International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-8 Issue-5, March 2019

Underwater bearings-only tracking using particle filter

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***Abstract:** Underwater target tracking is a pivotal area in the present scenario. In this paper, passive target tracking is accomplished. By using bearings-only measurements, the parameters like range, course and speed of the target with respect to observer are calculated which helps in determining the target motion. This is called Target Motion Analysis (TMA). As bearings-only tracking is non-linear in nature, traditional Kalman filter which is linear filter, cannot be used. So, Particle filter (PF) which is non-linear filter is preferred. Since particles/samples are used, particle degeneracy or sample impoverishment may occur. To avoid sample impoverishment, resampling of the particles is done after every iteration. So, stratified resampling which can give greater precision is used to reduce the sample impoverishment problem. For improved performance of the filter, PF is combined with Modified Gain Extended Kalman filter (MGEKF). The algorithm is simulated using different scenarios in MATLAB to evaluate its sensitivity. Estimating the performance of the algorithm depending on their convergence time is carried out.*

***Index Terms**: Modified Gain Extended Kalman Particle Filter (MGEKPF), Particle Filter (PF), Stratified Resampling, Sample Impoverishment, Target Tracking.*

I. INTRODUCTION

In underwater surroundings, two-dimensional tracking of a target using bearings-only measurement is widely prominent in recent years. The angle made from the reference axis to the line of sight (from observer to target) in clockwise direction is known as bearing. By using bearings-only measurements, the parameters like range, course and speed of the target with respect to observer are calculated which helps in determining the target motion. This is known as Target Motion Analysis (TMA) [1-2]. Course is the angle made by an object with respect to a reference axis in clockwise direction. For the TMA to be observable the it is assumed that the target is moving with constant speed and, the observer is modelled such that it follows 'S' maneuver [3]. The bearing measurements obtained within the state vector are non-linear. Kalman filter which is a linear filter is not suitable. There are extensions of Kalman filter. The most popular of them are Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF). EKF relies on linearization of the system. So, it is difficult to tune when the non-linearities are high. Coming to UKF, it reduces the linearization errors of EKF using Unscented transformation which is an approximation technique. Even if EKF and UKF can reduce the non-linearities, they are just extensions of linear filter and are not completely non-linear. So, Particle filter (PF) which can cope up with non-linearities and non-Gaussian noise is chosen for the application. But the improved performance comes with increased level of complexity [4-9]. If the increased computational efforts are worth of improved performance, then onlyPF will be chosenaccordingly depending on the requirements of the application and system dynamics. Using PF can sometimes lead to sample imporvishment and particle degeneracy.To avoid particle degeneracy, high number of particles are considered. But increase in number of particles means high computations efforts and hence eventually lead to particle degeneracy. So, resampling of particles after every measurement is performed. Stratified resampling with increased statistical efficiency is considered in this paper [10-14]. To improve PF, one proposed method is to combine PF with other filters like EKF, UKF or MGEKF resulting in each particle to be updated after taking measurement with a bank of Kalman filters. Performance ofPF combined with MGEKF (MGEKPF) is analyzedin this paper[15].

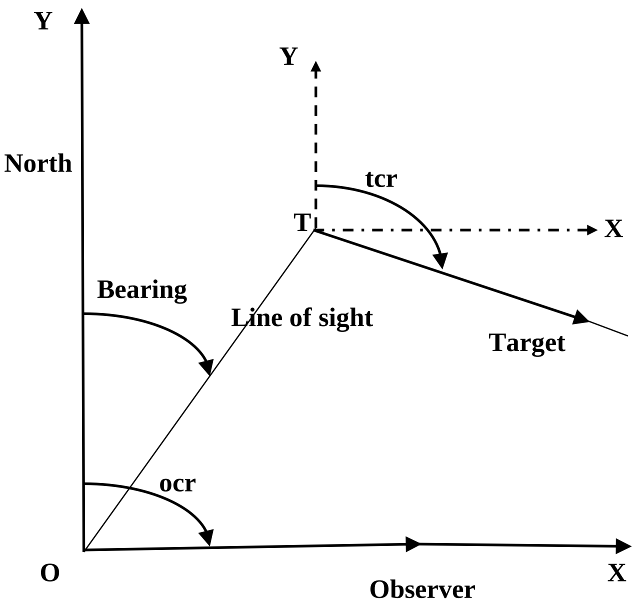

Fig 1: Observer Target Scenario

In figure 1, the observer target scenario is represented. Here, the observer is staring at position 'O' and target is at position 'T'. To satisfy the observability conditions,

