Application of PFMGBEKF for Bearings-only Tracking

L. Sandeep, S. Koteswara Rao, Kausar Jahan

Abstract: ***Detection and estimation of object in motion is crucial issue in tracking. In underwater object tracking, object parameters like course, range and speed of the object are estimated using passive mode operation of the sonar. In this paper particle filter combined with modified gain bearings-only extended Kalman filter (PFMGBEKF) and residual sampling are used. One of the main assumptions is that the object is moving with constant velocity. Bearing measurements are nonlinearly related to the state of the object and sub-optimal filter for a nonlinear approach is unscented Kalman Filter (UKF). But UKF is unreliable under non-Gaussian noise environment. Particle filter is an advanced filter that processes nonlinear data in non-Gaussian noise environment but hassample degeneracy problem. So, PFMGBEKF is applied and the operation is analysed based on the solution convergence time. Simulation of algorithms on numerous scenarios which are close to reality is done using MATLAB.***

Index Terms: ***Bearings-only tracking, Modified gain bearings-only extended Kalman filter, Particle filter, Residual sampling, Statistical signal processing.***

I. INTRODUCTION

Inactive sonar is used to take bearing measurements in x-y plane. The analysis of the movement of the object is termed as Object Motion Analysis (OMA). The general complication in OMA is estimation of velocity and path of the object from the sensor data tampered by noise [1]. Tracking with the help of bearings-only measurements, popularly known as bearing-only tracking (BOT), has a wide range of applications like underwater object tracking, aircraft surveillance, etc. Bearing is the angle made by the line of sight from observer to object, measured with respect to some reference axis in clock wise direction. Using this bearing, object motion parameters like course, range and speed are estimated. Course is angle made by heading direction of an object with respect to some reference axis measured in clock wise direction. Range is the distance between observer and object. Bearing is obtained from Hull mounted sensor of the observer. In the considered scenario of the ocean environment, the observer monitors passively sonar bearing from the single object which is presumed to have a constant velocity. This type of system is intuitively nonlinear. From the measurements prior to an observer manoeuvre, a few of the object states are unobservable [2]. In this work, commonly used S – manoeuvre is followed for tracking the object.

Manuscript published on 30 March 2019.
***Correspondence Author(s)**
L. Sandeep, Electronics and Communication Engineering, Koneru Lakshmaiah Education Foundation, Guntur district, India,
S. Koteswara Rao, Electronics and Communication Engineering, Koneru Lakshmaiah Education Foundation, Guntur district, India,
Kausar Jahan, Electronics and Communication Engineering Koneru LakshmaiahEducationFoundation,Guntur district, India,

BOT has been studied extensively since 1980's [2-3]. The tracking process is unobservable if observer follows straight path and constant speed. By making use of S-manoeuvre the process is made observable. For underwater scenarios, the observability criteria are discussed in [4-6]. Kalman filter is an appropriate filter for linear system. In practical approach bearing data are nonlinearly related to object state vector. Assumption that object is moving with constant velocity is made. According to S. C. Nardone and V. J. Aidala one can't estimate the object parameters unless the observer makes changes in its course or speed which is called manoeuvring [5]. Although the object velocity assumption reduces nonlinearity, the disturbance in object velocity is assumed as white Gaussian noise.The intrinsic challenges encountered in this object motion parameters estimation are nonlinearity in the estimation process, obtain signal to noise ratio as low as possible, baffling of sensors, reduction of fading in the acoustic signals, presence of high clutter, uniqueness in the properties of state Observability [7-9]. Extended Kalman filter (EKF) is an appropriate filter for nonlinear model [10]. In this EKF, by using Taylor series nonlinear system is converted to a linear system and then Kalman filter is applied. This approach of EKF is arduous to adjust and regularly gives unpredictable estimates if the system nonlinearity in the measurements is severe. Recursive way of estimating output using individual particle is followed in this approach. Unscented Kalman Filter (UKF) is only an approximate nonlinear estimator. Based on the small set of trail points UKF is not a truly global approximation.The systems with covariance matrices that tend to singular values, the performance of UKF is not well enough. It can only be applied to models driven by Gaussian noises. The particle filter (PF) is an advanced stochastic filtering algorithm for estimation and often preferable at times where conventional Kalman filter fails [10]. This approach overcomes the EKF by considering the properties of a set of particles instead of taking individual particle properties. In this particle filtering algorithm, based on initial probability distribution function (PDF), randomly N state vectors called particles are generated. These particles are updated based on the bearing measurements. Updated particles are assigned with weights based on the PDF of particles. The estimated particles aresampled to reduce sample impoverishment and sample degeneracy problem.One method to reduce sample impoverishment is to combine PF with other filters. In this paper PF is combined with Modified Gain Only Bearing Extended Kalman Filter (PFMGBEKF). In this paper performance of PFMGBEKF is evaluated in the presence of Gaussian noise. Mathematical modelling for this algorithm is discussed in detail in section II.

