

On Lucky Edge Labeling of Splitting Graphs and Snake Graphs

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Abstract: Let G be a simple graph with vertex set $V(G)$ and Edge set $E(G)$ respectively. Vertex set $V(G)$ is labeled arbitrary by positive integers and let $E(e)$ denote the edge label such that it is the sum of labels of vertices incident with edge e . The Labeling is said to be lucky edge of labeling if the edge set $E(G)$ is a proper coloring of G , that is if we have $E(e_1) \neq E(e_2)$ whenever e_1 and e_2 are adjacent edges. The least integer k for which a graph G has a lucky edge labeling from the set $\{1, 2, \dots, k\}$ is the lucky number of G denoted by $\eta(G)$. In this paper Lucky edge labeling of Splitting graph, and Snake graphs has shown.

Index Terms: Lucky edge labeling, Lucky Number, Splitting Graph, Alternate Triangular Snake, Alternate Quadrilateral Snake, Double Alternate Triangular Snake.

I. INTRODUCTION

A graph G is a finite non-empty set of objects called vertices together with a set of pairs of distinct vertices of G which is called edges. Each edge $e = \{u, v\}$ of vertices in E is called an edge or a line of G . [2,3].

II. PRELIMINARIES

Let G be a simple graph with vertex set $V(G)$ and Edge set $E(G)$ respectively. Vertex set $V(G)$ is labeled arbitrary by positive integers and let $E(e)$ denote the edge label such that it is the sum of labels of vertices incident with edge e . The Labeling is said to be lucky edge of labeling if the edge set $E(G)$ is a proper coloring of G , that is if we have $E(e_1) \neq E(e_2)$ whenever e_1 and e_2 are adjacent edges. The least integer k for which a graph G has a lucky edge labeling from the set $\{1, 2, \dots, k\}$ is the lucky number of G denoted by $\eta(G)$.

A graph which admits lucky edge labeling is the lucky edge labeled graph. [1,2].

For a graph G the Split graph is obtained by adding to each vertex V a new vertex V' such that V' is adjacent to V in G . The resultant graph is denoted as $Spl(G)$ [5].

An alternate triangular snake is obtained from a path u_1, u_2, \dots by joining u_i and u_{i+1} to a new vertex. That is every alternate edge of path is replaced by [4]

An alternate quadrilateral snake $A(Q_n)$ is obtained from a path by u_1, u_2, \dots joining u_i, u_{i+1} to new

vertices v_i, v_{i+1} respectively and then joining and that is every alternate edge of path is replaced by [4] A double alternate triangular snake $DACT_n$ consists of two alternate triangular snakes that have a common path. That is, double alternate triangular snake is obtained from a path u_1, u_2, \dots, u_n by joining u_i and u_{i+1} to new vertices v_i and w_i [4].

III. RESULTS

Theorem 3.1

Splitting graph of Star graph admits a Lucky edge labeling.

Proof:

Let v_1, v_2, \dots be the vertices of star graph $K_{1,n}$ and v_1', v_2', \dots be the newly added vertices with K_1 to form G [5].

We define $f : v \rightarrow \{1, 2, 3, \dots, n\}$ by

i) $f(v) =$

ii) $f(v') =$

iii) $f(v_i) = i, 1 \leq i \leq n$

when $n = 2$, then

$f(v') = j + 2, \text{ for } i = 1, j = 1 \text{ and for } i = 2, j = 2$

Similarly, for $n = 3, 4, 5, \dots$ the vertex labeling has tabulated below

n	i	j
2	1,2	1,2
3	1,2,3	2,3,4
4	1,2,3,4	3,4,5,6
5	1,2,3,4	4,5,6,7,8

We define Edge labeling as $f^* : E(G) \rightarrow \{2, 3, 4, \dots, 2n + 1\}$ labeling of the Edges has tabulated below where n denotes the number of splitting

	$f^*(v_i, v_{i+1})$	$f^*(v_i, v)$	$f^*(v, v_{i+1})$
When $n = 2$	3,4	2,3	4,5
When $n = 3$	3,4,5	2,3,4	5,6,7

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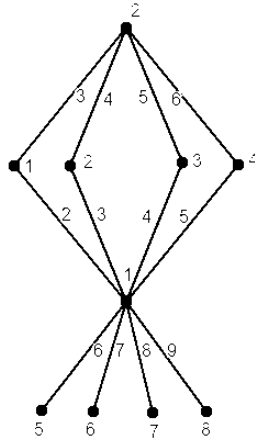
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When n = 4	3,4,5,6	2,3,4,5	6,7,8,9
When n = 5	3,4,5,6,7	2,3,4,5,6	7,8,9,10,11

Illustration:

Lucky Edge labeling of S (A_n is shown below



The Lucky edge number of splitting graph of star graph is $2n+1$, when $n = 2,3,4, \dots$ respectively.