Manuscript published on 30 March 2019.
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the observer follows 'S' maneuvering. Mathematical modeling for system dynamics, PF, MGEKF and stratified re-sampling are given in sectionII. The simulation results and the different scenarios on which the simulation is done are mentioned in SectionIII. Graphs are plotted for the obtained results and analyzed in tables. The total work done is summarized and concluded in Section IV

II. MATHEMATICAL MODELING

A. Target Motion Analysis

Consider that the observer is at position 'O' initially and the target is moving with constant speed and course. The state vector at time sample 'b' of the observer is as given by

$$X_o(b) = [v_{xo}(b)\ \ v_{yo}(b) r_{xo}(b) r_{yo}(b)]^T \quad (1)$$

where $r_{xo}(b)$, $r_{yo}(b)$, $v_{xo}(b)$, $v_{yo}(b)$, are the range and velocity components of the observerin x and y coordinates respectively. The change in the position of the observer can be acquired from its speed and course as

$$dr_{xo}(b) = v_{xo}(b) * sin\ ocr * t \quad (2)$$

$$dr_{yo}(b) = v_{yo}(b) * \ cos\ ocr * t \quad (3)$$

where the change in in x and y coordinates of the observer are $dr_{xo}(b), dr_{yo}(b)$ respectively, ocr is the course of the observer and t is time interval of one second. Similarly, target state vector is as follows

$$X_t(b) = [v_{xt}(b)\ v_{yt}(b) r_{xt}(b) r_{yt}(b)]^T \quad (4)$$

where $r_{xt}(b)$, $r_{yt}(b)$, $v_{xt}(b)$, $v_{yt}(b)$, are the range and velocity components of the target in x and y coordinates respectively. The change in the position of the target can be acquired from its speed and course as

$$dr_{xt}(b) = v_{xt}(b) * sin\ tcr * t \quad (5)$$

$$dr_{yt}(b) = v_{yt}(b) * \ cos\ tcr * t \quad (6)$$

where the change in x.coordinate and y.coordinate of the observer are $dr_{xt}(b), dr_{yt}(b)$ respectively, tcr is the course of the observer and t is time period of one second. The relative state vector of the target with respect to observer is given as

$$X(b) = [v_x(b) v_y(b) r_x(b) r_y(b)]^T \quad (7)$$

where $r_x(b)$, $r_y(b)$, $v_x(b)$, $v_y(b)$, are the relative components of the range and velocity of the targetin x and y directions. The relative state vector for the next time second based on the present time state vector is obtained as

$$X(b+1) = \emptyset(b)X(b) + D(b+1) + \ \omega * c(b) \quad (8)$$

where $\emptyset(b)$ is the system dynamics matrix computed as

$$\emptyset(b) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix} \quad (9)$$

$c(b)$ is the process noise and ω is computed as

$$\omega = \begin{bmatrix} t & 0 \\ 0 & t \\ t^2/2 & 0 \\ 0 & t^2/2 \end{bmatrix} \quad (10)$$

$D(b)$ is the deterministic matrix which is calculated as

$$D(b+1) = \begin{bmatrix} 0 \\ 0 \\ -(r_{xo}(b+1) - r_{xo}(b)) \\ -(r_{yo}(b+1) - r_{yo}(b)) \end{bmatrix}^T \quad (11)$$

The process noise covariance is calculated as

$$Cov(b) = E[(\omega * c(b))(\omega * c(b))^T] \quad (12)$$

$$Cov(b) = \sigma^2 \begin{bmatrix} t^2 & 0 & t^3/2 & 0 \\ 0 & t^2 & 0 & t^3/2 \\ t^3/2 & 0 & t^4/4 & 0 \\ 0 & t^3/2 & 0 & t^4/4 \end{bmatrix} \quad (13)$$

where $\sigma^2$is the variance in the process noise.

