Modeling and Performance Analysis of Zbr and DBR in Mobile Cellular Network

Jong Hun Park, Jihee Jung, Hee-Seon Jang, Jang Hyun Baek

Abstract This paper presents a mobility model to analyze the zone-based registration (ZBR) and distance-based registration (DBR). The number of registrations for the ZBR and the DBR are also calculated in order to compare their performances. Additionally, a combined registration method called DIR (DBR with implicit registration) is presented as a means to improve the performance of the DBR. The performance of the proposed DIR is analyzed using the mobility model presented in this paper and compared with the performance of the ZBR, which is widely used in mobile communication systems. To accurately evaluate the registration and paging costs of the ZBR and the DBR, a real network hierarchy, that displays a mobile network as being made up of many visitor location register (VLR) areas, is considered. The numerical results for the various surroundings show accurate evaluations of the ZBR and the DBR in a real network hierarchy. The computational results demonstrate that, considering the network hierarchy, the proposed DIR is superior to the ZBR in most cases.

Index Terms: distance-based registration; implicit registration; mobility model; registration; zone-based registration

I. INTRODUCTION

For mobile cellular networks, the network should record the location of a mobile station (MS) to successfully connect an incoming call to it. When an MS enters a new location, it registers its location to a network database by a process known as Location Registration (LR). There have been many studies on efficient LR schemes such as zone-based registration (ZBR), distance-based registration (DBR), movement-based registration and so on [1, 3-8, 10-12].

In this paper, the ZBR and DBR are considered. In ZBR, an MS registers whenever it enters a new zone [3-8]. Zones are groups of adjacent base stations within a given system. The implementation and management of registration and paging is relatively simple for ZBR since a mobile communication network consists of fixed zones (or location area). However, zigzag movement of the mobile subscriber near the border of zones is a potential source of frequent registrations (called ping pong phenomena) and the registration occurs only at the border cells of the fixed zone.

On the other hand, DBR results in an MS registering whenever the distance between the current base station and the base station where it was last registered exceeds a threshold [4-5, 13]. Generally, DBR experiences more frequent registrations than ZBR [4-5]. However, several unnecessary registrations along the border of zones can be avoided and the registration occurs evenly in every cell of the location area.

Revised Manuscript Received on December 22, 2018.
Jong Hun Park, Business School, Daegu Catholic University, Republic of Korea.
Jihee Jung, CAMTC Advanced Mechatronics Technology Institute, Republic of Korea.
Hee-Seon Jang, Department of Convergence Software, Pyeongtaek University, Republic of Korea.
Jang Hyun Baek, Department of Industrial & Information Systems Engineering and the RCIT, Chonbuk National University, Republic of Korea.

In this paper, new versions of analysis for the ZBR and DBR are presented. Following this introduction, Section 2 explains the mobility model and the performance analysis using our mobility model. Performance analysis under VLR network architecture is discussed in Section 3 and numerical results are given in Section 4. Finally, concluding remarks are given in Section 5.

II. MOBILITY MODEL AND PERFORMANCE

To analyze the expected number of registrations, we need the mobility model of a mobile subscriber. Unlike using random walk model in most previous studies, this study uses 4-directional (4D) mobility model. Under 4D model, the following assumptions are applied [3-6].

- A mobile subscriber moves straight, until he reaches the turning point.
- When he reaches the turning point, a mobile subscriber has four directions, \( n\times90^\circ \) (\( n = 0, 1, 2, 3 \)), with the same probabilities
- The distance, denoted by \( X \), between two successive turning points is exponentially distributed with mean \( \theta \). Further, let’s define some random variables and parameters.

\( X \): the distance between two successive turning points
\( S \): the distance from the turning point to the border of location area
\( K \): the number of registrations between two points
\( N \): the number of registrations per subscriber between two incoming calls
\( L \): the moving distance between two incoming calls

A. Zone-based registration

We assume that the shape of a location area (LA or zone) is a square in which the length of each side is \( 2d \), as shown in Fig. 1 and the movement path is parallel to or perpendicular to the border of the zone.

