Numerical Solution of PDE Using Two Dimensional Chebyshev Wavelet Collocation Method

V.Sumathi, S. Hemalatha, B.Sripathy

***Abstract:** In this current work, we investigate a new computational scheme to solve a system of Partial Differential Equations. To handle this method, we initially construct a Two Dimensional Chebyshev wavelet which is used to transform the PDE's to a linear system of algebraic equations. We approximate the obtained algebraic equations using collocation method. This algorithm can be easily implemented to solve PDE with boundary conditions. We illustrate with examples to analyze the convergence using this Two Dimensional Chebyshev collocation method. Finally, we show the validity, efficiency and applicability of this new technique with some Numerical Examples.*

***Keywords:** Two Dimensional Chebyshev Wavelets, Operational matrices, Collocation points.*

I. INTRODUCTION

In the Universe of Mathematics, It is pretty known that the numerical methods have played a vital role in solving (PDES). Wavelets are efficient tool in numerical modeling of physical problems and to build a bridge to avoid the communication gap. Wavelet theory developed mostly over the last 30 years which has generated a tremendous interest in many areas of research not only in mathematics but also in physics, computer science and Engineering. However most of the applications of wavelets have focused on data analyzing and data compression. The main advantage of this mathematical tool is to diminish the communication gap before the emerging researchers in this infancy field of wavelets. The fundamental logic behind wavelet is to represent a function both in time and frequency domains. We can emphasize that Numerical collocation methods based on wavelets will have good resolution. Wavelets were developed exclusively in the field of quantum physics, Electrical Engineering, Seismic geology, Image Compression, Musical tones and de-noising noisy data. Wavelets are functions that satisfy certain mathematical requirements and are used in representing data or other functions. Other applied fields that make use of wavelets include Astronomy, Acoustics, Nuclear Engineering sub-band coding, Signal Processing, Neuro physiology, Magnetic resonance Imaging, Speech discrimation, Optics, Fractals Turbulence, Earthquake prediction, Radar human vision etc.

There are many researchers in literature using Chebyshev series or Chebyshev polynomial approximations to solve Linear ODE's and PDE's using approximate methods, spectral methods, Galerkin and Collocation methods. [6, 7,9,10] In Literature there are several approximate methods are discussed such as Legendre wavelet method, Chebyshev – tau method, Homotopy perturbation method, etc, [2, 3] Nowadays the emerging researchers shows more enthusiasm in solving various numerical methods using several types of wavelets. [4,5,8] A crucial role in design of such numerical methods in PDE's plays a good localization property in our proposed method in this article.

The structure of our article is delivered as follows.

We give brief reviews of Chebyshev wavelets their approximation properties, function approximation in section 2. Newly constructed operational matrices of Chebyshev wavelets are introduced in section 3. In Section 4 We make use of our proposed two dimensional Chebyshev. Wavelets collocation method and solve some numerical examples. In Section 5, We conclude our proposed method of our article.

Section:2

Chebyshev wavelet and their properties

Chebyshev wavelet (CW)

$\phi^{C}_{n,m}(t) = \phi^{C}(k,\hat{n},m,t)$ have four arguments;

$\hat{n} = 2n-1, k \in N,\ n = 1,2,....,2^{k-1},$

and m is the degree of the Chebyshev polynomial of the first kind and t is the normalized time.

$$\phi^{C}_{n,m}(t) = \phi^{C}(k,\hat{n},m,t) = \begin{cases} 2^{k/2}\,\hat{T}_m(2^k t-\hat{n}) & if\ \dfrac{\hat{n}-1}{2^k} \le t < \dfrac{\hat{n}+1}{2^k}; \\ 0 & Otherwise \end{cases}$$

Where,

$$\hat{T}_m(t) = \begin{cases} \dfrac{1}{\sqrt{\pi}} & if\ m=0; \\ \sqrt{\dfrac{2}{\pi}}\,T_m(t) & if\ m>0. \end{cases}$$

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M = 0,1,…,M-1, and M – 1, and M is a fixed positive integer.