Theorem 3.2

Alternate triangular snake A_n graph admits Lucky edge labeling.

Proof:

Let A_n be alternate triangular snake obtained from a path u_1, u_2, \dots by joining u_i and u_{i+1} alternatively to new vertex v_i . [4] Number of vertices is equal to the twice of n . A_n has $3i+1$ vertex, and $4i+3$ edges, where $i = 1,2,3, \dots$

Let $f: V(G) \rightarrow \{1,2\}$ and $f*: E(G) \rightarrow \{3,4,5\}$ defined by,

Case(i) When n is an odd

- i) $f(u_1) = 1$
- ii) $f(u_n) = 2$
- iii) $f(v_i) = 1$

iii) Rest of the vertices (u_1, u_n) pair can be labeled as pair of 1's and pair of 2's alternatively

Edge labeling must be given as follows

- iv) $f*(u_i, u_{i+1}) = 3, \text{ when } i = 1,3,5, \dots$
- v) $f*(u_i, u_{i+1}) = 2, \text{ when } i = 2,4,6, \dots (n - 1)$
- vi) $f*(u_{4i-3}, v_j) = 5 \text{ when } \begin{cases} i = 1,2,3, \\ j = 1,3,5, \end{cases}$

respectively

$$f*(u_{4i}, v_j) = 5 \text{ when } \begin{cases} i = 1,2,3, \\ j = 2,4,6, \end{cases}$$

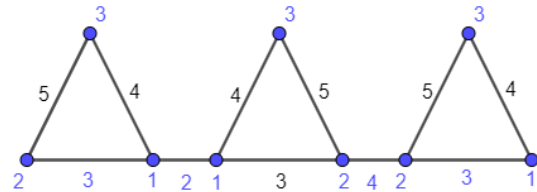
vii) respectively

viii) $f*(u_{4i-2}, v_j) = 4 \text{ when } \begin{cases} i = 1,2,3, \\ j = 1,3,5, \end{cases}$ respectively

ix) $f*(u_{4i-1}, v_j) = 4 \text{ when } \begin{cases} i = 1,2,3, \\ j = 2,4,6, \end{cases}$ respectively

Illustrations:

Lucky edge labeling of graph A_n is shown in the following figure



Lucky number of A_n is 5.

Case(ii) When n is an even

- i) $f(u_1) = 1$
- ii) $f(u_n) = 2$
- iii) $f(v_i) = 1$

iv) Rest of the vertices can be labeled as pair of 1's and pair of 2's alternatively.

Edge labeling must be given by

- iv) $f*(u_i, u_{i+1}) = 3, \text{ when } i = 1,3,5, \dots$
- v) $f*(u_i, u_{i+1}) = 2, \text{ when } i = 2,4,6, \dots (n - 1)$
- vi) $f*(u_{4i-3}, v_j) = 5 \text{ when } \begin{cases} i = 1,2,3, \\ j = 1,3,5, \end{cases}$

respectively

$$f*(u_{4i}, v_j) = 5 \text{ when } \begin{cases} i = 1,2,3, \\ j = 2,4,6, \end{cases}$$

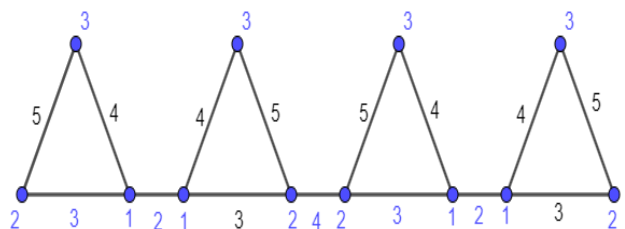
vii) respectively

viii) $f*(u_{4i-2}, v_j) = 4 \text{ when } \begin{cases} i = 1,2,3, \\ j = 1,3,5, \end{cases}$ respectively

ix) $f*(u_{4i-1}, v_j) = 4 \text{ when } \begin{cases} i = 1,2,3, \\ j = 2,4,6, \end{cases}$ respectively

Illustration

Lucky edge labeling of the graph A_n is shown in the following figure



Theorem 3.3

An alternate quadrilateral snake admits Lucky edge labeling.

