A Comparative Study on Maximum Matching Concept to Identify Crucial Components within A Large Complex Network

S.V.S. Santhi, P. Padmaja, Sri Harshitha Palla

Abstract: Previous studies mostly concentrated on the structural properties of the networks rather than behavioural properties. In real world, the networks are large, complex and sparse. To study the behavioural properties, we need to identify the crucial components of large complex networks. So it is necessary to identify driver nodes which provide control over the networks. Hence, it is important to analyze the controllability of these large and complex networks of the real world. For a complete study, we analyzed the maximum matching concept and suggest a comparatively more efficient approach. This paper puts-forth a delineate analysis of maximum matching concept using an example network.

Index Terms: Complex Networks, Controllability, Maximum Matching, Matched Nodes, Driver Nodes.

I. INTRODUCTION

A complex system is termed as a network with significant topological properties that will not occur in elementary networks like random graphs but frequently occur in real world systems. Complex systems are the systems whose formation is unpredictable, complex and dynamically developing in time. The main focus of these networks moved from small networks to large networks by considering the behavioural properties because the network topology plays an important role in complex networks to understand the behaviour of the system.

The study of complex networks is a young and active area of scientific research. It is in progress with the recent ideas and metrics to constitute the topography of the real world networks. This includes the underlying principles and properties of the real world network. It is an interdisciplinary area of research, which provides abundant scope for physicists, mathematicians, biologists, engineering, and several others.

Complex frameworks delineate systems of high technological and academic significance, like the internet, World Wide Web, social, biological and chemical, neural and communication systems.

The information portrayal in complex systems grants us in bringing together the structural complexity, vertex and link diversities.

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Governance of complex systems is a vital part of several applicants. It is termed as governable if it is propelled from an initial condition to a desired condition in an exceedingly finite duration. In the past, researches considered a complex system as the elementary model to examine the topological framework, developing model and its dynamic behaviour.

Controllability is a function of

i. Network structures.
ii. Dynamical interactions between its components and
iii. System parameters (Liu et.al, 2011).

The notation of structural controllability was presented by Lin [1974] and was newly adopted by Liu.et.al [2011] for directed complex systems. Depending on the maximum matching approach, structure for complex network is developed by identifying the minimum range of unmatched nodes that require attaining full controllability over the framework. The number of nodes which are not matched is considered as driver nodes and this count decides the controllability of the system.

Maximum matching issue is a core issue in the theory of matching (Bang Jensen.J. and Gutin. G. 2007) that might be useful to grasp the framework of a graph. Matching concept is tightly coupled with real time issues in work organization, resource allocation, and data transformation etc. These issues can be resolved using matching concept. Hence, the study on matching concept has significant hypothetical essentialness and a great range of application foundation.

Previous studies concentrated on undirected graphs or networks. Yet, a little analysis has been done on directed networks. Ongoing analysts began studies on directed networks by considering the strategy called maximum matching utilizing bipartite graphs.

II. RELATED WORK

Liu Y. et.al (2011), built up a methodical tool for investigation of controllability of a complex directed system, by mapping the structural controllability of a directed system into a matching issue. The unmatched nodes, expected to govern with the end goal to totally deal with over the whole network. Using these methodology creators recognized the arrangement of driver nodes with which will control the whole system dependent on time. The authors implemented this concept to many real-world systems for finding the driver node count. Hence it is necessary to implement the same concept rapidly and efficiently is a crucial issue to be resolved.

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Zhengzhong. Yuan. et.al. (2013), presented a correct controllability model stand to the maximum multiplicity to find the minimal quantity of driver nodes needed to achieve entire governance of systems consisting random framework and link-weights. The system recreates the extra governance of directed systems described using basic metrics. A productive and exact approach is suggested to evaluate the manageable of frameworks which are large, sparse and dense. Thus, this method empowers a complete comprehension of the effect of system behaviour on controllability, a key issue about the utmost power of complex frameworks.

Xizhe Zhang et.al. (2014), clarified the job of least driver nodes in structural controllability of complex systems depend on preferential matching algorithm. Current studies have shown that least set of driver nodes have a tendency to keep away from high-degree nodes. Nevertheless, this perception depends on the study of a count of driver nodes, in light of the fact that counting the majority of the MDSs of a system is a #P issue. In this way, previous examinations are not adequate to land at an influencing end. So, authors proposed a preferential algorithm to recognize least count of driver nodes having a specific degree property. The nodes obtained by this approach can be made out of high and medium degree nodes, closely associated to the edge direction of the framework.

Yunyun Yang and Gang Xie(2016), explored on matching concept in graph theory supported on largest geometric multiplicity. Authors emerged an effective technique for locating maximal matching in a di-graph. In a selected di-graph, count of maximal matched vertices having near association having larger geometric multiplicity for transpose of the adjacency-matrix. Besides, basic column transformations, we are able to get the matching nodes and associated matching links. Specifically, every node in the digraph is matched then it is known as perfect matching.