The probability density function of the random variable \( S \) is [5,9]

\[
f_s(s) = \begin{cases} \frac{1}{2d} & 0 < s \leq 2d \\
0 & \text{otherwise} \end{cases}
\]

Fig 1 Mobility of Mobility of a subscriber in ZBR
The expected number of registrations between two successive turning points, \( E(K) \), can be written as
\[
E(K) = \int_{-\infty}^{\infty} P(s \leq X < s + 2d) f_s(s) ds + \int_{s}^{\infty} P(s + 2d \leq s + d) f_s(s) ds + \ldots
\]
\[
= \frac{e^{-r^2/2}}{2} \sqrt{1 - \frac{r^2}{2}}
\]
Therefore, the expected number of registrations per subscriber between two incoming calls is [5]
\[
E(N) = \frac{E(L)}{E(X)} = \frac{E(L)}{2d}
\]

B. Distance-based registration

With the DBR, the MS registers when the distance between the current base station and the last registered base station exceeds a threshold, \( r \) [4-5, 13]. Therefore, it is assumed that the shape of location area is circular in form with the base station where the MS last registered at the center of the circle, and the radius of the circle is the threshold, \( r \), as shown in Fig. 2.

Fig 2 Mobility of Mobility of a subscriber in DBR

If we assume that MS is uniformly distributed, the probability density function of the random variable \( S \) is [2,9]
\[
f_s(s) = \frac{2}{\pi r} \sqrt{1 - \left(\frac{s}{r}\right)^2}, \quad 0 < s \leq 2r
\]
Note that, in distance-based registration, when a call arrives at a mobile station, location area of the mobile station is reset so that current cell becomes the center of a new location area. So, in this case, the distance to the border of location area is clearly \( r \). As a result, letting \( K_s \) be the number of registrations between an incoming call occurrence point and next turning point, \( E(K_s) \), can be written as
\[
E(K_s) = \int_{-\infty}^{\infty} P(s \leq X < 2r) f_s(s) ds + \int_{2r}^{\infty} P(s \leq X < 2r) f_s(s) ds + \ldots
\]
\[
= \frac{2}{\pi r^2 (1 - e^{-r^2/2})} e^{-r^2/2} \sqrt{1 - \frac{r^2}{2}} \int_{s}^{\infty} e^{-s^2/2} ds
\]

On the other hand, the expected number of registrations between two successive turning points can be written as
\[
E(K) = \int_{-\infty}^{\infty} P(s \leq X < s + 2r) f_s(s) ds + \int_{s}^{\infty} P(s + 2r \leq s + d) f_s(s) ds + \ldots
\]
\[
= \frac{2}{\pi r^2 (1 - e^{-r^2/2})} e^{-r^2/2} \sqrt{1 - \frac{r^2}{2}} \int_{s}^{\infty} e^{-s^2/2} ds
\]

Therefore, the expected number of registrations per subscriber between two incoming calls is
\[
E(N) = \begin{cases} 
\frac{E(L)}{E(X)} E(K_s) & \text{if } E(L) \leq E(X) \\
E(K_s) + \frac{E(L)}{E(X)} E(K_s) & \text{if } E(L) \geq E(X) 
\end{cases}
\]

C. Distance-based registration with implicit registration

In this study, we consider a combined registration method called DIR (DBR with implicit registration) to improve the performance of the DBR. In DIR, if the distance between the current base station and the last registered base station exceeds a threshold, then the registration occurs. Additionally, if a call arrives at the MS or the MS originates a call, the MS can implicitly notify the system of its location by the call set up messages instead of using a registration message [3, 9]. This type of registration is called implicit registration.

As shown in Fig. 3, when an MS notifies the system of its location by the implicit registration, it does not affect the radio channels because it is performed by the call set up messages. Therefore, this method has less registration load than original DBR.

