

# Back-Stepping Controller Design for Stable Solenoid Valve System

Yongho Jeon, Shinwon Lee

**Abstract:** The solenoid valve system has strong nonlinear characteristics. In order to overcome the nonlinear characteristics, the controller is designed by linearization near the operating point. This is because there is a difficulty that the system mathematical model changes as the operating point changes and the control gain also changes. The controller of the solenoid valve system is constructed by using the back stepping control technique which can obtain the asymptotic stability of the system and can set the gain more simply than the gain of the general PID controller. In order to estimate the non - observable state of the controller configuration, it designed the Luenberger state observer and estimate the state simply. This system improves the tracking performance of the system and achieves asymptotic stability. Since the position error of the state estimator is within 0.1 [mm], the result of accurate state estimation can be obtained, and the back-stepping controller can obtain the result that the steady state position error is within 0.1 [%].

**Index Terms:** Back-stepping, equilibrium, Nonlinear, Plunger, Solenoid valve

## I. INTRODUCTION

The solenoid valves are one of the simplest devices to convert electrical energy into mechanical energy. The solenoid valve is a device that regulates the pressure, flow rate and direction of fluid by opening and closing the valve by supplying or cutting off current by winding a coil around the outside of the plunger, called a moving iron core. In many places in the industrial field, it is widely used from simple opening and closing operations to precise position control. [1-13]

In an actual industrial field, an actuator composed of an electric motor among the actuators used is mostly used for precise position control, these structures are more complicated than solenoid valves, and they are expensive to manufacture and require maintenance costs. On the other hand, the solenoid valve system is not complicated in structure and has an advantage that it can be easily applied where simple operation is required.

The simple ON / OFF control of the solenoid valve does not require many units of the controller configuration, but there is a great problem such as noise and vibration due to the impact of the plunger during opening and closing operation. The proportional solenoid valve is applied to the solenoid valve with a position controller, which is a good way to overcome the problem of noise and vibration.

Prior to designing the position controller, the structure of the solenoid valve system includes a plunger with mechanical motion, a coil wrapped around the plunger to generate an electro-magnetic force, the support structure (frame), and the spring. It provides a restoring force corresponding to the magnetic force. Based on this, the

mathematical model of the solenoid valve system has a nonlinear second-order nonlinear differential equation for the position of the flange and a first-order nonlinear differential equation for the current flowing through the coil. The PID controller is most used in the industrial field. The PID controller should be linearized around the operating point for nonlinearities, and three controller gains for the mechanical part and three gains for the electrical part. This requires a lot of effort to obtain precise control performance.

In this work, a mathematical model of the system is proposed to design a precise position controller of a solenoid valve system with high nonlinear characteristics.

By using integral control with feedback linearization technique, precise position control is possible. By setting three control gains one by one for each state variable, it is possible to obtain steady state response of the system by setting control gain easily.

## II. BACK-STEPPING CONTROLLER

### A. Position back-stepping controller design

The dynamic equation of the solenoid valve is expressed by Equation (2.4) in the following equation (2.1).

$$\dot{v} = \frac{aL'}{2M(x+a)^2}i^2 - \frac{b}{M}v - \frac{K}{M}x - g_r \quad (2.1)$$

$$\frac{di}{dt} = \frac{x+a}{L'x} \left( U - iR - iv \frac{a}{(x+a)^2} \right) \quad (2.2)$$

$$L' = \frac{\pi d \mu_0 a N^2}{g} \quad (2.3)$$

Where  $x$  and  $v$  are the position and velocity of the plunger, and  $i$  is the current through the coil.  $M$  is the mass of the plunger,  $b$  is the coefficient of friction between the plunger and the guide tube,  $a$  is the length of the gap,  $L'$  is the constant of the inductance, and  $g_r$  is the gravitational acceleration. And  $R$  is the resistance of the coil and  $U$  is the input voltage supplied to the coil.