Daniel Leitold. et.al. (2017), showed on the system hypothesis dependent on controllability and observability. They understood that most applications are not related to dynamical frameworks, and essentially the topology of the frameworks is examined without more contemplation. Here, they draw concentration to the significance of elements within and between state variables by adding practical relationship characterized edges to the actual topology. Authors described the common association composes and feature how the topologies change the count of the necessary sensors and actuators in paradigm systems. Moreover, they offer a work process for dynamical framework examination and also introduced a technique for creating the least count of essential actuator and sensor focuses in the framework.

YunyunYang at.el. (2017), proposed a Two-Way-Page-Rank technique based on Page-Rank for finding essential nodes in directed weighted complex networks. In complex networks, mining essential nodes became a latest research area. Recent studies focused on undirected un-weighted complex frameworks. These methods are not suitable to directed weighted complex networks. Hence authors mainly focused on the recurrence of contact among the nodes and the period of time of contact between the vertices or nodes. Further, they considered the in-degree and out-degree of the vertices. In these approach vertices plays a major role. Authors explored the effect of variation of some parameters on node as critical indicators to check the performance. At long last the authors have confirmed the exactness and legitimacy of the strategy through observational system information.

The rest of the paper is organized as follows. In Section III, the Controllability of large complex networks is presented and explained using different methods by evaluating an example network. In Section IV, result analysis of the considered approaches is presented. In Section V, conclusion for the analytical study is mentioned. Finally, the scope of the future work mentioned in Section VI.

III. METHOD

A. Controllability of Large Complex Networks

Dynamical associations are caught through administering condition depicting the worldly advancement of state variables. A standout amongst the most demanding issue in engineering and science is controlling complex systems.

Generally, because of the inescapability of nonlinearity in nature, one must think about the governing of complex arranged frameworks with nonlinear elements. In any case, at present there is no broad edge work to address this issue as a result of the to a great degree convoluted interchange between system topology and nonlinear dynamical procedures, in spite of the advancement of nonlinear control in certain specific circumstances, for example accord, communication, traffic and device systems.

To build a frame work to regulate the complex systems with linear elements, there exists a well developed hypothetical frame work of governability for linear dynamical frameworks in the conventional area of planning (Ching-Tai Lin, 1974).

B. Controllability using Mapping of Matched Nodes

Most real-world frameworks are driven by nonlinear method, yet the governing of nonlinear frameworks is fundamentally same as linear frameworks in numerous aspects.

According to control theory a system is governable with proper selection of inputs; it can be operated from any primary state to any required final state within limited period (Kalman, 1963). This explanation accord with our intuitive notation of control shows a capacity to guide systems behaviour towards a desired condition via suitable control over some inputs.

As stated by control theory, the states of linear dynamics can be found by using differential equation as follows.

$$\frac{dx(t)}{dt} = \mathrm{A}x(t) + \mathrm{B}u(t)$$

In the event that we need to control a framework, initially we have to distinguish the arrangement of nodes, which might provide full authority over the system is known as driver nodes. It is to identify least number of driver nodes, indicated by N_D, is adequate to governing the system dynamics.

The customary method to handle the controllability issue is to get appropriate control matrix B of size N x M, comprising of least count of driver nodes in order to fulfil Kalman's rank status.

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As indicated by Kalman's condition, framework can be governed from any first position to any last position in limited time, if and just if

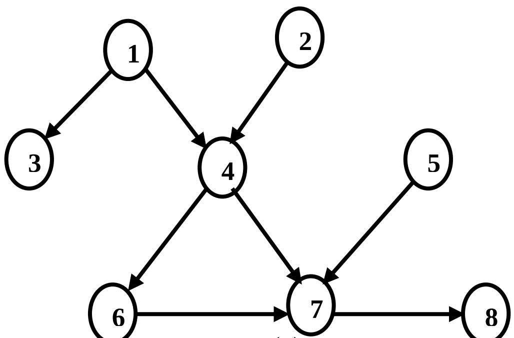
$$\text{rank}(C) = \text{rank}\,[B, AB, A^2B, \ldots\ldots A^{N-1}B] = N$$

Where

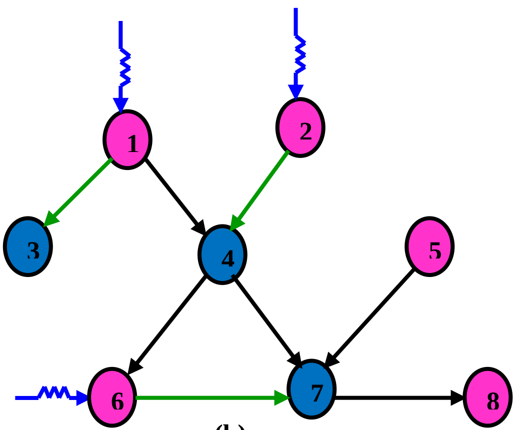
C= size of the matrix controllable = N x NM

According to Liu Y. et.al (2011) method, mapping of matched nodes by using maximum matching approach is analysed. In this methodology, the link pointing to the matched node is termed as matched edge. By considering the example network as in Figure. 1(a) and possible matching's are represented as in the figure 1. Matched nodes are represented in blue colour and unmatched nodes or driver nodes are represented in pink colour and the matched edge is represented with green colour.