Fig 3 Mobility of Mobility of a subscriber in DIR

In this case, let’s obtain the expected number of registrations per subscriber between two calls. A call (from or to a mobile station) occurrence makes the mobile station the center cell of a new location area, regardless of call types, the expected number of registrations per subscriber between two calls is as follows:
\[
E(N) = \begin{cases} 
\frac{E(L)}{E(X)} E(K_s) & \text{if } E(L) \leq E(X) \\
E(K_s) + \frac{E(L)}{E(X)} E(K_s) & \text{if } E(L) \geq E(X) 
\end{cases}
\]

Resultantly, assuming incoming call rate is \( \lambda_i \) and outgoing call rate is \( \lambda_o \), the expected number of registrations per subscriber between two incoming calls is simply
\[
E(N) = \frac{\lambda_i + \lambda_o}{\lambda_i} E(K_s).
\]

For example, if \( E(L) = 2 \), incoming call rate \( \lambda_i = 2 \), outgoing call rate \( \lambda_o = 3 \), and \( E(N) = 4 \), then the expected number of registrations per subscriber between two incoming calls is
\[
E(N) = \frac{\lambda_i + \lambda_o}{\lambda_i} E(L) = \frac{5}{2} \times 4 = 10
\]

III. MODELING AND PERFORMANCE ANALYSIS

A VLR area is composed of many LAs in general. Therefore, from the viewpoint of VLR, location registrations (LRs) can be classified as follows:

- o intra-VLR LR: LR between LAs in the same VLR
- o inter-VLR LR: LR between LAs in the different VLRs
Based on the two cases, let’s define LR Processing Capacity of VLR. LR Processing Capacity of VLR is defined as the number of reference LR (RLR) that VLR should process in an hour. In our study, an intra-VLR LR, a most popular LR, is selected as an RLR. Assuming that an inter-VLR LR load is 3 times of an intra-VLR LR load [4-5] VLR, LR Processing Capacity of VLR, C, can be obtained as follows:

\[ C = \left[ W_0 \times P[E_1] + W \times P[E_2] \right] E(U) = \left[ P[E_1] + 3 \times P[E_2] \right] E(U) \]

\( P[E_1] \): probability that MS performs intra-VLR LR

\( P[E_2] \): probability that MS performs inter-VLR LR

\( W_0 \): an intra-VLR LR load

\( W \): an inter-VLR LR load (=3W_0)

In the above, \( E(U) \) is average number of LR that occurs in a VLR area in an hour and we let \( W_0 = 1 \) since an intra-VLR LR is RLR.

A. Zone-based registration

In this study, it is assumed that, as shown in Fig. 4, VLR area is square of length \( D \).

![Fig 4 16LAs of VLR area and 2 types of LR in ZBR](image)

First the probability that an MS is located in a random point of a VLR area is assumed to be \( 1/D^2 \) regardless of the point of a VLR area. Then, average number of LR that occurs in a VLR area in an hour is as follows:

\[ E(U) = E(N)pD^2 = \frac{E(L)pD^2}{2d} \]

\( D^2 \): area of a VLR area

\( p \): average number of subscribers per unit area

Let’s calculate \( P[E_1] \) and \( P[E_2] \) in ZBR. The probability that MS performs inter-VLR LR, \( P[E_2] \), can be obtained as follows [4-5] since \( P[E_2] \) is composed of 3 types as shown in Fig. 5.

\[ P[E_2] = \sum_{i=1}^{2} P[E_i | L_i] P[L_i] = \frac{1}{D^2} \left[ 16d^2 \times \frac{2}{4} + 4(D-4d)2d \times \frac{3}{4} \right] = \frac{2d}{D} \]

\( Li \): (L1) (L2)

\( P[Li] \): 16d2/D2 4(D-4d)2d/D2

\( P[E2|Li] \): 2/4 1/4

The probability that MS performs intra-VLR LR, \( P[E1] \), is as follows:

\[ P[E_1] = \sum_{i=1}^{2} P[E_i | L_i] P[L_i] = \frac{1}{D^2} \left[ \frac{1}{16d^2} \times \frac{2}{4} + 4(D-4d)2d \times \frac{3}{4} \right] = \frac{d^2}{D^2} \]

