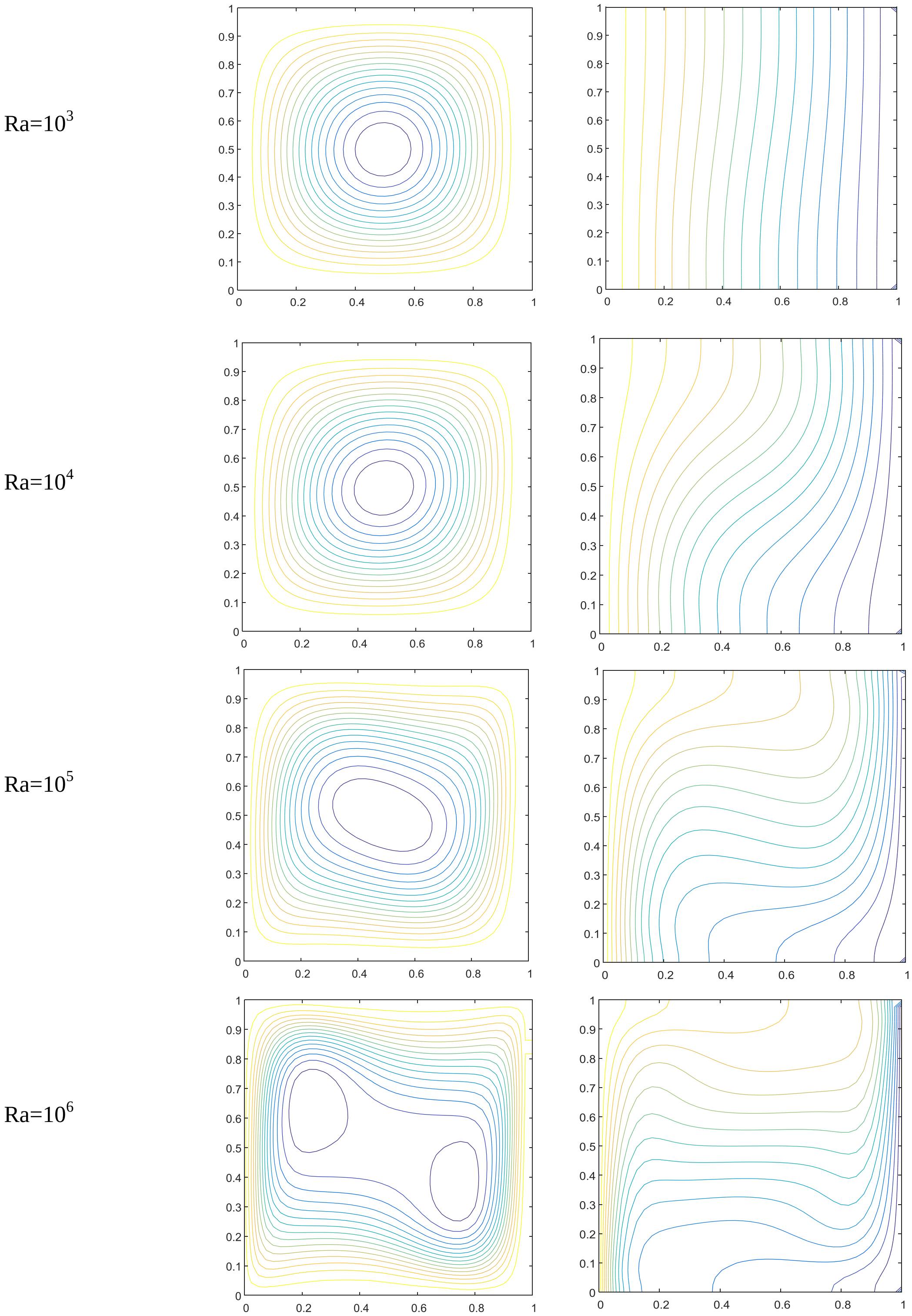

Fig. 3 Streamlines and isotherms for Nr=1, Pr= 0.71, K=1

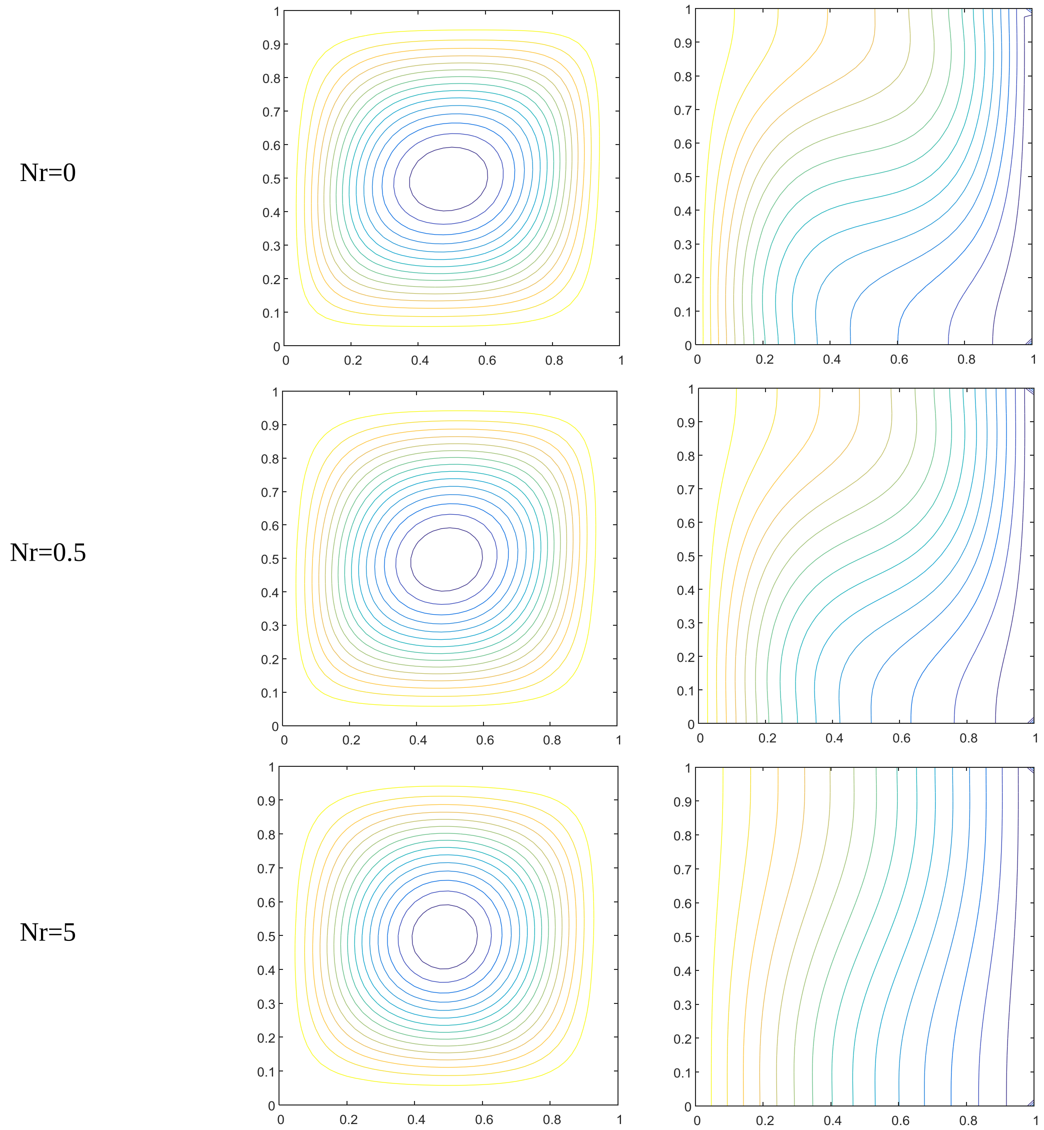

Fig. 4 Streamlines and isotherms for Ra=10^4, Pr= 0.71, K=1

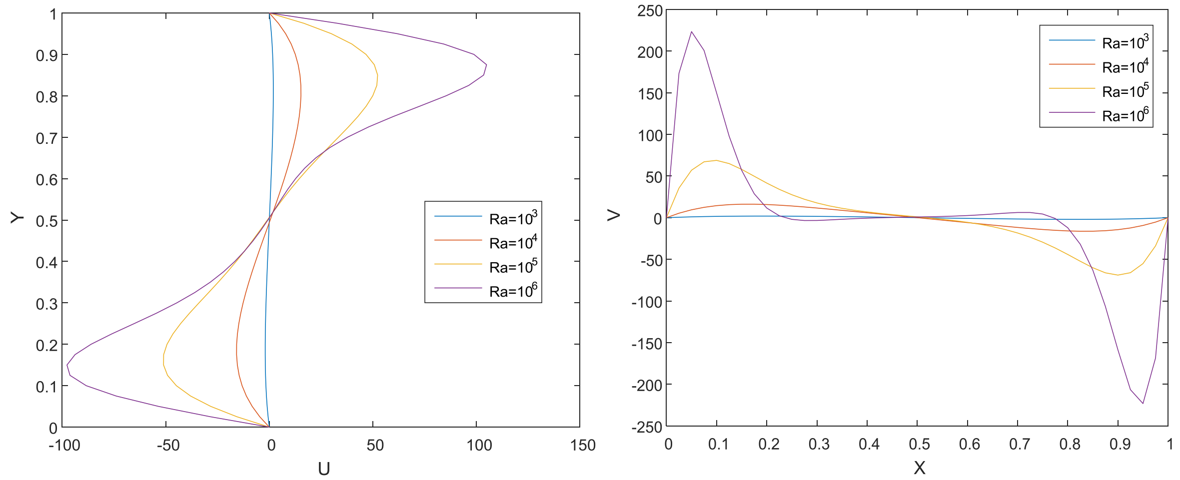

Fig. 5 Midsection velocity of U and V for Nr=1, Pr= 0.71, K=1

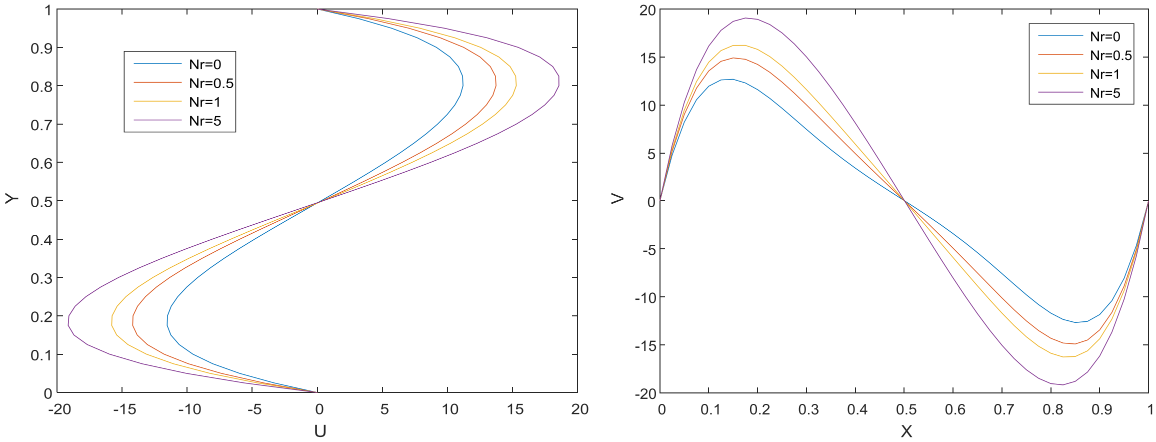