For the example network shown in figure 1(b), the inputs are given to the nodes v1, v2 and v6. Implementing the Liu.et.al (2011) process, Matched nodes = {3, 4, 7}; Unmatched nodes or driver nodes = {1, 2, 5, 6, 8};

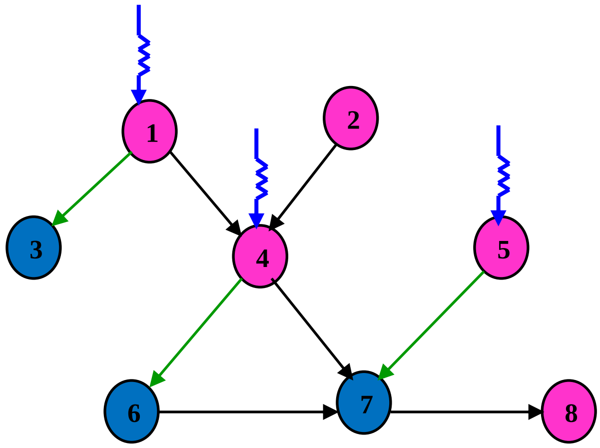


(a)

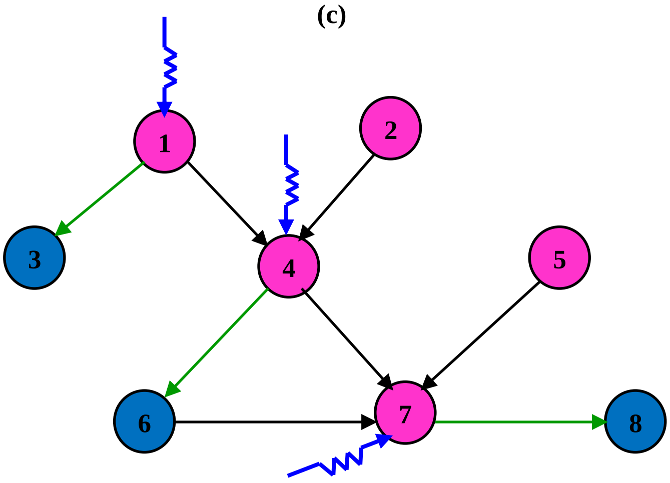


(b)

For the example network shown in figure 1(c), the inputs are given to the nodes v1, v4 and v5. Then the Matched nodes = {3, 6, 7}; Unmatched nodes = {1, 2, 4, 5, 8}; similarly for figure 1(d), the Matched nodes = {3, 6, 8}; and Unmatched nodes = {1, 2, 4, 5, 7};



(c)



(d)

Figure. 1. (a). Sample Network, (b). Network with matched nodes {3, 4, 7}, (c). Network with matched nodes {3, 6, 7}, (d). Network with matched nodes {3, 6, 8}.

Time Complexity:

The Time complexity of controllability of complex networks (Liu.et.al.2011) is

$$= O(N^{1/2}E)$$
$$= O(N^{1/2} * N^2)(E=V^2 \text{ for sparse graphs})$$
$$\mathbf{=O(N^{5/2})}$$
$$= \mathbf{O(N^{2.5})}$$

Where,

N = nodes of the network.

E = edges of the network.

C. Controllability Using Maximum Multiplicity

The perfect governable framework allows understanding of the effect of system properties on governability and extraordinary control of complex systems. This methodology acquainted a proficient precise tool to acquire the governability of large sparse and dense networks. It replicates the structural governability of directed networks specified by structural matrices.

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This strategy is utilized to discover least arrangement of driver nodes dependent on maximum multiplicity and to acquire full management of systems with absolute structures and edge-weights.

In exact controllability method, it is proved that minimal check of independent driver nodes or outside controllers is equivalent to the minimum geometric multiplicity of all eigenvalues of the network matrix when the system framework is diagonalizable.

The exact controllability can be proficiently estimated by numerical calculations. The least count of driver nodes can be located by basic transformations supported the precise governability of frame-work.

To implement the controllability by using exact method, a new strategy known as PBH rank condition is considered.

PBH-Rank condition

Let

$$X=Ax+Bu \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

A system i.e. equation (1) is said to be completely controllable when the equation (2) is satisfied.

$$\text{Rank }(CI_N - A \cdot B) = N \ldots\ldots\ldots\ldots (2)$$

Where,

C = a complex number.

I_N = the identity matrix of dimension N.

If the eigenvalue ‘λ’ belonging to matrix ‘A’ fulfils condition (3),the system is totally controllable. The fundamental objective is to locate an arrangement of 'B' relating to the minimum range of driver nodes, N_D necessary to govern the complete network.