![Fig 5 LR types of inter-VLR LR in ZBR](image)

Finally, LR processing capacity of VLR, C, can be obtained as follows:

\[ C = \left[ (1 - \frac{2d}{D}) + \frac{3 \times 2d}{D} \right] E(U) = \left( 1 + \frac{4d}{D} \right) \frac{E(L)pD^2}{2d} \]

B. Distance-based registration

In DBR, it is not easy to obtain exact \( P[E2] \). In fact, it was obtained in a previous study [4], but it is not as accurate since LR occurs immediately when a mobile station enters a new VLR. Based on this, we shall instead calculate the correct probability that MS performs inter-VLR LR, \( P[E2] \) and final LR processing capacity of VLR, C.

In DBR, it is also assumed that, as shown in Fig. 4, VLR area is square of length \( D \).

![Fig 6 2 types of LR in DBR](image)

When DBR is adopted, average number of LR that occurs in a VLR area in an hour is as follows:

\[ E(U) = E(N)pD^2 = \frac{E(L)}{\theta} \frac{2pD^2}{\pi^2} \int_{1}^{\infty} e^{-\frac{s^2}{2}} \left( \sqrt{1 - \frac{1}{2} s^2} \right) ds \]

\( D^2 \): area of VLR area

\( \theta \): average number of subscribers per unit area

Similar to Fig. 5 in ZBR, \( P[E2] \) is composed of 3 types as shown in Fig. 7. 3 cases of occurring inter-VLR LR to obtain each probability that next LR is an inter-VLR LR are then considered.
Modeling and Performance Analysis of ZBR and DBR in Mobile Cellular Network

\[ P[L_i] = \frac{4\pi r (D-2r)}{D^2} \]

Fig 7 LR types of inter-VLR LR in ZBR

Case 1: the case that next VLR where next LR can occur is only one

\[ r\mathbf{D}\mathbf{r} \cdot \mathbf{D} \mathbf{r} = (2\pi - 2\theta - \alpha) r + r \cos \theta + r \sin \alpha + r \cos \alpha + r \sin \theta \]

Since p.d.f. (probability density function) of \( \theta \) is \( g(\theta) = \sin \theta, 0 \leq \theta \leq \pi/2 \), the probability that next LR is an inter-VLR LR is

\[ P[E_2|L_1] = \int_{\theta=0}^{\pi/2} \sin \theta \, d\theta \]

Fig 8 An example of case 1

Case 2: the case that next VLR where next LR can occur is two

Similar to case 1, the probability that next LR is an inter-VLR LR is

\[ r\mathbf{D}\mathbf{r} \cdot \mathbf{D} \mathbf{r} = (3\pi - 2\theta - \alpha) + \cos \alpha + \sin \theta + \cos \theta + \sin \alpha \]

Since p.d.f. of \( \theta \) is \( g(\theta) = \sin \theta, 0 \leq \theta \leq \pi/2 \) and p.d.f. of \( \alpha \) is

\[ h(\alpha) = \sin \alpha, 0 \leq \alpha \leq \pi/2 \]

Then, the probability that next LR is an inter-VLR LR is

\[ P[E_2|L_2] = \int_{\theta=0}^{\pi/2} \frac{\sin \theta + \sin \alpha}{\pi/2 + 2\alpha} \sin \alpha \, d\alpha \, d\theta \]

Fig 9 An example of case 2

Case 3: the case that next VLR where next LR can occur is three

Similar to both case 1 and case 2, the probability that next LR is an inter-VLR LR is

\[ \text{length of line EA + length of line AC} \]

\[ = \left(2\pi - \frac{\pi}{2} - \theta - \alpha\right) r + r \cos \theta + r \sin \alpha + r \cos \alpha + r \sin \theta \]

\[ = (3\pi - 2\theta - \alpha) + \cos \alpha + \sin \theta + \cos \theta + \sin \alpha \]