Here, $\{T_m(t), m \in N \cup \{0\}\}$ is the set of well known chebyshev polynomials of degree m which are orthogonal with respect to the weight function $w(t) = \frac{1}{\sqrt{1-t^2}}$ on the interval [-1,1] and satisfy the following recursive formula:

$$T_0(t) = 1, T_1(t) = t, T_{m+1}(t) = 2tT_m(t) - T_{m-1}(t), \quad m = 1,2,.....$$

We should note that in dealing with Chebyshev polynomials the weight function

$$\hat{w}(t) = w(2t-1)$$

have to be dilated and translated as

$w_n(t) = w(2^k t - 2n + 1)$ to get orthogonal wavelets.

Two –dimensional Chebyshev wavelet

Two-dimensional CW can be written in the product of one-dimensional CW as follows:

$$\phi^C_{n,m,n',m'}(t,x) = \phi^C_{n,m}(t)\phi^C_{n',m'}(x)$$

Where $\phi^C_{n,m}(t)$ *and* $\phi^C_{n',m'}(x)$ *are defined* $.n' = 1,2,.......2^{k'-1}$ *and* $m' = 0,1,2,....,M'-1.$

II. OPERATIONAL MATRICES

Chebyshev wavelets operational matrix of differentiation with respect the variable x

Let $\phi^C(t,x)$ be two dimensional CW vector defined and then derivative matrix as follows:

$$\frac{\partial}{\partial x}\phi^C(t,x) = \frac{\partial}{\partial t}(\phi^C(t) \otimes \phi^C(x))$$
$$= D^C_t \phi^C(t,x)$$

Where ,

$D^C_t = D^C \otimes I$ the matrix of order $2^{k-1}2^{k'-1}MM'$ and also I is the identity matrix. Similarlly,

Chebyshev wavelet operational matrix of differentiation with respect to variable t

Let $\phi^C(t,x)$ be two dimensional CW vector defined in Eq (10) then derivative matrix as follows:

$$\frac{\partial}{\partial t}\phi^C(t,x) = \frac{\partial}{\partial t}(\phi^C(t) \otimes \phi^C(x))$$
$$= D^C_t \phi^C(t,x)$$

Where

$D^C_t = D^C \otimes I$ is the matrix of order $2^{k-1}2^{k'-1}MM'$ and also I is the identity matrix.

Function approximation

Suppose that $f(t,x)$ is an arbitrary function in $L^2(\Omega)$ then it can be approximated as follows:

$$f(t,x) = \sum_{n=1}^{2^{k-1}}\sum_{m=0}^{\infty}\sum_{n'=1}^{2^{k-1}}\sum_{m'=0}^{\infty} f_{nmn'm'}\psi_{nmn'm'}(t,x)$$

If the above infinite series (2) is truncated for $m = M-1, m' = M'-1,$

Then approximation of (2) can be represented as in the following form.

$$f(t,x) = \sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1}\sum_{n'=1}^{2^{k'-1}}\sum_{m'=0}^{M'-1} f_{nmn'm'}\psi_{nmn'm'}(t,x) = F^T\psi(t,x)$$

Where, $F \; and \; \psi \; are \; 2^{k-1}2^{k'-1}M\,M' \times 1,$ vector given as follows:

$$F = [f_{1,0,1,0,}\ldots, f_{1,0,1,M'-1}, f_{1,0,2,0}\ldots, f_{1,0,2,M'-1},\ldots, f_{1,0,2^{k'-1},0},\ldots,$$
$$f_{1,0,2^{k'-1},M'-1},\ldots, f_{1,M-1,1,0},\ldots, f_{1,M-1,,1,M'-1}, f_{1,M-1,2,0}, f_{1,M-1,2,M'-1,}$$
$$f_{1,M-1,2^{k'-1},0},\ldots, f_{1,M-1,2^{k'-1},M'-1}, f_{2,0,1,0},\ldots, f_{2,0,1,M'-1}, f_{2,0,2,0},\ldots,$$