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

Section III gives thesimulation and results obtained from MATLAB environments. Overall summary of work is given in section I

II. MATHEMATICAL MODELING

Consider an observer with $s_{xr}(l), s_{yr}(l)$as velocities in X and Y direction and$g_{xr}(l),\ g_{yr}(l)$as ranges. Initially the object is at 'O' position and is moving with constant speed and course. The state vector of the observer at time instant $l$is represented as

$v_r(l) = [\, s_{xr}(l) \quad s_{yr}(l) \quad g_{xr}(l) \quad g_{yr}(l)]^T$ (1)

As the observer follows 'S' manoeuvring there will be change in position of observer and change in position can be obtained by speed and course as

$dg_{xr}(l) = s_{xr}(l) * \sin(ocr) * t$(2)

$dg_{yr}(l) = s_{yr}(l) * \cos(ocr) * t$(3)

The change in x coordinate and y coordinate is given by $dg_{xr}(l)$and $dg_{yr}(l)$and the observer course angle is given by $ocr$and $t$is the time period of one second.

Now consider an object with velocities $s_{xb}(l)$and $s_{yr}(l)$ in x and y direction and $g_{xb}(l),\ g_{yb}(l)$ as ranges. The state vector of the object at time instant $l$is represented as

$v_b(l) = [s_{xb}(l) \quad s_{yb}(l) \quad g_{xb}(l) \quad g_{yb}(l)]^T$(4)

The change in object position in x and y coordinate is given by $dg_{xb}(l)$and $dg_{yb}(l)$ and $bcr$is object course angle and t is the time period of one second

$dg_{xb}(l) = s_{xb}(l) * \sin(bcr) * t$(5)

$dg_{yb}(l) = s_{yb}(l) * \cos(bcr) * t$(6)

The relative state vector for object with reference to observer is given as

$$v_v(l) = [\, s_x(l) \quad s_y(l) \quad g_x(l) \quad g_y(l)]^T \quad (7)$$

Where $s_x(l),\ s_y(l),\ g_x(l),\ g_y(l)$ are the relative complements of velocity and range in x and y directions for object.

For the next time period based on the present time state vector the relative state vector is given as

$v_v(l+1) = P(l)v_v(l) + wR(l)$(8)

Where $P(l)$ represents state matrix and the value is given as

$$P(l) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix} \quad (9)$$

and $R(l)$ represents process noise and w is given as

$$w = \begin{bmatrix} t & 0 \\ 0 & t \\ \frac{t^2}{2} & 0 \\ 0 & \frac{t^2}{2} \end{bmatrix} (10)$$

The covariance of the process noise is given as

$Q(r) = E[(wR(r))(wR(r))^T]$ (11)

$$Q(r) = \rho^2 \begin{bmatrix} t^2 & 0 & t^3/2 & 0 \\ 0 & t^2 & 0 & t^3/2 \\ t^3/2 & 0 & t^4/2 & 0 \\ 0 & t^3/2 & 0 & t^4/2 \end{bmatrix} (12)$$

where ρ^2 is the variance for the process noise.