Since p.d.f. of \( \theta \) is \( g(\theta) = \sin \theta, 0 \leq \theta \leq \pi/2 \) and p.d.f. of \( \alpha \) is

\[ h(\alpha) = \sin \alpha, 0 \leq \alpha \leq \pi/2 \]

Then, the probability that next LR is an inter-VLR LR is

\[ P[E_2|L_3] = \int_{\theta=0}^{\pi/2} \frac{\sin \theta + \sin \alpha}{\pi/2 + 2\alpha} \sin \alpha \, d\alpha \, d\theta \]

Finally, MS performs inter-VLR LR, \( P[E_2] \) can be obtained as follows:

\[ P[E_2] = \sum_{i=1}^{3} P[E_i|L_i]P[L_i] = \frac{4\pi r (D-2r)}{D^2} \int_{\theta=0}^{\pi/2} \frac{\sin \theta}{\sin \theta} \, d\theta \]

Finally, LR processing capacity of VLR, C, can be obtained as follows:

\[ C = \left( P[E_1|L_1] + 3 \times P[E_2] \right) E(U) \]

Using the above results, it is possible to accurately evaluate the total signaling cost of each registration scheme, considering a real network hierarchy where a mobile network is made up of many VLRs.
IV. NUMERICAL RESULTS

Let us obtain the performance measures of ZBR and DBR. If the area of location area of ZBR and DBR is assumed to be the same, paging load for an incoming call is also the same. Letting the length of a side of a square location area in ZBR be $2d$ and the radius of a circle location area in DBR be $r$, then the condition $r = \frac{2d}{\sqrt{\pi}}$ is met, provided that the area of location area of ZBR and DBR is the same. The following are also assumed to analyze the performance.

$$d = 5 \text{ km, } E(L) = 20 \text{ km, } D = 30 \text{ km, } \rho = 10$$

Fig. 11 shows the number of LRs for various distances between two successive turning points, $\theta$. In the case of $E(L)=20$, DBR is inferior to ZBR for all $E(X)$ from 1 to 10. On the other hand, in the case of $E(L)=20/3$ that calls occur more often, DBR is superior to ZBR when $E(X)$ is less than 4. In other words, considering the implicit registration, no of LRs in DBR is less than that of ZBR in some cases.

Fig. 12 shows the LR processing capacity for various distances between two successive turning points. In the case of $E(L)=20$, even if the number of LRs in DBR is more than that of ZBR for all $E(X)$ from 1 to 10, the LR processing capacity of DBR is less than that of ZBR if $E(X)$ is less than 4.

In the case of $E(L)=20/3$ that calls occur more often, DBR is superior to ZBR when $E(X)$ is less than 8 from the viewpoint of LR processing capacity. In other words, considering the VLR based network architecture, LR processing capacity of DBR is less than that of ZBR in many cases.

Similar results can be obtained for different sizes of location areas (LAs). Fig. 13 shows the number of LRs for various sizes of LAs. In Fig. 13, $d=3$ means that there are $102^2=100$ zones in a VLR area while $d=7.5$ means that there are $42^2=16$ zones in a VLR area. From Fig. 13, it is shown that DBR is inferior to ZBR when the number of zones is larger than or equal to 62=36 (or $d$ is smaller than or equal to 5) but DBR is superior to ZBR when the number of zones is smaller than $5^2=25$ (or $d$ is larger than or equal to 6).

Fig. 14 shows the LR processing capacity for various inter-VLR LR load ($W$) assuming $E[L]=10$. In the case of $W=4$, DBR is superior to ZBR in most cases while DBR is superior to ZBR in most cases in the case of $W=1$. From the figure, it is shown that, DBR is superior to ZBR in more cases as inter-VLR LR load increases, which means that portion of the inter-VLR LR load in DBR is larger than that of ZBR.
From the figures, it is established that, considering the implicit registration and VLR based network architecture, DBR may be superior to ZBR in most cases.

**ACKNOWLEDGMENT**

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT and Future Planning (2017R1E1A1A03070134) and by the Ministry of Education (2016R1D1A1B01014615).

**REFERENCES**