Minimize Aggregate Measure of Waiting Times and Queue Lengths in M/G/1 Queue

Roweda M.A. Mahmuod, Rahela Rahim

Abstract: The classic optimization techniques are helpful to find the optimum resolution or free maxima or minima of continuous and differentiable functions. The target of this study is to develop a replacement formula to solve the pedestrian congestion problem based on queuing theory and optimization that will lead to an efficient algorithm for pedestrian flow in optimizing allocation of pedestrian and service capacity subject to limited buffer space by show a mathematical model that minimizes an aggregate measure of waiting times and queue lengaths for a group of arrivals by using Lagrange multiplayer. Ls (expected variety of consumers within the i^{th} system, Ws (expected time employment spends within the i^{th} system), Lq(expected variety of consumers within the queue in the i^{th} system) ,Wq(Expected time employment spends in the queue in the i^{th} system) decreased compared with the naïve value of λi. A queuing model calculator used to calculate the optimal values.

Keywords: Queuing Network, Optimization, M/G/1 queue.

I. INTRODUCTION

Queuing theory will be outlined as a mathematical study of looking forward to lines or queues. Models from queuing theory are widely used as useful tools toward controlling congestion delays and queuing issues are most common options not only in way of life things like as a bank or post workplace, ticketing workplace, in publicly transportation or during a traffic congestion but additionally in more technical environments like in producing, Pc networking and telecommunications [1]. The origin of queuing theory come back to early within the last century once Erlang, a Danish engineer, applied the idea to review the behavior of telephone networks., Erlang developed more queuing that is considered the backbone of queuing until today [2]. From that time until now so many studies were done using queueing theory [3,6]. One of the most important works has done on the theory of queuing systems and their computer applications done by [3] once planned a queueing discipline for a single-server queue within which customers from totally different categories accumulate priority as linear functions of their waiting time. At the moment that a server becomes free, it selects the waiting client with the best accumulated priority, as long as the queue is nonempty. He developed a formula for scheming the expected waiting time for every category. [4] Considers queuing phenomena with relation to its applications in performance analysis of Pc and communication systems. Moreover, [5] focus on analyzing the performance and optimization of queueing networks. Queueing network models are recognized as necessary helpful tools for evaluating the performance of Pc systems and also the communication network [6]. The Queueing Theory may be a mathematical study that describes, analyze and optimize of a queueing system [8]. The queuing system focuses on some key performance measures, like queue lengths and waiting times either within the queue or within the system. Thanks to the random nature of the arrival and repair processes, and of the routing method of jobs through a network of queues, the most performance measures are random variables therefore it will use the multiple queues multiple server models to represent a central job routing system [9]. [10] Given a model that minimizes an aggregate measure of waiting times and queue lengths for a group of workstations. Showed that the naïve allocation of arrivals among workstations in proportion to their process times doesn't end in an optimized system. This study has been used Lagrange multiplayer technique to optimize the queuing system by minimizing waiting time and queue lengths in M/G/1 allocation The analysis aim is to propose a mathematical modeling supported on queuing theory and optimization of a pedestrian sp as to systemize the doorway and exit of facilities to avoid congestion. The most important contribution of this analysis to make new arrival allocation model for optimizing pedestrian flow by improved allocation mechanism during a pedestrian walking system. This work will be used as a general methodology and may be applied on another sort of queueing in several functions.

II. METHODOLOGY

In this paper we use Lagrange multiplayer optimization method to develop a new algorithm to solve the pedestrian congestion problem based on M/G/1 queuing system.

Let Q denote a group of n M/G/1 queueing systems and outline the subsequent notation:

n= number of M/G/1 queuing systems belongs to Q

λi = Average arrival intensity at n customers/jobs within the i^{th} system

μi = Average service intensity for the system once there are n customers/jobs within the i^{th} system.

ρ = The utilization factor for the service facility.

Lsi = Expected range of consumers within the i^{th} system

Lqi = Expected range of consumers within the queue in the i^{th} system.