N_D is defined in terms of B as the structural controllability condition is

$$N_D = \min\{\text{rank}(B)\}$$

$$N_D = \max_i\{\mu(\lambda_i)\} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\mu(\lambda_i) = \dim V\lambda_i = N - \text{rank}(\lambda_i I_N - A)$$

Where,

$\mu(\lambda_i)$ = maximum geometric multiplicity.

λ_i= distinct eigen values of A [i=1. . . . . l]

For undirected networks with random edge weights, N_D is calculated by utilizing maximum algebraic multiplicity of λ_i is

$$N_D = \max_i\{\delta(\lambda_i)\} \ldots\ldots\ldots\ldots (4)$$

Where,

$\delta(\lambda_i)$ is the eigenvalue degeneracy of matrix A.

Identification of driver nodes:

The least number of driver nodes, N_D can be calculated by maximal geometric multiplicity $\mu(\lambda^M)$ of the eigenvalue λ^M.

a. substituting λ^M for complex number C in equation (2) to ensure full control over the matrix B, as
rank $[\lambda^M I_N - A \cdot B] = N$ . . . . (8)
b. Basic column transformation operation on the matrix $[\lambda^M \cdot I_N - A]$ or $[A - \lambda^M I_N]$ results, linearly dependent rows that violate the full rank condition i.e. equation (8).
c. The nodes relative to the linearly dependent rows are the driver nodes with number N.
d. Maximum geometric multiplicity, $\mu(\lambda^M) = \text{rank }[\lambda^M I_N - A]$

Considering the example network as in figure 2,and finding the adjacency matrix A for the network. Also find the eigen values for the matrix A. To find the matched nodes of the network, calculate the column canonical form for $[A - \lambda^* I_N]$. Hence, from the column canonical form matrix, identify the rows which are independent to the network. Thus, the independent nodes are known as matched nodes and other or unmatched or driver nodes.

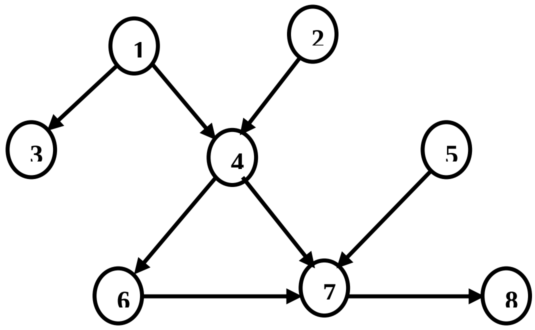

Figure. 2. Example network.

Adjacency Matrix, A is

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Eigen values

$$\begin{vmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{vmatrix}$$

Transpose of matrix A, A^T is

$$\begin{bmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 1&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&1&1&1&0&0\\ 0&0&0&0&0&0&1&0 \end{bmatrix}$$

Column canonical form of matrix $[A - \lambda * I_N]$ is

$$\begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0 \end{bmatrix}$$

In this example (figure 2), network is having 8 nodes. The rank of $[A - \lambda * I_N]$ is 5. Maximum geometric multiplicity $\mu(\lambda^M)$ is (N- rank of $[A - \lambda * I_N]$). Therefore, the maximum geometric multiplicity is (8 – 5) = 3 nothing but the count of driver nodes. Hence, the nodes which are unmatched or driver nodes based on exact controllability process is 3.

From column canonical form of matrix consider the nodes 3, 8 as matched nodes in combination with other any three nodes. The combinations are {3, 8, 4, 6, 7} and {1, 2, 3, 5, 8}. But, 1, 2, 5 has no in degree, hence not considered for matched nodes. Therefore, Driver nodes = {1,2,5};

Time Complexity:

The time complexity of exact controllability of complex networks (Zhengzhong. Yuan. et.al.2013), to determine matched nodes by using the fundamental column transformations, by adopting the elimination method is $(N^2 (\log N)^2)$.

Time complexity = $\mathbf{(N^2 (\log N)^2)}$.

D. Controllability using Preferential Matching

From the previous studies, it is clear that the matched nodes are not unique and higher degree nodes can be neglected while finding the driver nodes.

Preferential matching process **(**Xizhe Zhang et.al. 2014), enhances the importance of the low degree nodes. In this approach, there is a maximum possibility that the low degree nodes become as driver nodes.

For a directed graph G^d, V(G) is the group of vertices and E(G) is the group of edges with N=|V| and l=|E|. A group of edges in G^d is termed as matching 'M' if no two edges in 'M' share a node. And also a vertex 'v_i' is matched by M if there is an edge of 'M' pointing to v_i, otherwise v_i is unmatched.

Coordination with the more number of nodes is termed as maximum matching M* and the coordination with entire nodes of the graph 'G' are matched by M.

Maximum input theorem [Smith DA and White DR (1992)] demonstrates that, the count of driver nodes is one then it is termed as perfect matching otherwise not a correct matching and contains unmatched nodes, termed as driver nodes.

The scope of the maximum coordination M* is denoted as |M*| and the least count of driver nodes is represented as

$N_D = \max \{ N - |M^*|, 1 \}$

Where,

N_D = Count or number of driver nodes,

N = Count of nodes,

M^* = Count of matched nodes.