Fig. 6 Midsection velocity of U and V for Ra=10^4, Pr= 0.71, K=1

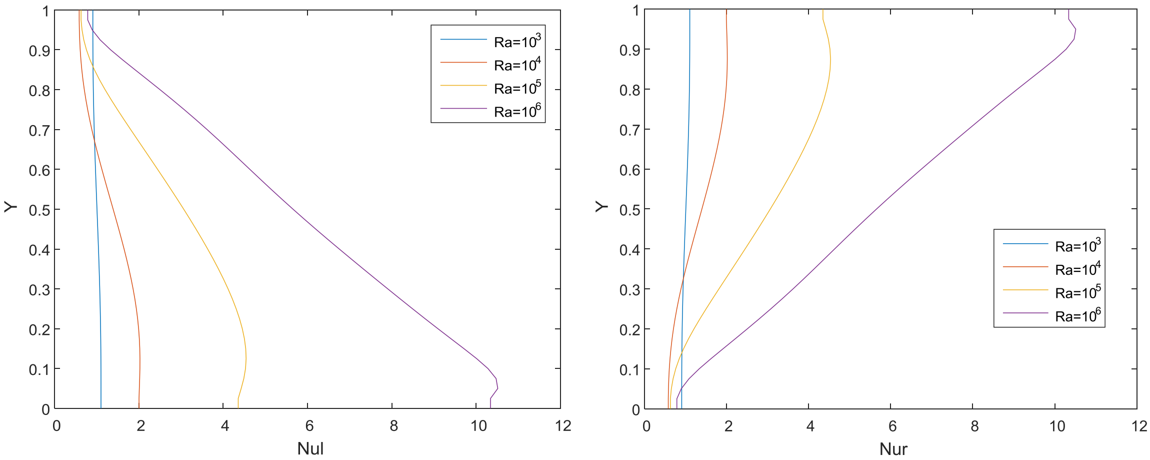

Fig. 7 Local Nusselt number of hot and cold walls for Nr=1, Pr= 0.71, K=1

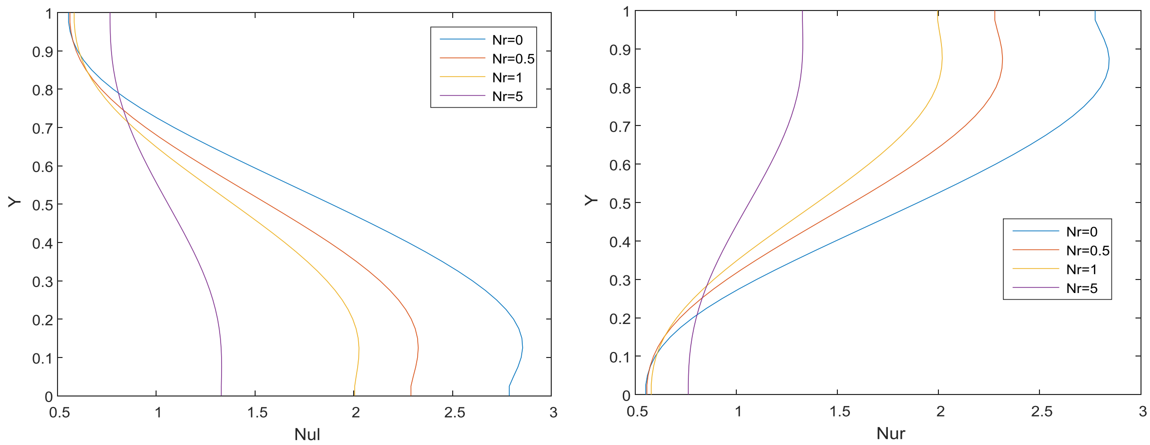

Fig. 8 Local Nusselt number of hot and cold walls for Ra=10^4, Pr= 0.71, K=1

Fig.3 depicts the flow patterns and temperature contours for various values of Rayleigh number Ra with $Nr=1$, $Pr=0.71$, $K=1$. The fluid flow circulates in the enclosure at middle and the corresponding isotherms are distributed whole enclosure, the temperature contours are parallel to vertical wall at $Ra=10^3$. The flow patterns and the isotherms are changed slightly with increasing of Rayleigh number $Ra=10^4$. We observe the isotherms are parallel neighbouring the bottommost portion of left wall and highest portion of right wall only for $Ra=10^4$. From the hot wall, the heat has been transferred to cold wall for increases more for enhancing the Rayleigh number.