The measurement equation for this application has only bearing angles which is denoted as $\beta(b)$.

$$\beta_m(b) = \tan^{-1}(r_x(b)/r_y(b)) + \Upsilon_k \quad (14)$$

where $\Upsilon_k$is the noise measurement that is assumed to be following Gaussian distribution with variance σ_B^2.

B. Particle Filter

Let the probability distribution function(pdf) of initial state be$p(b)$. Assume that it is known. Now initially generate N particles based on the pdf $p(b)$ randomly.N is chosen by the user based on the computational effort and estimation accuracy

The system and measurement equations are:

$$X_{b+1} = f_b(X_b, W_b) \quad (15)$$

$$\beta_m(b) = h_b(X_b, V_b) \quad (16)$$

W_b and V_b are the independent white noise processes with known pdf'.

Priori estimation: Perform the time propagation step.Calculate for Nparticles.

$$X_{b,i}^- = f_{b-1}(X_{b-1,i}^+, W_{b-1}^i) \quad (17)$$

where $i = 1,2, \ldots, N$.

Posteriori estimation: After obtaining the measurement at time b, the relative likelihood q_i for every particle of $X_{b,i}^-$ conditioned on the measurement, $\beta_m(b)$ is calculated by evaluating the pdf $p(\beta_m(b)|X_{b,i}^-)$ based on nonlinear computation equation and themeasurement noise pdf.

Scale the relative likelihood obtained as

$$q_i = \frac{q_i}{\sum_{j=1}^{N} q_j} \quad (18)$$

Now, the sum of all the resulting likelihoods is one.

Then, generate a set off posteriori particles $X_{b,i}^+$ based on the relative likelihoods q_i. This is called resampling.

Now, the set of particles $X_{b,i}^+$ that are distributed accordingly based on the conditional pdf $p(\beta_m(b)|X_{b,i}^-)$, helps us to compute any designed statistical measure of pdf.

C. Particle Filter Combined with Modified Gain Extended Kalman Filter

For improving the performance of PF, it is combined with MGEKF. It is like processing a bank of NKalman filters for each particle before the re-sampling step.

The state equations are:

Priori estimation covariance:

$$P(b+1,b)_i = \emptyset(b+1,b)_i P(b,b)_i \emptyset^T(b+1,b)_i + \omega * c(b+1)\omega^T \quad (19)$$

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Kalman Gain is

$$K(b+1)_i = P(b+1,b)_i H^T(b+1)_i [H(b+1)_i P(b+1,b)_i H^T(b+1)_i + \sigma_B^2]^{-1} \quad (20)$$

The updated state equation:

$$X(b+1,b+1)_i = X(b+1,b)_i + K(b+1)_i[\beta_m(b+1) - M(b+1, X(b+1,b)_i)] \quad (21)$$

where $M(b+1, X(b+1))$ is the predicted bearing measurement estimate obtained at time $(b+1)$.

Posterior estimation covariance:

$$P(b+1,b+1)_i = [I - K(b+1)_i g(\beta_m(b+1), X(b+1,b)_i] P(b+1,b)_i * [I - K(b+1)_i g(\beta_m(b+1), X(b+1,b)_i]^T + \sigma_B^2 K(b+1)_i K^T(b+1)_i \quad (22)$$

where g represents the modified gain function given as

$$g = \left[0 \quad 0 \quad \left(\frac{\cos\beta_m}{r_x \sin\beta_m + r_y \cos\beta_m}\right) \quad \left(\frac{-\sin\beta_m}{r_x \sin\beta_m + r_y \cos\beta_m}\right)\right] \quad (23)$$