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$$f_{2,0,2,M'-1},\ldots, f_{2,0,2^{k'-1},0},\ldots, f_{2,0,2^{k'-1},M'-1},\ldots, f_{2,M-1,1,0},\ldots, f_{2,M-1,1,M'-1,}$$
$$f_{2,M'-1,2,0},\ldots, f_{2,M-1,2,M'-1}, f_{2,M-1,2^{k'-1},0},\ldots, f_{2,M-1,1,0},\ldots, f_{2,M-1,2^{k'-1},M'-1,},\ldots,$$
$$f_{2^{k-1},0,1,0},\ldots, f_{2^{k-1},0,1,M'-1}, f_{2^{k-1},0,2,0},\ldots, f_{2^{k-1},M-1,2^{k'-1},M'-1}]^T$$

Section:3

Two-Dimensional Chebyshev wavelet operational matrices

In this section, We introduce some new operational matrices of derivative which is derived from the concepts mentioned in the section:2

Therefore, We construct

Case-(i) For $k=1,\ n=1,\ m=0,1,2\ then\ \ F\ and\ \ \phi\ \ are\ 9\times 1\ \ vector.$

Thus, we obtain u(x,t) as

$$u(x,t)=\left(f_{1010}f_{1011}f_{1012}f_{1110}f_{1111}f_{1112}f_{1210}f_{1211}f_{1212}\right)\begin{pmatrix} \frac{2}{\pi} \\ \frac{2\sqrt{2}}{\pi}(2x-1) \\ \frac{2\sqrt{2}}{\pi}(8x^2-8x+1) \\ \frac{2\sqrt{2}}{2}(2t-1) \\ \frac{4}{\pi}(2x-1)(2t-1) \\ \frac{4}{\pi}(8x^2-8x+1)(2t-1) \\ \frac{2\sqrt{2}}{\pi}(8t^2-8t+1) \\ \frac{4}{\pi}(2x-1)(8t^2-8t+1) \\ \frac{4}{\pi}(8t^2-8t+1)(8x^2-8x+1) \end{pmatrix}$$

Therefore with the heip of the above constructed operational matrix we can able to find the derivatives of u(x,t) with respect to x and t partially.

Case(ii): Using the preliminaries in section2, we also obtained

For $k=2,\ n=1,\ m=0,1,2\ then\ \ F\ and\ \ \phi\ \ are\ 9\times 1\ \ vector.$

Thus, we get u(x,t) as follows

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$$u(x,t)=\left(f_{1010}f_{1011}f_{1012}f_{1110}f_{1111}f_{1112}f_{1210}f_{1211}f_{1212}\right)\begin{pmatrix}\frac{4}{\pi}\\ \frac{4\sqrt{2}}{\pi}(4x-1)\\ \frac{4\sqrt{2}}{\pi}(32x^2-16x+1)\\ \frac{4\sqrt{2}}{\pi}(4t-1)\\ \frac{8}{\pi}(4t-1)(4x-1)\\ \frac{8}{\pi}(4t-1)(32x^2-16x+1)\\ \frac{4\sqrt{2}}{\pi}(32t^2-16t+1)\\ \frac{8}{\pi}(4x-1)(32t^2-16t+1)\\ \frac{8}{\pi}(32t^2-16t+1)(32x^2-16x+1)\end{pmatrix}$$

Section:4

Numerical Experiment

To illustrate the description in Section:3 and to apply the method developed in Section:3, we have considered three numerical examples and solved them.