Using the bearing angle $\alpha(k)$ the measurement equation is represented as

$\alpha(l) = \tan^{-1}(g_x(l)/g_y(l))$(13)

The degraded bearing measurement due to noise is given as

$\alpha_m(l) = \alpha(l) + n(l)$(14)

Noise in the measurement is represented by $n(l)$ and the system measurement equation is given as

$N(l) = h(l)v_v(l) + \beta(l)$(15)

where $\beta(l)$ is the measurement noise matrix and $h(l)$ represents measurement model matrix

A. MGBEKF algorithm

The noises in plant and measurements are presumed to be independent to each other. Using Taylor series expansion, linearization of nonlinear equation is done. The measurement model matrix is calculated as

$$H(l+1) = \begin{bmatrix} 0 \\ 0 \\ g_y(l+1)/G^2(l+1) \\ g_x(l+1)/G^2(l+1) \end{bmatrix}^T (16)$$

Since the actual values of range will not be known, the estimated range values will be used in the above equation. The predicted covariance matrix is calculated as

$A(l+1) = \big(P(l+1)A(l)P^T(l+1)\big) + wR(l+1)\mathrm{w}^{\mathrm{T}}$ (17)

The Kalman gain is

$Y(l+1) = A(l+1)H^T(l+1)[\rho_B^2 + H(l+1)A(l+1)H^T(l+1)]^{-1}$ (18)

The updated state matrix is calculated as

$v_v(l+1) = v_v(l+1) + Y(l+1)\big[\alpha_m(l+1) - M\big(l+1, v_v(l+1)\big)\big]$(19)

where $M\big(l+1, v_v(l+1)\big)$ is the bearing measurement obtained from predicted estimate at time index $(l+1)$. The updated covariance matrix follows the below equation

$A(l+1) = \big[I - Y(l+1)m\big(\alpha_m(l+1), v_v(l+1)\big)\big]P(l+1)\big[I - Y(l+1)m\big(\alpha_m(l+1), v_v(l+1)\big)\big]^T + \rho_B^2 Y(l+1)Y^T(l+1)$(20)

where$m$is the modified gain function and iscomputed as follows

$$m = \begin{bmatrix} 0 \\ 0 \\ \left(\frac{\cos\alpha_m}{g_x \sin\alpha_m + g_y \cos\alpha_m}\right) \\ \left(\frac{-\sin\alpha_m}{g_x \sin\alpha_m + g_y \cos\alpha_m}\right) \end{bmatrix}^T (21)$$

B. Particle filter

Particle filter is a non-linear state estimator and more computational effort is required for higher performance of the particle filter [10]. For recursively calculating the posterior probability density function of a state vector gives system and measurement equations as follows

$x_{l+1} = f_l(x_l, w_l)$(22)

$y_l = h_l(x_l, v_l)$(23)

where $\{w_l\}$ and $\{v_l\}$ are independent white noise processes with known PDF's.

Generate N initial particles randomlybased on the assumed initial state PDF $p(x_o)$. These particles are denoted by $x_{0,i}^{+}(i = 1, \dots, N)$.

For l=1,2, . . ., do the following

(a) Using the process equation and known PDF of the process noise a time propagation step is performed to obtain a priori particles$x_{l,i}^{-}$.

$x_{l,i}^{-} = f_{l-1}\big(x_{l-1,i}^{+}, w_{l-1}^{i}\big)\ \ (i = 1, \dots, N)$(24)

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

w_{l-1}^{i} is the randomly generated noise vector.

(b) The relative likelihood q_i of each particle $x_{l,i}^{-}$ conditioned on the measurement y_l is computedby evaluating the PDF $p(y_l|x_{l,i}^{-})$ and the PDF of the measurement noise

(c) Scaling therelative likelihoods obtained in the above step as follows:

$$q_i = \frac{q_i}{\sum_{j=1}^{N} q_j} \quad (25)$$

d) Based on the above relative likelihoods a set of posteriori particles $x_{l,i}^{+}$ are generated. This is the re-sampling techniques in particle filter. Residual re-sampler is used as the re-sampling technique.

C. Particle filter combined with modified gain bearings-only extended Kalman filter (PFMGBEKF):

Improving particle filtering is done by combining particle filter with another filter like MGBEKF. In this methodat the measurement time each particle is updated using the MGBEKF, then using the measurement residual re-sampling is performed [10]. This is like running a bank of N number of Kalman filter (one for each particle) and then adding a residual re-sampling step after measurement. The state $v_v(l+1,l)$ is updated to $v_v(l+1,l+1)$ according to the following MGBEKF equations.