Algorithm :

Step 1: Perform sorting operation on nodes $\{v_0, v_i, ... , v_n\}$ by considering the degree.

Step 2: Represent the count of matchings as 'm'.

Step 3: Generate the sub-graphs $\{S_0, S_1, ...S_n\}$ with lowest-degree ranked as first.

Step 4: At every step i, the subgraph S, iteratively extended by joining the node with the i^{th} rank, and with maximum matching of S_{i-1}.

Step 5: Redo the process till the subgraph S_i is equal to the original network.

Considering an example network as in figure.3, and implement the preferential matching process for this example. Initially, calculate the in degree and out degree for this network and is represented in the table 3.3.1. Arrange the k_{out} in descending order and is represented in table 3.2. Represent the eight subgraphs $S_0, S_1, ... S_7$ for this network and eight bipartite graphs $B_1, B_2, ... B_8$ for those subgraphs as shown in figures 4 and 5. From the figure 4 the nodes which are represented in red colour are known as un- matched or driver nodes and the remaining nodes are known as matched nodes.

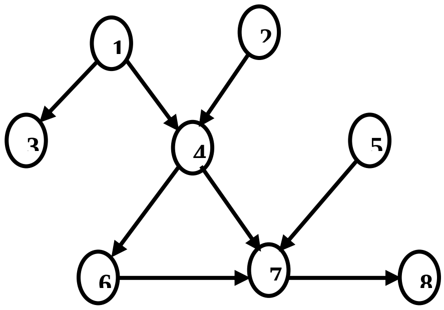

Figure. 3. Sample Network.

Table.1: Network In and out-degrees.

Node	k_{in}	k_{out}
1	0	2
2	0	1
3	1	0
4	2	2
5	0	1
6	1	1
7	3	1
8	1	0

Table.2: Degree of Nodes (k_{out}) in Descending Order.

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Node	k_{out}
1	2
4	2
2	1
6	1
7	1
5	1
8	0
3	0

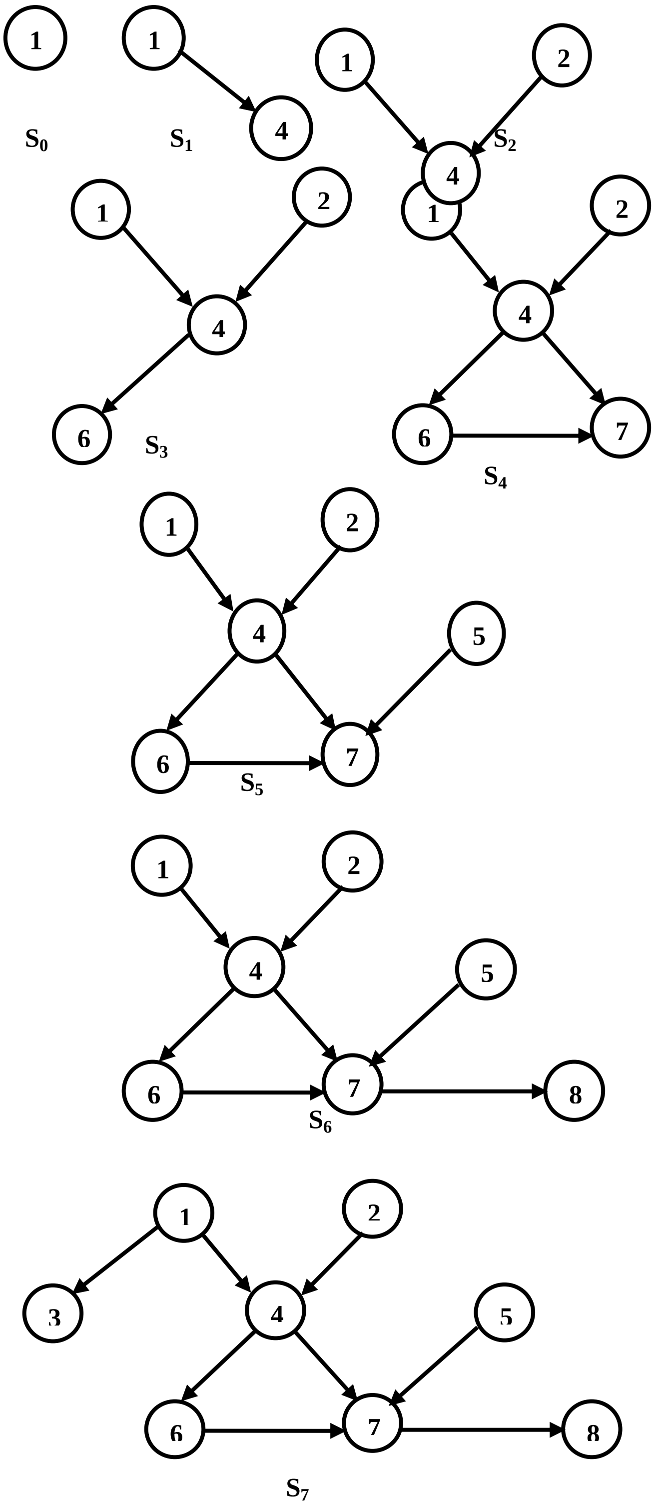

Figure. 4: Subgraphs for the Example Network.