When the reference position for position tracking is  $x_r$ , the position error and the change rate with respect to time are expressed by the following equation (2.5) and (2.6).

$$e_x = x_r - x \quad (2.5)$$

$$\dot{e}_x = \dot{x}_r - \dot{x} \quad (2.6)$$

Using the equation (2.5), the evaluation function to judge stability and the rate of change of time can be obtained by the following equation (2.7) and (2.8).

$$V_1 = \frac{1}{2}e_x^2 \quad (2.7)$$

$$\dot{V}_1 = e_x \dot{e}_x \quad (2.8)$$

Revised Manuscript Received on December 22, 2018.

Yongho Jeon Department of Aviation Maintenance Engineering  
Jungwon University, Republic of Korea.

Shinwon Lee, Department of Computer Engineering Republic of Korea.



If the error is defined by using the virtual reference speed  $v_r$ , the virtual input  $v_r$ , which is equal to the following equation (2.9), and if the value of the equation (2.8) becomes negative, the virtual reference speed  $v_r$  is expressed by the following equation (2.10).

$$e_v = v_r - v = v_r - \dot{x} \quad (2.9)$$

$$v_r = \dot{x}_r + k_x e_x \quad (2.10)$$

Using the equations (2.9) and (2.10), the following equation (2.11) is substituted for the equation (2.8).

$$e_v = v_r - v = v_r - \dot{x} \quad (2.9)$$

$$v_r = \dot{x}_r + k_x e_x \quad (2.10)$$

Using the equations (2.9) and (2.10), the following equation (2.11) is substituted for the equation (2.8).

$$\begin{aligned} \dot{V}_1 &= e_x(\dot{x}_r - (v_r - e_v)) \\ &= -k_x e_x^2 + e_x e_v \end{aligned} \quad (2.11)$$

The gain  $k_x$  of the equation (2.11) is set to an arbitrary positive value. Next, the evaluation function to stabilize the velocity equation is as shown in the following equation (2.12), and the rate of change with respect to time is given by equation (2.13).

$$V_2 = \frac{1}{2} e_x^2 + \frac{1}{2} e_v^2 \quad (2.12)$$

$$\dot{V}_2 = e_x \dot{e}_x + e_v \dot{e}_v \quad (2.13)$$

The rate of change of velocity error with respect to time in the equation (2.13) is given by the following equation (2.14).

$$\dot{e}_v = \dot{v}_r - \dot{v} \quad (2.14)$$

If this system designs the virtual input  $I$  so that (2.13) becomes negative by replacing  $\dot{v}^2 = I$  in (2.2), then the following equation (2.15) is obtained.

$$I = \frac{2M(x+a)^2}{aL} \left( \frac{b}{M} v + \frac{K}{M} x + g_r + \dot{v}_r + k_v e_v + e_x \right) \quad (2.15)$$

The equation (2.15) is a value designed to be negative in the equation (2.13). The equation (2.16) is substituted into the equation (2.13) by using the equation (2.14) and the equation (2.2) together.

$$\begin{aligned} \dot{V}_2 &= -k_x e_x^2 + e_x e_v + e_v (\dot{v}_r - \dot{v}) \\ &= -k_x e_x^2 + e_x e_v \\ &\quad + e_v \left( \dot{v}_r - \left( \frac{aL}{2M(x+a)^2} I - \frac{b}{M} v - \frac{K}{M} x - g_r \right) \right) \\ &= -k_x e_x^2 - k_v e_v^2 \end{aligned} \quad (2.16)$$

If any gains  $k_x$  and  $k_v$  are set to positive values, (2.16) will have a negative value. The equation (2.15) can be used as a current reference for current control as a control input for position control.

### B. Current back-stepping controller design

The result for the position control of the plunger is designed by Eq. (2.15). If the current reference is constructed by using it, it is shown in the following equation (2.17).

$$i_r = \sqrt{\frac{2M(x+a)^2}{aL} \left( \frac{b}{M} v + \frac{K}{M} x + g_r + \dot{v}_r + k_v e_v + e_x \right)} \quad (2.17)$$

Using the current reference in equation (2.17), the error equation for the current and the rate of change with respect to time are shown in the following equations (2.18) to (2.19).

$$e_i = i_r - i \quad (2.18)$$

$$\frac{d}{dt} e_i = \frac{d}{dt} i_r - \frac{d}{dt} i \quad (2.19)$$

Using the equation (2.18), by constructing the evaluation function, the following equation (2.20) is obtained.