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Wsi = Expected time employment spends within the i^{th} system.
Wqi = Expected time a employment spends within the queue in the i^{th} system

Where the single server queue with Poisson arrivals and unlimited capacity M/G/1 can be defined as:

$\rho=\lambda i\mu i$ (1)

$Ls=\rho+2(12+2)2(1+\rho)=\rho+2(1+22)2(1-\rho)$ (2)

$Ws=1+\lambda(12+2)2(1-\rho)$ (3)

$Wq=\lambda(12+2)2(1-\rho)$ (4)

$Lq=2(12+2)2(1+\rho)$ (5)

We drive the model for Lsi forced improvement problem:

Minimize

$i=1nLsi=i=1n\rho+2(12+2)2(1+\rho)=i=1n\lambda i\mu i+i2(1i2+2)2(1+\lambda i\mu i)$ (6)

Subject to:

$i=1n\lambda i=M$ (7)

Where M is constant specified M•∑μ

Optimization drawback resolved by using Lagrange technique as forward m is Lagrange multiplayer then the Lagrange operate outlined as:

$L1,2,\ldots,m=i=1n\lambda i\mu i+i2(1i2+2)2(1+\lambda i\mu i)-m(i=1n\lambda i-M)$ (8)

The solution to the improvement operate is found by setting:

$\partial Li=0$ and $\partial Lm=0$

This yields to the subsequent equation:

$\partial Li=1+\lambda(22+1)(2\mu-\lambda)2\mu(\mu-\lambda)2-m=0$ (9)

The solution to the on top of equation is:

$i=\mu+A\mu-2(1+A)1+A$ (10)

Where

$A=m.2222+1$

For Ws the target operate and best resolution is that the same as Ls since:

$Ws=1+\lambda(12+2)2(1+\rho)=1+Ls$ (11)

$Wsi=i=1n1i+i=1nLsiM$ (12)

$i=1nLsiM=-i=1nWsii$

Next, we tend to derive the model for Lqi and Wqi considering the subsequent strained improvement problem:

$i=1nLqi=i=1nLsi-Mi=1ni$ (13)

$i=1nWqi=i=1nWsi-1i=1ni$ (14)

III. RESULTS

Sample Case for General Service distribution

To demonstrate the pertinence of the results obtained within the previous sections, numerical results are conferred to check between naïve and best model for various values of n and λis. Queuing system calculator has been accustomed calculate the best values of model for varied objective functions together with Ls, Ws ,Lq and Wq . Table 1 display the end performance measures for the queuing systems. In these case (n=1,2) the best allocation of arrivals decreases the combination Ls, Ws, Lq, Wq compared to the naïve allocation of arrivals.

Table. 1 Performance Measures for the Queuing Systems

∑i	Naïve		Optimal		Ls	Lq	Ws	Wq	Queue's Performance Percentage
	λ1	λ2	λ1	λ2					
2.8	1.9	0.9			37.12	35.16	23.7	22.2	
			1.84	0.96					
					35.66				1.01%
						34.48			1.02%
							14.75		1.61%
								13.25	1.68%
2.7									
	1.9	0.8			37.008	35.158	23.675	22.175	
			1.81	0.89	21.17				1.75%
						19.375			1.81%
							8.391		2.82%
								7.891	2.81%
2.4									
	1.8	0.6			15.324	13.824	9.69	9.11	
			1.58	0.82					
					9.24				1.66%
						7.64			1.81%
							6.04		3.19%
								1.531	5.95%
=2,1	^{2} = 0.5								

Figure 1, Figure 2 shows the Ls and Ws prices for various value of λi and therefore the same $\sigma^2 = 0.5$, During this case wherever σ^2 is constant the worth of best model is decrease compare with naïve allocation of arrivals.