$$A(l+1,l)_i = (P(l+1,l)_i A(l,l)_i P^T(l+1,l)_i) + wR(l+1)w^T \quad (26)$$

$$Y(l+1)_i = A(l+1,l)_i H^T(l+1)_i [\rho_B^2 + H(l+1)_i A(l+1,l)_i H^T(l+1)_i]^{-1} \quad (27)$$

$$v_v(l+1,l+1)_i = v_v(l+1,l)_i + Y(l+1)_i [\alpha_m(l+1) - h(l+1, v_v(l+1,l)_i)] \quad (28)$$

$$A(l+1,l+1)_i = [I - Y(l+1)_i m(\alpha_m(l+1), v_v(l+1,l)_i)] P(l+1,l)_i \times [I - Y(l+1)_i m(\alpha_m(l+1), v_v(l+1,l)_i)]^T + \rho_B^2 Y(l+1)_i Y^T(l+1,l)_i \quad (29)$$

Where $Y(l+1)$ represents the Kalman gain, $A(l+1,l)$ represents priori estimation error covariance for the i^{th} particle and m(.) is modified gain function. m(.) is given by

$$m = \begin{bmatrix} 0 \\ 0 \\ \left(\frac{\cos\alpha_m}{\hat{g}_x \sin\alpha_m + \hat{g}_y \cos\alpha_m}\right) \\ \left(\frac{-\sin\alpha_m}{\hat{g}_x \sin\alpha_m + \hat{g}_y \cos\alpha_m}\right) \end{bmatrix}^T \quad (30)$$

D. Re-sampling:

Particle filter mainly involve three steps, propagation of particles, calculation of particle weights and sampling of particles based on weights. The first two operations are discussed in part ‘c’ of section II. In re-sampling, it replaces one set of particles and their weights of another set. The re-sampling step isglobal and generally state dimension free. Without re-sampling particle filter will produce a degenerate set of particles, i.e. a set in which a few particles dominate the rest of particles with their weights. Due to this deterioration the obtained estimates will be inaccurate and will have unacceptable large variances. To avoid this problem re-sampling techniques are high essential for particle filter. Re-sampling methods has more importance and different re-sampling methods are viewed [11-15]. Out of different methods in re-sampling in this paper residual re-sampling approach has been used. Residual re-sampling depends on set of particles instead taking the entire particles as the process followedin sampling methods and it is a different approach to re-sample. In this method many particle progenies n_i can be regulated without any sorting of the random particles, which is done by considering the integer part of $N\omega_i$. In order to maintain the native number of particles, more particles are added called residual particles which are selected randomly from the original particles with modified weights[10].

Allocate $n_l^{'} = [N\omega_l]$ copies of the particle v_l to the new distribution and re-sample $m = N - \sum n_l^{'}$ particles from $\{v_l\}$ by making $n_l^{''}$ copies of particle v_k where the probability for selecting v_l is proportional to $\omega_l^{'} = N\omega_l - n_l^{'}$ using one of the re-sampling methods.

III. SIMUATION AND RESULTS

MATLAB PC environment is used for the performance analysis of the particle filter. Assumption is made such that estimation measurements are available regularly for every second. The observer is manoeuvring in its course. So, the observer initially has the course of 90^0 for two minutes and then turns 180^0 in order to attain first leg in manoeuvring and has a course of 270^0.The observer is considered to take it for four minutes for complete manoeuvre of 180^0. The object is assumed to be having different initial ranges, speeds and courses in different scenarios, which is given in the Table I. The object state vector’s initial estimate for PF combined with MGBEKF is taken as $v_v(0,0) = [5 \quad 5 \quad 5000\, sin\alpha_m \; 5000\, cos\alpha_m]$ (31)

The availability of only angle measurement makes difficult for the prediction of velocity components of the object. So, they are each assumed as 5m/s. The object initial position is calculated based on the Sonar Range of the Day (SRD), which is assumed to be 5000m. The initial state covariance matrix can be taken as a diagonal matrix if the uniform distribution of the initial state estimate is considered and given as

$$Q(0,0) = diagonal \begin{bmatrix} 4s_x^2(0,0)/12 \\ 4s_y^2(0,0)/12 \\ 4g_x^2(0,0)/12 \\ 4g_y^2(0,0)/12 \end{bmatrix} \quad (32)$$

Table I Scenarios for PFMGBEKF

Scenario	Parameters				
	Range (m)	Bearing (degrees)	Object Speed (m/s)	Object Course (deg)	Observer Speed (m/s)
1.	5000	0	8	135	12
2.	3500	0	12	110	8
3.	3000	0	12	135	8
4.	3000	0	12	140	8

The simulation and filtering process for 100 particles with

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

1200 samples are carried out for above-mentioned scenarios in MATLAB software. The performance is analyzed based on the Root-Mean-Squared (RMS) error of the object parameters. The convergence time is obtained and listed in Table II, based on the below acceptance criteria

The acceptance criteria for this particle filter combined with MGBEKF is assumed as

1. Range error estimate <= (8%)/3 of the original range
2. Course error estimate <=1^0.
3. Speed error estimate <=0.33m/s.