$$V_3 = \frac{1}{2} e_x^2 + \frac{1}{2} e_v^2 + \frac{1}{2} e_i^2 \quad (2.20)$$

If the evaluation function of Eq. (2.20) is changed to the rate of change with respect to time, the following equation (2.21) is obtained.

$$\dot{V}_3 = e_x \dot{e}_x + e_v \dot{e}_v + e_i \dot{e}_i \quad (2.21)$$

If the control input is designed so that Eq. (2.21) becomes negative, the following equation (2.22) is obtained.

$$U = iR + iv \frac{a}{(x+a)^2} + \frac{Lx}{x+a} \left( \frac{d}{dt} i_r + k_i e_i \right) \quad (2.22)$$

In equation (2.22),  $k_i$  is set to a suitable positive value as the current gain. The designed input becomes the voltage supplied to the coil. Equation (2.19) and Equation (2.22) are substituted into Equation (2.21) and summarized as the following Equation (2.23).

$$\begin{aligned} \dot{V}_3 &= -k_x e_x^2 - k_v e_v^2 + e_i \left( \frac{d}{dt} i_r - \frac{d}{dt} i \right) \\ &= -k_x e_x^2 - k_v e_v^2 \\ &\quad + e_i \left( \frac{d}{dt} i_r - \frac{x+a}{Lx} \left( U - iR - iv \frac{a}{(x+a)^2} \right) \right) \\ &= -k_x e_x^2 - k_v e_v^2 - k_i e_i^2 \end{aligned} \quad (2.23)$$

The three gains  $k_x$ ,  $k_v$ ,  $k_i$  are the appropriate positive values so that Eq. (2.23) is negative.

### C. Current back-stepping controller design

In order to apply previously designed controllers, the position  $x$ , velocity  $v$ , and the current  $i$  of the plunger must be measurable. But this system does not measure the speed of the plunger and design the state observer and apply the estimated value to the controller. Using the Luenberger

observer as the state observer, it obtains from the following equation (2.24) to (2.28).

$$\frac{d}{dt} \begin{pmatrix} \hat{x} \\ \hat{v} \\ \hat{f} \\ \hat{i} \\ \hat{f}_i \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{v} \\ \hat{f} \\ \hat{i} \\ \hat{f}_i \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ g & 0 \\ 0 & 0 \\ 0 & g_i \\ 0 & 0 \end{pmatrix} \begin{pmatrix} I \\ U \end{pmatrix} + \begin{pmatrix} L_1 & 0 \\ L_2 & 0 \\ L_3 & 0 \\ 0 & M_1 \\ 0 & M_2 \end{pmatrix} \begin{pmatrix} x - \hat{x} \\ i - \hat{i} \end{pmatrix} \quad (2.24)$$

$$g = \frac{aL'}{2M(x+a)^2} \quad (2.25)$$

$$f = \Delta \frac{aL'}{2M(x+a)^2} I - \frac{b}{M} v - \frac{K}{M} x - g_r \quad (2.26)$$

$$g_i = \frac{x+a}{Lx} \quad (2.27)$$

$$\hat{f}_i = \Delta \frac{x+a}{Lx} U - iR \frac{x+a}{Lx} - iv \frac{a}{Lx(x+a)} \quad (2.28)$$

By setting the appropriate gains L1, L2, and L3 of the state observer above, the estimated state of position and velocity can be obtained. Also, by setting the appropriate gains M1 and M2, an estimated value of the current state can be obtained. The estimated state is used in equations (2.17) and (2.22) to construct the controller.

### III. RESULT

The parameters of the solenoid valve used to verify the designed controller are shown in Table 1.

Table 1. The Parameter of the Solenoid valve

Parameter	N(turns)	M(kg)	b(Ns/m)	K(N/m)	R(Ω)	L'(mH)
value	200	0.3	2	2667	1	23.69

The state estimation gain for the state estimation of the observer of equation (2.24) with respect to the position, velocity, and current state of the plunger is shown in Table 2 below.