Example:1 Consider the Klein-Gordan Equation

$u_{tt} - u_{xx} = u;\ 0<x<1$ *subject to the initial conditions*

$u(x,0) = 1+\sin x,\ u_t(x,0)=0$ *and the boundary*

conditions $u(0,t)=0$, $u(1,t)$ 0 , $t>0$ *with the exact solution is,* $u(x,t) = \sin x + \cosh t$

Using the above discussed constructed Two-Dimensional Chebyshev method , case(i) of Section 3 and also by collocation method we get the following equations such as,

$$-0.6366 f_{1010} - 13.50469 f_{1012} + 15.3053 f_{1210} = 0$$

$$0.6363 f_{1010} - 0.9 f_{1012} - 0.9 f_{1110} + 1.2732 f_{1112} + 0.9 f_{1210} - 1.2732 f_{1212} = 1.00872$$

$$18.9 f_{1110} - 2.546 f_{1112} - 7.2025 f_{1210} + 10.1859 f_{1212} = 0$$

$$0.6366 f_{1010} - 0.9003 f_{1011} + 0.9003 f_{1012} - 0.9003 f_{1210} + 1.2732 f_{1211} - 1.2732 f_{1212} = 0$$

$$0.6366 f_{1010} + 0.9003 f_{1011} + 0.9003 f_{1012} - 0.9003 f_{1210} - 1.2732 f_{1211} - 1.2732 f_{1212} = 0$$

$$14.42306 f_{1210} + 14.2768 f_{1211} - 14.3484 f_{1012} - 14.2412 f_{1112} - 0.6366 f_{1010} - 0.63022 f_{1011}$$
$$- 0.63022 f_{1110} - 0.62388 f_{1111} - 0.000509 f_{1212} = 0$$

$$0.6366 f_{1010} + 0.63022 f_{1011} - 0.0180 f_{1012} - 0.9003 f_{1110} - 0.89126 f_{1111} - 0.01782 f_{1112} + 0.9003 f_{1210} +$$
$$0.89126 f_{1211} - 0.02546 f_{1212} = 1.014834$$

$$1 - 800636 f_{1011} - 7.20253 f_{1012} - 1.8029 f_{1111} + 7.21161 f_{1112} + 0.0006437 f_{1211} - 0.02571 f_{1212} = 0$$

$$0.6366 f_{1010} - 0.63741 f_{1011} + 0.9003 f_{1012} - 0.6374 f_{1110} + 0.9014 f_{1111} - 0.9014 f_{1112} +$$
$$0.00227 f_{1210} - 0.003218 f_{1211} + 0.003218 f_{\ 1212} = 0$$

solving these system of 9 equations using Matlab we get

$$U^C = [1.11964\ \ 2.30190\ \ 1.67950\ \ 0.39651\ \ 0.72588\ \ 1.28632\ \ 1.52851\ \ 1.62773\ \ 0.66661]$$

Case(ii):

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$$1.2732\,f_{1010} + 113.4398\,f_{1012} - 117.041\,f_{1210} + 2.5464\,f_{1212} = 0$$
$$1.2732\,f_{1010} + 1.80063\,f_{1012} - 1.80063\,f_{1110} - 2.5464\,f_{1112} - 1.80063\,f_{1210} - 2.5464\,f_{1212} = 1.004363$$
$$7.20253\,f_{1110} - 10.18591\,f_{1112} - 28.81012\,f_{1210} + 40.7436\,f_{1212} = 0$$
$$1.2732\,f_{1010} - 1.80063\,f_{1011} + 1.80063\,f_{1012} + 2.54647\,f_{1211} - 2.54647\,f_{1212} = 0$$
$$1.2732\,f_{1010} + 5.4018\,f_{1011} + 30.6102\,f_{1012} - 1.8006\,f_{1210} - 7.639\,f_{1211} + 2.5464\,f_{1212} = 0$$
$$1.2732\,f_{1010} + 1.2726\,f_{1011} + 115.2404\,f_{1012} + 1.2726\,f_{1110} + 7.418\,f_{1111} + 115.1904\,f_{1112} - 115.2404\,f_{1210} - 115.1904\,f_{1211} = 0$$
$$1.27323\,f_{1010} - 1.800636\,f_{1011} + 1.800063\,f_{1012} + 1.2726\,f_{1110} - 1.79985\,f_{1111} + 1.79985\,f_{1112} - 0.0015665\,f_{1210} + 0.0022154\,f_{1211} + 0.0022154\,f_{1212} = 0$$
$$7.20253\,f_{1110} + 7.1994\,f_{1111} - 28.8101\,f_{1210} + 28.7976\,f_{1211} = 0$$
$$1.27323\,f_{1010} - 1.80063\,f_{1011} + 1.80063\,f_{1012} + 1.2726\,f_{1110} - 1.79985\,f_{1111} + 1.79985\,f_{1112} - 0.0015665\,f_{1210} + 0.0022154\,f_{1211} + 0.0022154\,f_{1212} = 0$$