Proof:

Let $A(Q)$ is obtained from a path u_1, u_2, \dots by joining u_i, u_{i+1} to new vertices v_i , respectively and then joining v_i, v_{i+1} and w_i, w_{i+1} . [4]

$A(Q)$ has $4i+4$ vertices and $5i+4$ edges, where $i = 1, 2, 3, \dots, n$

Let the function $f: v(G) \rightarrow \{1, 2\}$ and $f^*: E(G) \rightarrow \{3, 4, 5\}$

We consider the following two cases, for the given labeling

Case (i): When n is an odd

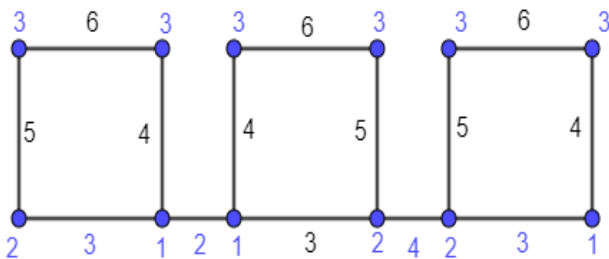
- i) $f(u_1) = 1$
- ii) $f(u_n) = 1$
- iii) $f(v_i) = 3, \text{ for } i = 1, 2, 3, \dots, n$
- iv) $f(w_i) = 3, \text{ for } i = 1, 2, 3, \dots, n$

v) Rest of the vertices in $u_1 - u_n$ path can be labeled as pair of 1's and pair of 2's alternatively

Edge labeling must be defined as,

- i) $f(u_i, u_{i+1}) = 3 \text{ for } i = 1, 3, 5, \dots$
- ii) $f(u_{4i-2}, u_{4i-1}) = 2 \text{ for } i = 1, 2, 3, \dots$
- iii) $f(u_{4i+1}, u_{4i}) = 4 \text{ for } i = 1, 2, 3, \dots$
- iv) $f(u_i, v_i) = 5 \text{ for } i = 1, 4, 5, 8, 9, 12, 13$
- v) $f(u_i, v_i) = 4 \text{ for } i = 2, 3, 6, 7, 10, 11$
- vi) $f(v_i, w_i) = 6 \text{ for } i = 2, 3, 4, \dots, n$

Illustration:



Lucky edge labeling of $A(Q)$ is 6.

Case (ii): When n is an even

- i) $f(u_1) = 1$
- ii) $f(u_n) = 1$
- iii) $f(v_i) = 3, \text{ for } i = 1, 2, 3, \dots, n$
- iv) $f(w_i) = 3, \text{ for } i = 1, 2, 3, \dots, n$

v) Rest of the vertices in $u_1 - u_n$ path can be labeled as pair of 1's and pair of 2's alternatively

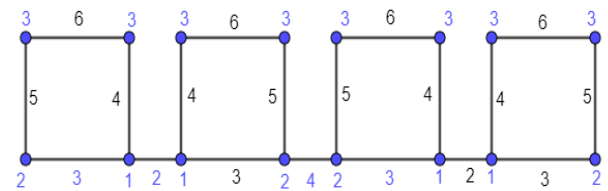
Edge labeling must be defined as,

- i) $f(u_i, u_{i+1}) = 3 \text{ for } i = 1, 3, 5, \dots$
- ii) $f(u_{4i-2}, u_{4i-1}) = 2 \text{ for } i = 1, 2, 3, \dots$

- iii) $f(u_{4i+1}, u_{4i}) = 4 \text{ for } i = 1, 2, 3, \dots$
- iv) $f(u_i, v_i) = 5 \text{ for } i = 1, 4, 5, 8, 9, 12, 13$
- v) $f(u_i, v_i) = 4 \text{ for } i = 2, 3, 6, 7, 10, 11$
- vi) $f(v_i, w_i) = 6 \text{ for } i = 2, 3, 4, \dots, n$

Illustration:

Lucky edge labeling of $A(Q)$ is shown below



THEOREM 3.4

A double alternate triangular snake $DA(n)$ which admits a Lucky edge labeling, where 'n' represents number of triangles.