Table 2 The gain of the observer

Gain	L1	L2	L3	M1	M2
value	3K	9000K	1G	200	10000

The result of the state estimation of the observer is shown in Fig. 1

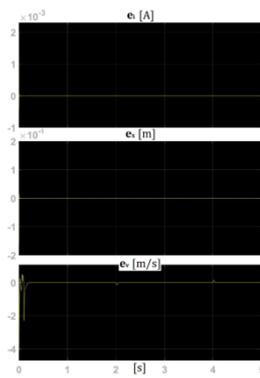


Fig. 1 State estimation error of observer

The first figure in Figure 1 shows that the error estimated from the actual current and the estimated current value is within 0.1 [mA] of the error, the second figure shows that the difference between the actual position and the estimated position is within 0.1 [mm]. The third figure shows the error between the actual speed and the estimated speed condition, which is within 2 [m / s] maximum. Therefore, estimation of the state of the observer can be obtained precise results. The control gain is used when constructing the back-stepping controller using the estimated state from the observer. The control gains used at this time are shown in Table 3 below.

Table 3 The gain of the back-stepping controller

Gain	k_x	k_v	k_i
value	10	100	100000

The results obtained by applying the control gain of Table 3 to Equation (2.22) are shown in Fig. 2. At this time, the target position was set to 10 [mm] at the initial start, the constant load was operated at 5 [Nm] at the time 2 seconds, and the constant load was removed at the time 4 seconds.

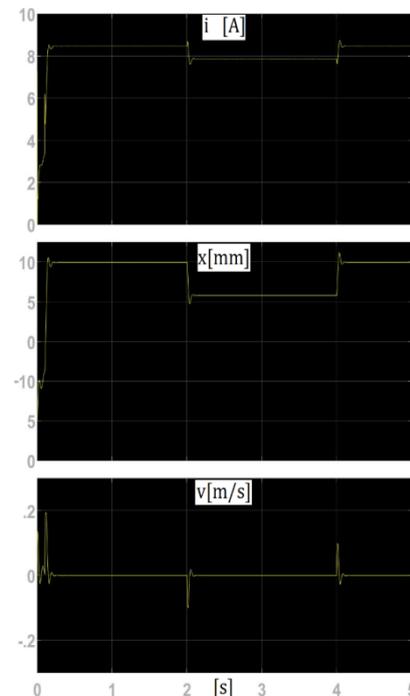


Fig 2 Output State of the back-stepping Controller I

In Figure 2, the first picture shows current control, and overshoot occurs within about 5%. The second figure shows that the position of the plunger is controlled and overshoot occurs within about 5%. The third figure shows the result that maximum 0.2 [m / s] of vibration occurs in the velocity state and all of them reach the steady state within 0.2 second. The steady state error occurs due to the constant load operation at time 2 seconds, and the steady state error disappears again due to the constant load disappearing at 4 seconds. In the previous results, the control gain set to reduce the vibration of the position state is shown in Table 4 below.

## Back-Stepping Controller Design For Stable Solenoid Valve System

gain	kx	k <sub>v</sub>	ki
value	50	200	100000

The first figure in Figure 3 shows the control result for the current, the second figure shows the position state, and the third figure shows the state where the speed state is controlled. Compared with the results in Fig. 2, the gain of the position error is increased 5 times, so that the overshoot of the position state disappears and the critical damping becomes. Therefore, it can be judged that the impact and noise will disappear due to the vibration of the plunger.

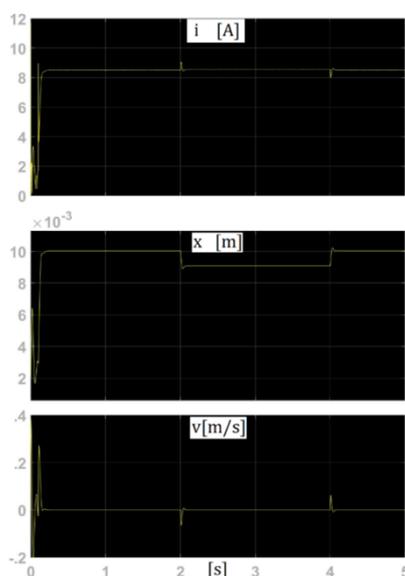


Fig. 3 Output State of the back-stepping Controller II

### IV. CONCLUSION

The solenoid valve system generally uses a method of designing a controller by linearizing the equilibrium point with a system having a strong nonlinear characteristic. However, the proposed controller included feedback linearization and designed in back-stepping control scheme. The designed back-stepping controller proposed to estimate the state using the Luenberger state observer and apply it to the controller.