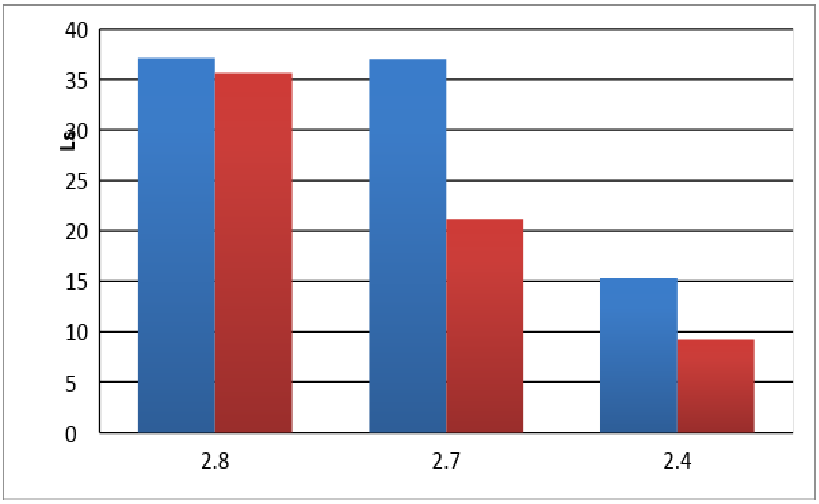

Fig. 1 Shows the Ls values for different value of λi and the same $\sigma^2 = 0.5$

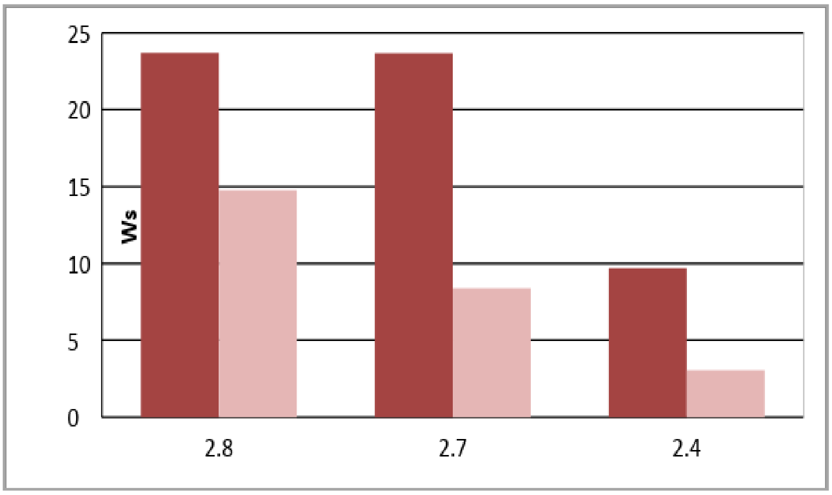

Fig. 2 Shows the Ws values for different value of λi and the same $\sigma^2 = 0.5$

In Figure 3 show the prices of Ls in best and naïve value with totally different values of σ^2. The price of best model decrease compare with naïve allocation of arrivals and therefore the deference increase with σ_2 value. From table 1, Fig.1, Fig.2 and Fig.3. We are able to see clearly the distinction in (Ls,Lq,Ws,Wq) between a normal queuing model and planned one.

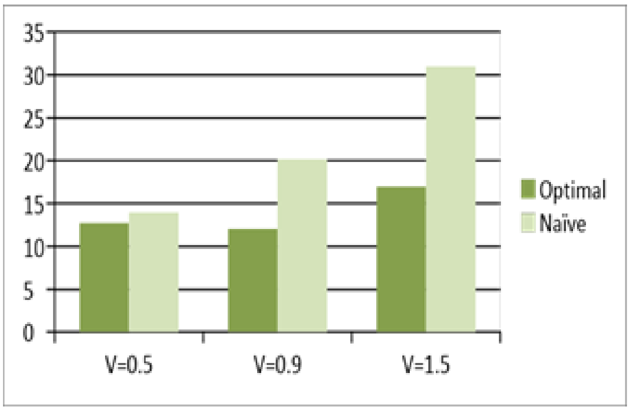

Fig. 3 Shows the Ls values for different values of σ^2

IV. CONCLUSIONS

The minimizes of an combination measure of waiting times and queue lengths for a set of arrivals by using Lagrange multiplayer may be a terribly helpful technique that can reduce Ls,Ws,Lq,Wq. This mathematically technique may be accustomed reduce the arrogate measure to a special kind of queuing models. The result obtained from our analysis could also be applied to several totally different universe things. The results of our analysis is easy enough to use to several differing types of parallel workstations in allocating restricted range of servers.

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