Proof

Let $DA(n)$ is obtained from a path u_1, u_2, \dots by joining u_i, u_{i+1} and u_{i+1}, u_{i+2} to new vertices v_i and w_i [4]

$DA(n)$ has $4i+4$ vertices, and $6i+5$ edges, where $i = 1, 2, 3$

Let $f: v(G) \rightarrow \{1, 2, 3, 4\}$ and $f^*: E(G) \rightarrow \{2, 3, 4, 5, 6, 7\}$

We consider the following cases for labeling

Case(i) When $n = 4k + 2$, where $k = 1, 2, 3$

- i) $f(u_i) = 1$
- ii) $f(u_n) = 1$

iii) Rest of the vertices in $u_1 - u_n$ path can be labeled as pair of 1's and pair of 2's alternatively

- iv) $f(v_i) = 4, \text{ for } i = 1, 2, 3, \dots, n$
- v) $f(w_i) = 2, \text{ for } i = 1, 2, 3, \dots, n$

Edge labeling must be given

- i) $f(u_i, u_{i+1}) = 4, \text{ for } i = 1, 3, 5, \dots$
- ii) $f(u_i, u_{i+1}) = 2, \text{ for } i = 2, 4, \dots, (n - 1)$
- iii) $f(u_i, v_i) = 7, \text{ for } i = 1, 3, 5, \dots$
- iv) $f(u_n, v_n) = 5, \text{ Here 'n' be the last vertex}$

Number of vertices in $u_1 - u_n$ is equal to twice the vertices in $u_1 - u_n$ Rest of the edges of $u_i v_i$ can be labeled as pair of 5's and pair of 7's alternatively

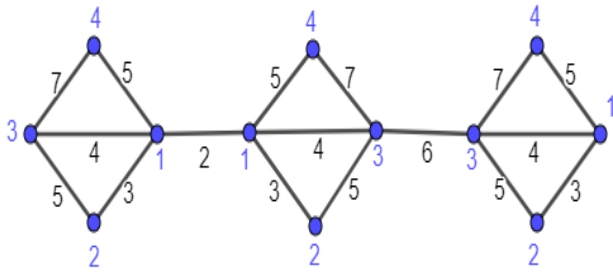
- v) $f(u_i, w_i) = 3, \text{ for } i = 1, 2, 3, \dots, n$
- vi) $f(u_n, w_n) = 3 \text{ Here 'n' be the last vertex}$

Rest of the edges of $u_i w_i$ can be labeled as pair of 3's and pair of 5's alternatively

Illustrations:

Double Alternate Triangular Snake $DA(n)$





Lucky number of the above graph is 7.

Case(ii): When $n = 4k, \text{ where } k = 1, 2, 3, \dots$

i) $f(u_1) =$

ii) $f(u_n) =$

iii) $f(v_i) = 4 \text{ for } i = 1, 2, \dots, k$

iv) $f(w_i) = 4 \text{ for } i = 1, 2, \dots, k$

v) Rest of the vertices in $u_1 - u_n$ path can be labeled as pair of 1's and 3's alternatively.

Edge labeling must be given

i) $f(u_i, u_{i+1}) = 4, \text{ for } i = 1, 3, 5, \dots$

ii) $f(u_i, u_{i+1}) = 2, \text{ for } i = 2, 4, \dots, (n - 1)$

iii) $f(u_i, v_i) = 7, \text{ for } i = 1 \text{ and } i = k$

Here 'n' be the last vertex

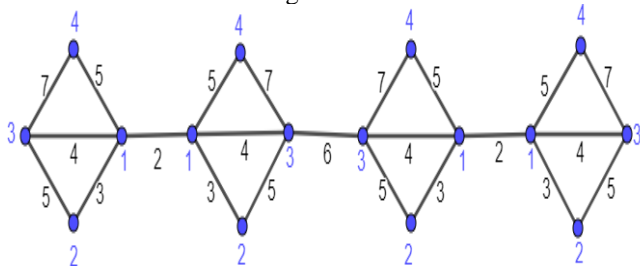
Rest of the edges of $u_i v_i$ can be labeled as pair of 5's and pair of 7's alternatively

iv) $f(u_i, w_i) = 5, \text{ for } i = 1 \text{ and } i = k$

Rest of the edges of $u_i w_i$ can be labeled as pair of 3's and pair of 5's alternatively.

Illustration:

Double alternate Triangular snake $DA(4)$



Lucky number of the above graph is 7.

IV. FINDINGS

Here we proved that the lucky edge number of splitting graph of star graph is $2n+1$, when $n = 2, 3, 4, \dots$ respectively.

Lucky edge number of alternate triangular snake is 5.

Lucky edge number of Alternate Quadrilateral Snake is 6.

Lucky edge number of Double alternate triangular snake is 7.

V. CONCLUSION

In this paper, it is proved that Lucky edge labelling of certain type of graphs. It is of interest to extend these polygons and regular graphs. And also find the existence of proper lucky edge labeling of corona operations on some graphs.

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