The mono flow eddy is developed diagonally and the corresponding isotherms clustered at bottom portion of hot wall and top portion of cold wall for $Ra=10^5$, the horizontal walls in the middle of the enclosures are isotherms patterns and they are parallel each other at the Rayleigh number $Ra=10^5$. At $Ra=10^6$, the fluid flow is driven diagonally in middle of the enclosure and also developed two eddies near the hot and cold walls.

The isotherms are more clustered near the hot wall and cold wall, the middle of the enclosure the contours of temperature is parallel to the horizontal walls. The thermal boundary layer is formed at hot and cold walls for the influence of Rayleigh number $Ra=10^5$. The formed thermal boundary layer of a hot wall is decreases along from bottom to top and the opposite influence is observed on a cold wall.

Fig.4 presents the inspiration ofnumerous values of thermal radiation parameter on flow pattern and isotherm contours for $Ra=10^4$, $Pr= 0.71$, $K=1$. The fluid flow circulation is gradually changed with increasing of thermal radiation parameter *Nr*. The variation of isotherm contours are affected by different values of *Nr*. The temperature contours inclination is noticed high at the absence of thermal radiation parameter *Nr* and this nature is decreases with increasing of thermal radiation. The convective heat transfer in the enclosure is transformed into transmission mode with effect of enhancing the thermal radiation parameter.

Fig.5 and 6 illustrates the central line velocity profiles of U and V underneath the stimulus of Rayleigh number and thermal radiation parameter respectively. The U and V are the velocity fields which are increases with increases of thermal Rayleigh number with fixed fluid parameters *Nr=1,Pr= 0.71, K=1*are shown Fig.5.The mid – section velocity profile of U and V exhibits sinusoidal nature with the affected by thermal radiation parameter with $Ra=10^4$, $Pr= 0.71$, $K=1$. The speed profiles are step by step increments with the expanding warm radiation parameter, it is portrayed in Fig.6 for. The nearby Nusselt number of dynamic vertical dividers with the impact of warm Rayleigh number and warm radiation parameter are exhibited Fig.7 and 8 individually. The warmth exchange rate is increments with expanding of Rayleigh number for Nr=1, Pr= 0.71, K=1 are portrayed in Fig.7. The expanding of warm radiation parameter Nr then the rate of warmth exchange is diminishes for $Ra=10^4$, $Pr= 0.71$, $K=1$ are shown in Fig.8.

V. CONCLUSIONS

Characteristic convection in a hole loaded up with micropolar liquid affected by warm radiation has been considered numerically by Marker and Cell (MAC) strategy. The left mass of the cavity warmed consistently while right divider is cooled at a steady temperature and best and base dividers are thermally protected. The warm radiation is consider in course of X and Y headings. The impact of warm radiation parameter and Rayleigh number parameter on midriff speed profiles is watched, these impact is increment the liquid stream rate. The nearby Nusselt number is seen at the left and right dividers, the ascent of warm Rayleigh number builds the warmth exchange rate and the liquid disseminations changes bit by bit. The warmth move rate in cavity is diminishing on expanding the warm radiation parameter.

Nomenclature

a_R = Rosseland mean spectral absorption coefficient
C_p = specific heat
j = micro –inertia density
k = vortex viscosity
K = dimensionless vortex viscosity parameter
L = length of the enclosure
n = micro-gyration parameter
N = non -dimensional micro rotation angular velocity
Nr = non –dimensional thermal radiation parameter
Nu= local Nusselt number
Nul = local Nusselt number of left vertical wall
Nur = local Nusselt number of left vertical wall
p = pressure
P = non-dimensional pressure
Pr = Prandtl number
q_{rx} = radiative heat flux in x-direction
q_{ry} = radiative heat flux in y-direction
Ra = Rayleigh number
t = time
T = dimensional Temperatre
(u, v) = dimensional velocity components
(U, V) = non- dimensional velocity components
(x, y) = dimensional coordinate axes
(X, Y) = non- dimensional coordinate axes