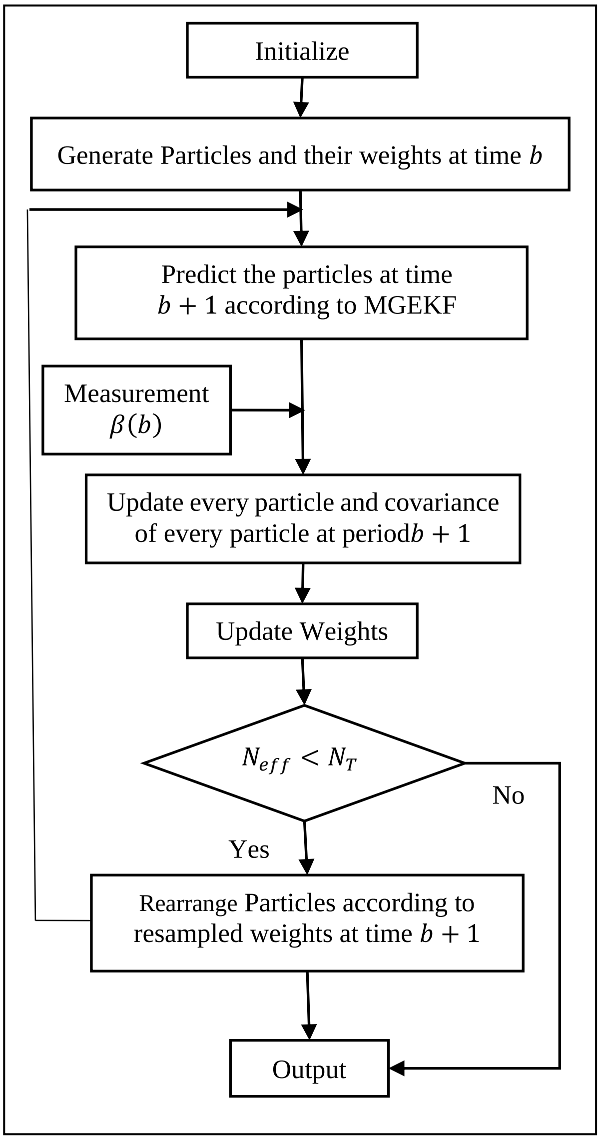

Fig 2: Flow chart for Particle Filter combined with MGBEKF

D. *Stratified Resampling*

The basic idea oresampling using stratified technique is to distribute the samples equally according to their respective variance into sub regions. Take N particles and assign weightsW_i for all these particles. The normalized weight of N particles is W_N.

$$u_i = \frac{(i-1)+\hat{u}_i}{N} \quad with\ \hat{u}_i \sim U[0,1) \quad (23)$$

where $\hat{u}_i \sim U[0,1)$ is standard uniform random number distribution.

AlloteS_N copies of the particle X_N to the new distribution, where

$$S_N = the\ number\ of\ u_i \in (\textstyle\sum_{i=1}^{N-1} w_i, \sum_{i=1}^{N} w_i] \quad (24)$$

Refine the set of $\emptyset(b+1)$ and $P(b+1)$ on the bases of u_i. Now, any desired statistical measure of this set of particles can be computed.

III. SIMULATION AND RESULTS

It is hypothetically considered that the measurements are obtained after each second. The observer is manoeuvring in its course. The observer initially has a course of 90°, after maintaining it for 120seconds it turns an angle of 180° resulting in a course of 270°. It takes a period of 240seconds to complete 180° maneuvre with a turn rate of 0.5° per/sec. To assess the performance of the algorithm, different scenarios containing of diverse initial range, speed and course are taken into consideration. For these scenarios, the simulation is carried out using MATLAB. These taken scenarios are mentioned in table 1.

The acceptance criteria of the above algorithm is

Acceptable error in Range <= 8% of true Range.

Acceptable error in Speed <=1m/s.

Acceptable error in Course <= 3̊.

The time from which a parameter satisfies its acceptance criteria is convergence time. The time from which all the parameters satisfy the convergence criteria is called overall convergence. For the considered scenarios, the convergence times of range, speed, course and overall convergene is mentioned in table 2.

Table 1: Scenarios taken for particle filter

Scenario	Parameters				
	R (m)	B (deg)	TS (m/s)	TC (deg)	OS(m/s)
1	3000	0	12	140	8
2	3500	0	12	110	8
3	5000	0	8	135	12

R:Range from observer to target at the start

B: Initial Bearing

TS: Speed of the Target

TC:Course of the Target

OS: Speed of the Observer

Table 2: Convergence time in seconds

Scenario	Convergence Time (sec)			
	Range	Speed	Course	Overall
1	260	298	348	348
2	347	348	422	422
3	432	503	665	665

The target can be tracked precisely after the overall convergence time is achieved. Till that time, the measurements are not accurate and not dependable for further use.