Table II Convergence in time in seconds for PFMGBEKF

Scenario	Convergence times in seconds			
	Range	**Course**	**Speed**	**Overall convergence time**
1	438	664	538	664
2	362	383	362	383
3	293	353	325	353
4	267	349	325	349

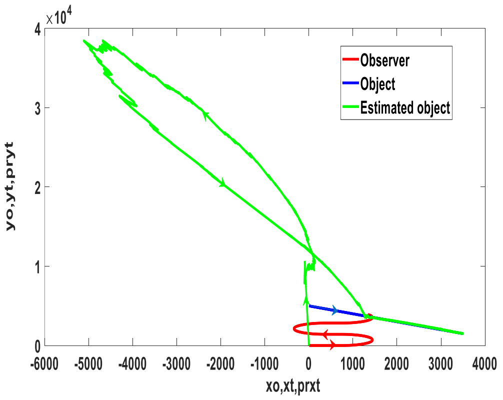

Figure 1 Observer and object movements

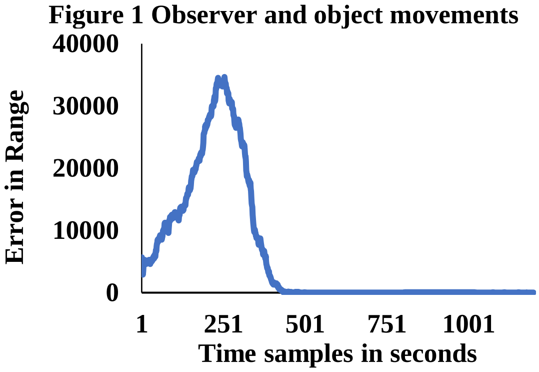

Figure 2 Error in estimates of range

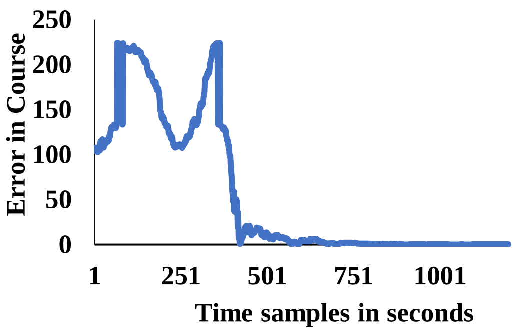

Figure 3 Error in estimates of Course

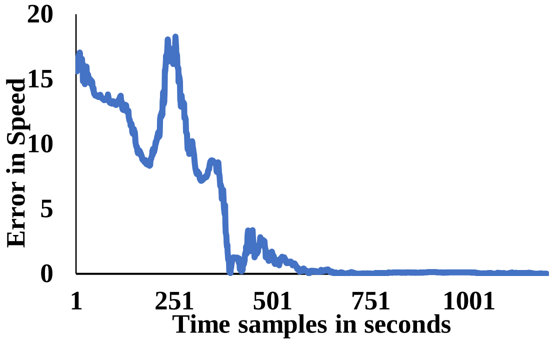

Figure 4 Error in estimates of Speed.

From Table 2 for scenario 1 within the acceptance criteria, the estimate range, estimated course and estimated speed of obtained from simulation at 438, 664 and 538 seconds respectively using PFMGBEKF. The overall convergence time for scenario 1 is 664 seconds.

Figure 1 shows the observer, object and estimated object movements in 2-dimensional plane and observer follows 'S' manoeuvring for the validation of observability criteria.

Figure 2-4 shows the RMS errors in the estimates of range, estimates of course and estimates of speed of the object for the PF combined MGBEKF

IV. CONCLUSION

In this paper, an effort is made to analyse PFMGBEKF for four different scenarios in the MATLAB environment. Taking into consideration of computational effort 100 particles are considered for simulation, which can be increased for better estimation of OMP. Better convergence time can be obtained if the number of particles is increased by compromising on consideration of computational effect.