solving these system of 9 equations for case(ii) using Matlab we get

$$U^C = [1.286074\ \ 1.982322\ \ -0.2002739\ \ 1.710701136\ \ 0.62598449\ \ 0.68939144\ \ -0.1858097819\ \ 0.77024777\ \ -0.26145208]$$

Example:2

Consider the 2 – Dimensional Wave – like equation

$$U_{tt} - \frac{x^2}{2}U_{xx} = 0\ ;\ 0 < x < 1,\ t > 0$$

subject to the initial conditions

$$u(x,0) = x,\ u(x,0) = x^2 \text{ and Boundary conditions } u(0,t) = 0,\ u(1,t)\ \ 1 + \sinh(t)\ ,\ t > 0$$

$$\text{exact solution is } u(x,t) = x + x^2 Sinh(t)$$

Case(i):

$$14.3992\,f_{1210} - 1.7999\,f_{1012} = 0$$
$$0.6363\,f_{1010} - 0.9\,f_{1012} - 0.9\,f_{1110} + 1.2732\,f_{1111} + 0.9\,f_{1210} - 1.2732\,f_{1212} = 0.5$$
$$0.6363\,f_{1010} - 0.9003\,f_{1011} + 0.9003\,f_{1012} - 0.9003\,f_{1210} + 1.2732\,f_{1211} - 1.2732\,f_{1212} = 0$$
$$0.6366\,f_{1010} + 0.9003\,f_{1011} + 0.9003\,f_{1012} - 0.9003\,f_{1210} - 1.2732\,f_{1211} - 1.2732\,f_{1212} = 1.52109$$
$$14.40506\,f_{1210} + 14.259\,f_{1211} - 5.20382\,f_{1012} - 5.151063\,f_{1112} = 0$$
$$0.6366\,f_{1010} + 0.63022\,f_{1011} - 0.0180\,f_{1012} - 0.9003\,f_{1110} - 0.89126\,f_{1111} - 0.01782\,f_{1112} + 0.9003\,f_{1210} + 0.89126\,f_{1110} - 0.02546\,f_{1212} = 0.85$$
$$0.6366\,f_{1010} - 1.2748\,f_{1011} + 0.0022759\,f_{1012} = 0.021316$$
$$0.6366\,f_{1010} - 0.63741\,f_{1011} + 0.9003\,f_{1012} - 0.6374\,f_{1110} + 0.9014\,f_{1111} - 0.9014\,f_{1112} + 0.00227\,f_{1210} - 0.003218\,f_{1211} + 0.003218\,f_{1212} = 0$$
$$0.6366\,f_{1010} + 0.9003\,f_{1011} + 0.9003\,f_{1012} - 0.4501\,f_{1110} - 0.6366\,f_{1111} - 0.6366\,f_{1112} - 0.4501\,f_{1210} - 0.6366\,f_{1211} - 0.6366\,f_{1212} = 1.252612$$

solving the above system of equations we obtain,

$$U^C = [1.55733\ \ 0.7606129\ \ -0.198388\ \ 0.589577\ \ 0.01955621\ \ -0.03314886\ \ -0.02479851\ \ -0.0593242\ \ 0.05838481\ ]$$