Greek Symbols

γ = spin-gradient viscosity
ρ = fluid density
α = thermal diffusivity
σ^* = Stephan Boltzmann constant
μ = dynamic viscosity
η = dimensional angular momentum
θ = dimensionless temperature
τ = dimensionless time

REFERENCES

1. A.C Eringen, Simple microfluids, Int. J. Eng. Sci. 2 (1964) 205-217.
2. A.C Eringen, Theory of micropolar fluid, J. Math. Mech. 16 (1966) 1-18.
3. A.C Eringen, Theory of thermomicrofluids, J. Math. Anal. Appl. 38 (1972) 480-496.
4. G. De Vahl Davis and P. Jones, Natural convection in a square cavity: A comparison exercise, International Journal for Numerical Methods in Fluids, 3 (1983) 227-248.
5. G. De Vahl Davis, Natural convection of air in a square cavity: A benchmark numerical solution, International Journal for Numerical Methods in Fluids, 3 (1983) 249-264.
6. Mikhail A. Sheremet, Ioan Pop, AnuarIshak,Time-dependent natural convection of micropolar fluid in a wavy triangular cavity, International Journal of Heat and Mass Transfer 105 (2017) 610–622.
7. Igor V.Miroshnichenko, Mikhail A.Sheremet, Ioan Pop, Natural convection in a trapezoidal cavity filled with a micropolar fluid under the effect of a local heat source, International Journal of Mechanical Sciences, 120 (2017) 182-189.
8. Hari R. Kataria, Harshad R. Patel, Rajiv Singh, Effect of magnetic field on unsteady natural convective flow of a micropolar fluid between two vertical walls, Ain Shams Engineering Journal 8 (2017) 87–102.

9. MatejZadravec, Matjazˇ Hribersˇek, LeopoldSˇ kerget, Natural convection of micropolar fluidinanenclosurewithboundary element method, Engineering Analysis with Boundary Elements 33 (2009) 485–492.
10. AhmadiG.Self-similarsolutionofincompressiblemicropolarboundarylaye flow overasemi-infiniteflatplate, IntJEngSci 14 (1976) 639–646.
11. A.J. Chamkha, T. Grosan, I. Pop, Fully developed free convection of a micropolar fluid in a vertical channel, Int. Commun. Heat Mass Transfer 29 (2002) 1119–1127.
12. A.C. Eringen, Microcontinuum Field Theories: II. Fluent Media, Springer-Verlag, New York, 2001.
13. T. Ariman, M.A. Turk, N.D. Sylvester, Microcontinuum field mechanics – a review, Int. J. Eng. Sci. 11 (1973) 905–929.
14. T. Ariman, M.A. Turk, N.D. Sylvester, Applications of microcontinuum field mechanics, Int. J. Eng. Sci. 12 (1974) 273–291.
15. G. Lukaszewicz, Micropolar Fluids, Theory and Application, Birkha¨ user, Basel, 1999.
16. Brown DL, Cortez R, Minion ML (2001),Accurate projection methods for the incompressible Navier–stokes equations. J ComputPhys 168:464–499
17. Ambethkar, Mohit Kumar Srivastava, Numerical Study of an Unsteady 2-D Incompressible Viscous Flow with Heat Transfer at Moderate Reynolds Number with Slip Boundary Conditions, International Journal of Applied Mathematics, Volume 25 No. 6 2012, 883-908.
18. Francis H. Harlow, J. Eddie Welch, Numerical calculation of time dependent viscous incompressible flow of fluid with free surface, Phys. Fluids, 8 (1965), 2182-2189.
19. Barrett R et al. (1994) Templates for the solution of linear systems: building blocks for iterative methods. SIAM Press, Philadelphia.
20. Tapas Ray Mahapatra, Dulal Pal, SabyasachiMondal, Effects of buoyancy ratio on double-diffusive natural convection in a lid-driven cavity, International Journal of Heat and Mass Transfer 57 (2013) 771–785.