REFERENCES

1. G. Lindgren, KAI F. Gong, "Position and velocity Estimation via Bearing Observations", IEEE Trans. Aerosp. Electron Syst., Vol. AES-14, No.4, pp 564-577.
2. S. C. Nardone, A. G. Lindgren and K. F. Gong, "*Fundamental properties and performance of conventional bearings-only target motion analysis*", IEEE Trans. Autom. Control, Vol. AC-29, No. 9, pp 775-787, September 1984.
3. J. Aidala, "*Kalman filter behavior in bearings-only tracking applications*", IEEE Trans. Aerosp. Electron. Syst., Vol.AES-15, No. 1, pp. 29-39, 1979
4. S. C. Nardone, V. J. Aidala, "Observability criteria for Bearing-Only Target Motion Analysis", IEEE Trans. Aerosp. Electron. Syst., Vol. AES-17, No.2, pp. 162-166, 1981
5. S. Koteswara Rao, "*Comments on Discrete-time observability and estimability analysis for bearings-only target motion analysis*," IEEE Trans. Aerospace Electronic Systems, Vol. 32, No.4 (1998) pp. 1361-1367.
6. Weiliang Zhu; Zhaopeng Xu; Bo Li; Zhidong Wu, "*Research on the Observability of Bearings-only Target Tracking Based on Multiple Sonar Sensors*", Second International Conference on Instrumentation, Measurement, Computer, Communication and Control; pp: 631-634, DOI: 10.1109/IMCCC.2012.154; IEEE Conference Publications, 2012.
7. T. Northardt, S. C. Nardone, "Track-Before - Detect Bearings - Only Localization Performance Complex passive Sonar Scenarios", IEEE Journal of Ocean Engineering, 2018.
8. J. Salmond and H. Birch, "A particle filter for track-before-detect", in Proc. Amer. Control Conf., 2001, vol. 5, pp. 3755-3760.

Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)

International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-8 Issue-5, March 2019

9. B. Ristic, S. Arulampalan, and N. Gordon, Beyond the Kalman Filter: Particle Filters for Tracking Applications. Norwood, MA, USA: Artech House, 2004.
10. Dan Simon, "Optimal State Estimation: Kalman, H∞ and Nonlinear Approximations", Wiley, 2006.
11. Tiancheng Li, P. M. Djuric, M. Bolic, "Resampling Methods for Particle Filtering: Classification, implementation, and strategies", IEEE Signal Processing Magazine, pp. 70-86, 2015.
12. Kitagawa, "Monte Carlo filter and smoother and non-Gaussian nonlinear state space models," J.Comput. Graph. Stat., vol. 5, no. 1, pp. 1–25, 1996.
13. J. Carpenter, P. Clifford, and P. Fearnhead, "An improved particle filter for nonlinear problems," IEEE Proc., Radar Sonar Navigat., vol. 146, no. 1, pp. 2–7, 1999.
14. E. R. Beadle and P. M. Djuric ´, "A fast-weighted Bayesian bootstrap filter for nonlinear model state estimation," IEEE Trans. Aerosp. Electron. Syst., vol. 33, no. 1, pp. 338–343, 1997.
15. J. Liu and R. Chen, "Sequential Monte-Carlo methods for dynamic systems," J. Amer. Statist. Assoc., vol. 93, no. 443, pp. 1032–1044, 1998

AUTHORS PROFILE

Lingutla SandeepstudyingB. Tech in Electronics and Communication Engineering stream from Koneru Lakshmaiah Education Foundation,Vaddeswaram, Guntur, AP, India.

S. Koteswara Rao former scientist "G" in NSTL, DRDO, Visakhapatnam is currently working as Professor in Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur, AP, India. He received BTech in 1977 at JNTU and ME in 1979, PSG college oftechnology, Coimbatore and Ph. D at Andhra university allin Electrical Engineering. He published severalpapers in International Conference & Journals in thefieldof signalprocessing. He is the fellow member of IETE.

Kausar Jahan research scholar in Koneru Lakshmaiah Educational Foundation, vaddeswaram, Guntur, AP. India. She completed her B. Tech in 2009 at JNTU Kakinada and M Tech in 2015 at JNTU Kakinada in ECE. She is presently working as women scientist WOS-A.