Case(ii):

$$115.2404\,f_{1210} - 3.601262\,f_{1012} = 0$$
$$1.2732\,f_{1010} - 1.80063\,f_{1012} - 1.80063\,f_{1110} + 2.5464\,f_{1112} + 1.80063\,f_{1210} - 2.5464\,f_{1212} = 0.25$$
$$1.2732\,f_{1010} - 1.80063\,f_{1011} + 1.80063\,f_{1012} + 1.80063\,f_{1210} + 2.54647\,f_{1211} - 2.54647\,f_{1212} = 0$$
$$0.6366\,f_{1010} + 0.9003\,f_{1011} + 0.9003\,f_{1012} - 0.4501\,f_{1110} - 0.6366\,f_{1111} - 0.6366\,f_{1112} - 0.4501\,f_{1210} - 0.6366\,f_{1211} - 0.6366\,f_{1212} = 0$$
$$115.2404\,f_{1210} + 115.1904\,f_{1211} - 10.49107\,f_{1012} - 10.482326\,f_{1112} = 0$$
$$1.2732\,f_{1010} + 1.27268\,f_{1011} - 0.0015665\,f_{1012} - 1.80063\,f_{1110} - 1.79985\,f_{1111} + 0.0022032\,f_{1112} + 1.800632\,f_{1210} + 1.79985\,f_{1211} - 0.00220912\,f_{1212} = 0.4267$$
$$1.27323\,f_{1010} + 1.27268\,f_{1011} - 0.0015665\,f_{1012} - 1.80063\,f_{1110} - 1.79985\,f_{1111} + 0.0022032\,f_{1112} + 1.800632\,f_{1210} + 1.79985\,f_{1211} - 0.00220912\,f_{1212} = 0.182072$$
$$1.27323\,f_{1010} - 1.80063\,f_{1011} + 1.80063\,f_{1012} + 1.2726\,f_{1110} - 1.79985\,f_{1111} + 1.79985\,f_{1112} + 0.0015665\,f_{1210} + 0.0022154\,f_{1211} + 0.0022154\,f_{1212} = 0$$
$$1.2732\,f_{1010} + 5.4018\,f_{1011} + 30.6102\,f_{1012} + 1.272664\,f_{1110} + 5.3993\,f_{1111} + 30.5965\,f_{1112} - 0.00162\,f_{1210} - 0.0687\,f_{1211} - 0.038959\,f_{1212} = 1.43976$$

solving the above system of equations we obtain,

$$U^C = [\ 34687812.32\ \ 2542279.556\ \ -3057850\ \ 5547409.714\ \ 20252315.28\ \ -2621378.718\ \ -95557.79923\ \ -421442.6348\ \ 12894519.12\ ]$$

Example:3

Consider the following variable co-efficient linear equation with Dirichlet boundary condition

$$x^2 u_{xx} + t^2 u_{tt} = 2x^2 t^2 e^{xt}$$

$$u(x,0) = 1,\ u(x,1) = e^x,\ u(o,t) = 1,\ u(1,t) = e^t$$

$$\text{exact solution is } u(x,y) = e^{xt}$$

Similarly solving this problem by our proposed method we get the solutions in both the cases as follows:

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$$U^C = [2.188155\ \ 0.514052\ \ 0.039915\ 0.507889\ 0.351478\ \ 0.104570\ \ 0.0046713\ \ 0.0591012\ \ \ 0.0287799]$$

$$U^C\ = [\ 0.883238\ 0.004130\ 1.115544\ -0.000133\ 0.070518\ -1.114270\ \ -1.114967\ 1.117485\ 1.1638801]$$

II. DISCUSSION

The objective of our article is to develop a Two-Dimensional Chebyshev Wavelet for obtaining the approximate solution of PDEs. We first find an operational matrix of our own from Two-Dimensional Chebyshev Wavelet and then by using the operational matrix of derivatives, we reduce the PDE into linear system of equations. Moreover the advantage of our method is that we use only small size operational matrix to provide the solution at high accuracy. Our numerical examples demonstrate that our method is very effective and useful technique for solving PDE's this method can also used for higher order PDE's.

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