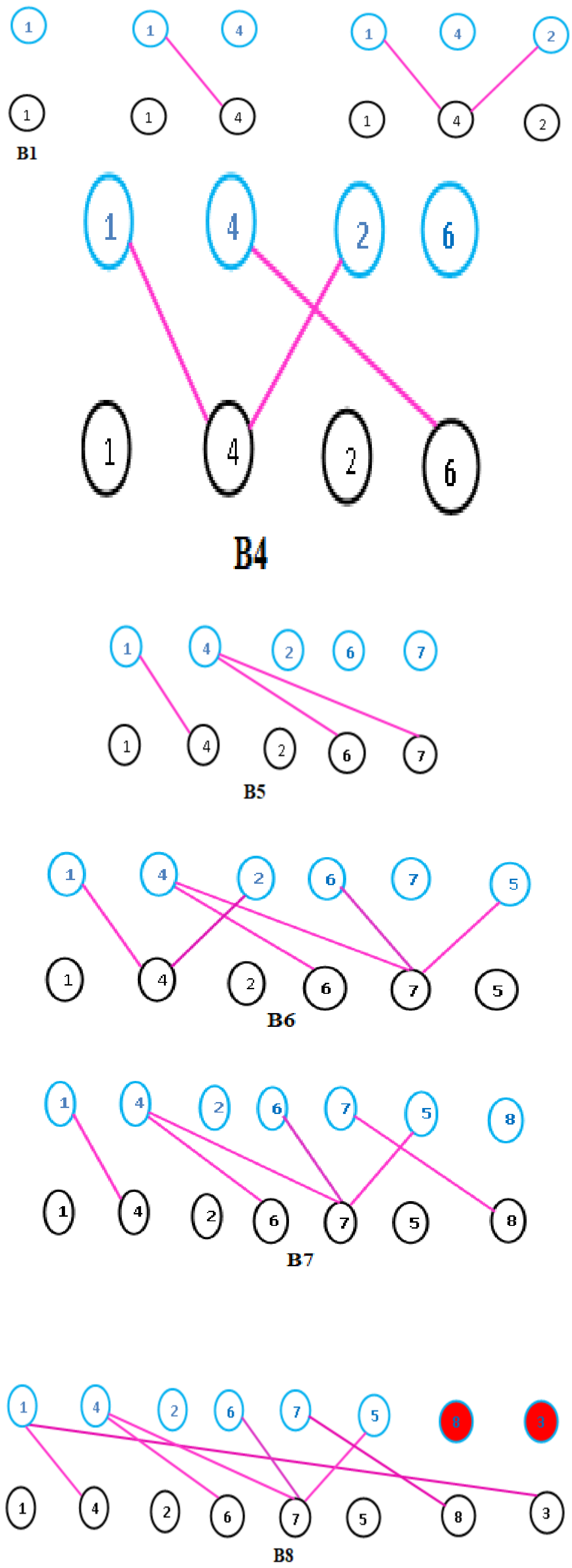

Figure. 5: Bipartite graphs for Example Network

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From the above process, the driver nodes = {8, 3} and are represented in red color

Time Complexity:

Time complexity of preferential matching algorithm is

$$= T_{node\ degree} + T_{node\ degree\text{-}sorting} + T_{Bipartite\ matching\text{-}graph}$$

$$= O(E + N) + O(N \log N) + O(N)$$

$$= O(N^2 + N) + O(N \log N) \text{ or } O(N^2) + O(N)$$

$$= \mathbf{O(N^2)}$$

E. Controllability based on In-Degree priority

Yang Yunyun and Xie Gang (2016), identified that there is still lack of efficiency in the matching theories even after considering the bipartite approach. Hence introduced the maximum matching based on In-Degree priority for finding driver nodes in a complex large network.

In this approach, a node having in degree 1 is known as matched node and in degree 0 as unmatched node. Initially, in this method we start with identifying the nodes, in-degree (K_{in}) and out-degree (K_{out}) of the nodes. Later the following operations are to be performed.

***Algorithm*:**

Step 1: Locate the set of nodes having in degree K_{in} is 1 and represent with v_i, and the value of 'i' is $0 \le i \le N$, as N represents number of nodes.

Step 2: Find the set of nodes pointing to v_i, and represent with v_j where $0 \le j \le N$.

Step 3: Find the vertex having maximum out degree and represent with v_a.

Step 4: Detach edges from v_a to other vertices to make the out-degree of v_a as 1.

Step 5: Redo the steps 1 to 4 for the remaining nodes which are not considered until the in-degree of every node come to 1 or 0.

Step 6: Find the nodes which acquire in-degree 1, 0 and represent the nodes within-degree 1 as matched nodes and nodes with in degree 0 as un matched nodes or driver nodes.