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From table 1 & 2, the convergence time is relatively high for scenario 3 compared to that of scenario 1 & 2. That is because of the higher speed of observer in scenario 3 compared to other scenarios. The passive tracking used here works based on the signals/noise produced in underwater. If the observer moves with high speed, the noise produced by ownship is more. So, it takes more time for tracking the target.

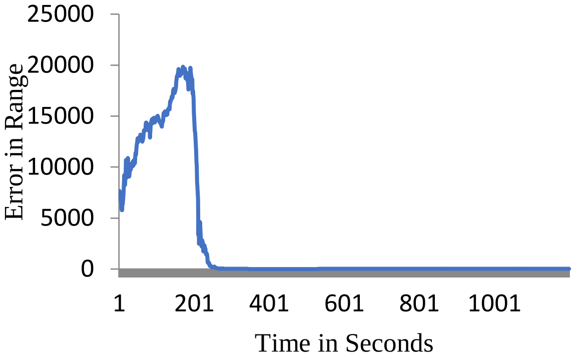

Fig 3: Range Estimate Error

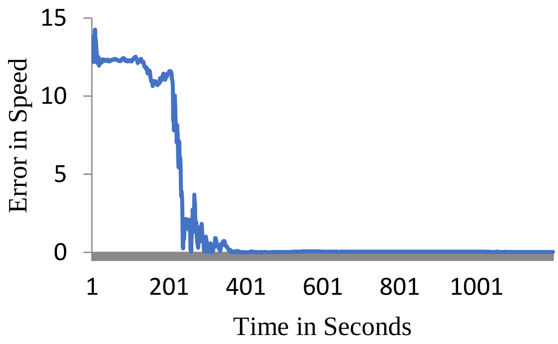

Fig 4: Speed Estimate Error

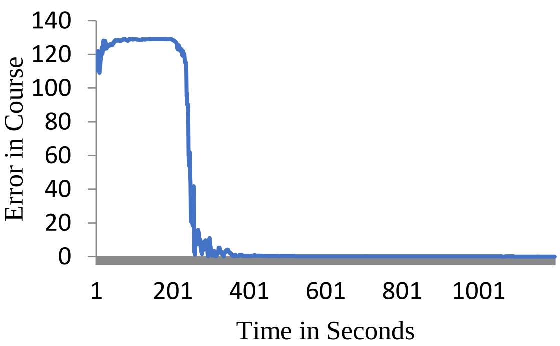

Fig 5: Course Estimate Error

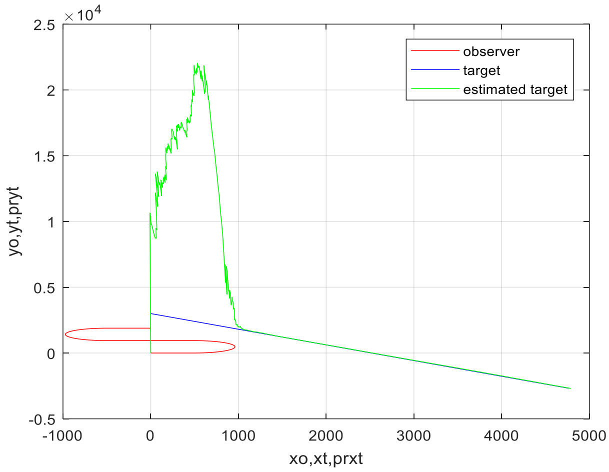

Fig 6: Observer and Target Movement

For the scenario 1, the error estimates of range, speed and course are shown in fig 3, 4 and 5. Fig 6 shows the trajectories of observer, target and the estimated target movement for scenario 1. The target is moving in straight line and the observer is 'S' maneuvering which is shown in fig 6.

IV. CONCLUSION

In practical scenarios, the non-linearities are generally high. This is the reason for considering the particle filter primarily because it is the most suitable one for nonlinear applications. For better performance of the filter, MGEKPF is used. Stratified Resampling technique is used. Different scenarios are considered to evaluate the performance of the algorithm. Solution is obtained faster with higher accuracy by using MGEKFPF. The simulated results show that the MGEKPF is relevant for bearings-only passive target tracking in underwater surroundings.

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