First, since the position error of the state estimator is within 0.1 [mm], the result of accurate state estimation can be obtained, and the back-stepping controller can obtain the result that the steady state position error is within 0.1 [%]. As a result, it can be seen that the state observer can replace the state measurement at low cost. The steady state error is obtained when the constant load is applied, and the design of the controller that can actively respond to the disturbance should be supplemented.

### REFERENCES

1. Sarang P Lunge., Dr. Shailaja Kurode., Mr. Bhaskar Chhibber., Proportional Actuator from On Off Solenoid Valve using Sliding Modes. Proceedings of the 1st International and 16th National Conference on Machines and Mechanisms (iNaCoMM2013), IIT Roorkee, India,
2. M.F. Rahman, N.C. Cheung, K.W. Lim "A Sensorless Position Estimator for a Nonlinear Solenoid Actuator" IEEE Industrial Electronics Society annual general meeting, IECON 1995

3. T. Kajima, Y. Kawamura, "Development of a high-speed solenoid valve : investigation of solenoids," IEEE Transactions on Industrial Electronics, Vol. 42, No. 1, pp 1-8, 1995
4. M. F. Rahman , N. C. Cheung and K. W. Lim, "Position estimation in solenoid actuators", IEEE Trans. Ind. Appl., vol. 32, no. 3, pp. 552-559, 1996
5. F. Malaguti, E. Pregolato "Proportional Control of on/off solenoid operated hydraulic valve by nonlinear robust controller", IEEE 2002
6. Nayyar, Anand, Vikram Puri, and Dac-Nhuong Le. "A Comprehensive Review of Semiconductor-Type Gas Sensors for Environmental Monitoring." Review of Computer Engineering Research 3, no. 3 (2016): 55-64.
7. G. F. Franklin, J. D. Powell, Feedback control of dynamic systems, 3rd edition Addison Wesley, pp. 191~196, 1994
8. Shoewu, O., G. Anuforonini, and O. Duduyemi. "Simulation of the Performance of CdTe/CdS/ZnO Multi-Junction Thin Film Solar Cell." Review of Information Engineering and Applications 3. 1 (2016): 1-10.
9. N. C. Cheung, K. W. Lim, "Modeling of Linear and Limited Travel Solenoid", IEEE Transactions, pp.1567-1572. 1993.
10. Nahar, Ali Kareem, Mohammed Moanes Ezzaldeen, Sabah A. Gitaffa, and Hussain K. Khleaf. "OFDM Channel Estimation Based on Novel Local Search Particle Swarm Optimization Algorithm." Review of Information Engineering and Applications 3. 2 (2016): 11-21.
11. M. C. Cheung, M. F. Rahman, K. W. Lim, "Simulations and experimental studies towards the development of a proportional solenoid", Australian Universities Power Engineering Conference, Vol.2, pp.582-587, Sept 1993.
12. Akram, H. M., and M. Zafarullah Khan. "Selection of Appropriate Equipment for Designing Effective Vacuum System." Review of Information Engineering and Applications 2. 2 (2015): 51-63.
13. K. Khalil, "Nonlinear Systems", 33d edition Prentice Hall, pp.485~498.
14. Ali, A., & Haseeb, M. (2019). Radio frequency identification (RFID) technology as a strategic tool towards higher performance of supply chain operations in textile and apparel industry of Malaysia. *Uncertain Supply Chain Management*, 7(2), 215-226.
15. Y. Jeon, S. Lee, C. Chae, M. Kim, "PI Controller Design using the scheduling of Control Gain", 6<sup>th</sup> International Symposium, ISAAC2018, Korea, 2018.