Now consider a small network to illustrate the process of above mentioned algorithm. It consists of eight nodes and eight edges and is represented in Figure.6.

Initially, determine the list of nodes v3, v6, v8 having in degree 1 and these nodes are represented with v_i. Next, determine the nodes pointing to v_i and represent as v_j. Therefore the nodes in the list v_j are v1, v4, v7. According to step3 of the algorithm, identify v_a, the maximum out degree. The two nodes v1 and v4 having out degree as 2, then select v_a randomly from v1,v4. Hence represent the node v1 as v_a and remove the edges from v_a to other nodes to make the out degree of v_a as 1. Repeat the above procedure until the in degree of the nodes becomes 1 or 0. In the next iteration of the process, node v4 becomes v_a and so on. By performing the entire process for the remaining nodes until the in degrees of all nodes becomes either 1 or 0. Finally, the nodes having in degree 1 are represented as matched nodes and those having in degree 0 are un matched or driver node for the considered network.

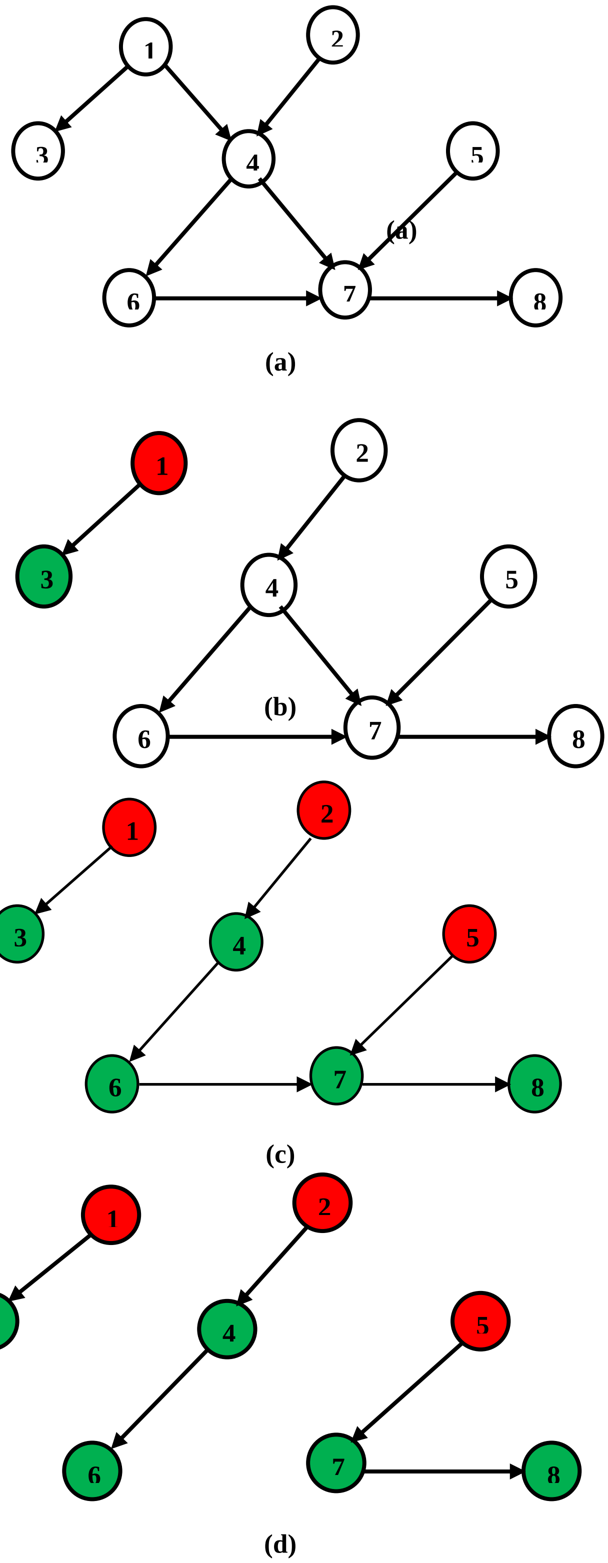

Figure 6: Illustration of mining maximum matchings of directed complex networks. (a) The network consists with eight nodes and eight edges. (b), (c), (d). The search process until the in-degrees of all nodes are 0 or 1. The matched (Green colour) and unmatched nodes (Red colour).

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Table.3: Details of In-degree(k_{in}) and Out-degree (k_{out}) for Nodes for network in figure.4 (a).

Node	k_{in}	k_{out}	
1	0	2	Va
2	0	1	
3	1	0	
4	2	2	
5	0	1	
6	1	1	
7	3	1	
8	1	0	

Table.4: Details of In-degree (k_{in}) and Out-degree (k_{out}) of figure. 4 (b).

Node	k_{in}	k_{out}	
1	0	1	
2	0	1	
3	1	0	
4	1	2	Va
5	0	1	
6	1	1	
7	2	1	
8	1	0	

Table.5: Details of In-degree (k_{in}) and Out-degree (k_{out}) of figure.4 (c).

Node	k_{in}	k_{out}	
1	0	1	
2	0	1	
3	1	0	
4	1	1	
5	0	1	
6	1	1	Va
7	1	1	
8	1	0	

Table.6: Details of In-degree (k_{in}) and Out-degree (k_{out}) of figure. 4 (d).

Node	k_{in}	k_{out}
1	**0**	1
2	**0**	1
3	1	0
4	1	1
5	**0**	1
6	1	0
7	1	1
8	1	0

For Unmatched nodes, k_{in} = 0 and the Driver nodes are {1, 2, 5}.

Time Complexity:

Initially we need to locate the node whose in-degree is 1 among 'N' nodes. Later, we need to search among (N-1) nodes and so on until number becomes 1. So, under a worst possible condition the whole time complexity of the algorithm is N^2.

Therefore, Time complexity of Mining maximum matching nodes using In-degree approach is **$O(N^2)$.**

IV. RESULT ANALYSIS

In this section, we present the results obtained by controllability methods by using controllability of mapping of matched nodes (Liu.et.al.2011), Exact controllability of complex networks (Zhengzhong.Y..et.al.2013), controllability of complex networks by using preferential matching (Xizhe.Z.et.al 2014), and controllability of complex networks by giving preference to in-degree Yunyun.Y and Gang.X 2016) for identifying the control nodes and comparative study details are represented in Figure 5 and Table 7.

In controllability of networks which are complex (Liu.et.al 2011), considered Kalman's controllability (1974), the driver nodes are not unique and changing with the change of input nodes they have considered. Also, the driver nodes obtained are more and the time complexity is $O(N^{2.5})$. In exact controlling of complex networks (Zhengzhong.Y.et.al. 2013), considered PBH rank condition for finding the driver nodes. In this method, the driver nodes are minimum but the process is having computational complexity due to usage of eigen values and geometric multiplicity calculation. The time complexity of this approach is $O(N^2 (\log N)^2)$. Accordingly, in preferential matching method (Xizhe.Z.et.al 2014), generates minimum count of driver nodes and the time complexity is $O(N^2)$. Even though, preferential process using bipartite graph results less driver nodes and best time complexity, the generated driver nodes are not matching with the previous results and the nodes which are identified as driver nodes are not having any out degree. In the process of matching approach by considering in-degree priority (Yunyun.Y and Gang.X 2016), generates driver nodes in less number and the time complexity is $O(N^2)$.

Table.7: Comparison of different methods for identifying driver nodes or control nodes

Method	Driver nodes	Time complexity
Liu.et.al	{1,2,5,6,8} or {1,2,4,5,8}or {1,2,4,5,7}	$O(N^{2.5})$
Zhengzhong.Y..et.al	{1,2,5}	$O(N^2(\log N)^2)$
Xizhe.Z.et.al	{3,8}	$O(N^2)$
Yunyun.Y and Gang.X	{1,2,5}	$O(N^2)$

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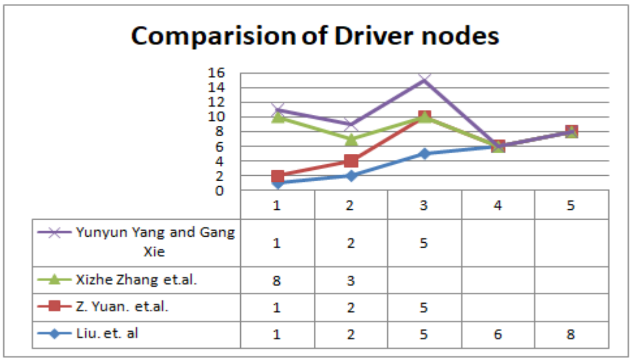

Figure. 5: Result analysis of all approaches.

V. CONCLUSION

In this paper, we had explored the significance of controllability of real-world networks which are large and complex by analyzing the maximum matching concept to identify the driver nodes. We had also analyzed different approaches of maximum matching by with an example network to find the driver nodes. Thus, we observed the time complexity for different methods and that the numbers of nodes are the same as driver nodes.

Now, from the above analysis performed by using an example network, we had come to the conclusion that the method which considers in-degree priority generates minimum count of driver nodes and gives better time complexity as $O(N^2)$.

From the above analysis, we suggest an improvised approach to reduce the time complexity for this method by considering in-degree priority for maximum matching.

In our previous studies of efficient and refined minimum spanning tree approach (S.V.S. Santhi and P. Padmaja, 2016), considered $\sqrt{N}$ partitions to perform operations on large graph or network. Hence, we consider the process for dividing the network into $\sqrt{N}$ partitions and thereby we can reduce the time complexity for finding maximum matching by considering in-degree priority approach to O(N). Thereby, the suggested method gives efficient results in comparison with other methods.

FUTURE SCOPE OF THE WORK

The suggested approach will be enhanced by the identification of crucial components in large real-world network using prioritization concept.

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