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On Lucky Edge Labeling of Splitting Graphs and Snake Graphs

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Abstract: ***Let Gbe a simple graph with vertex set V(G) and Edge set E(G) respectively. Vertex set V(G) is labeled arbitrary by positive integers and let E(e) denote the edge label such that it is the sum of labels of vertices incident with edge e. The Labeling is said to be lucky edge of labeling if the edge set E(G) is a proper colouring of G, that is if we have $E(e_1) \neq E(e_2)$ whenever e_1, and e_2 are adjacent edges. The least integer k for which a graph G has a lucky edge labeling from the set {1,2. . .k} is the lucky number of G denoted by η(G). In this paper Lucky edge labeling of Splitting graph, and Snake graphs has shown.***

Index Terms: ***Lucky edge labeling, Lucky Number, Splitting Graph, Alternate Triangular Snake, Alternate Quadrilateral Snake, Double Alternate Triangular Snake.***

I. INTRODUCTION

A graph G is a finite non-empty set of objects called vertices together with a set of parts of distinct vertices of G which is called edges. Each edge $e = \{u$ of vertices in E is called an edge or a line of G. [2,3].

II. PRELIMINARIES

Let G be a simple graph with vertex set V(G) and Edge set E(G) respectively. Vertex set V(G) is labeled arbitrary by positive integers and let E(e) denote the edge label such that it is the sum of labels of vertices incident with edge e. The Labeling is said to be lucky edge of labeling if the edge set E(G) is a proper coloring of G, that is if we have $E(e_1) \neq E(e_2)$ whenever e_1, and e_2 are adjacent edges. The least integer k for which a graph G has a lucky edge labeling from the set {1,2. . .k} is the lucky number of G denoted by η(G).

A graph which admits lucky edge labeling is the lucky edge labeled graph. [1,2].

For a graph G the Split graph is obtained by adding to each vertex V a new vertex V' such that V' is adjacent to V in G. The resultant graph is denoted as $SPl($[5].

An alternate triangular snake is obtained from a path $u_1, u_2, \dots$ by joining u_i and u to a new vertex. That is every alternate edge of path is replaced by [4]

An alternate quadrilateral snake $A(Q$ is obtained from a path by $u_1, u_2, \dots$ joining u_i, u to new vertices $v_i,$ respectively and then joining and that is every alternate edge of path is replaced by [4] A double alternate triangular snake $DA($ consists of two alternate triangular snakes that have a common path. That is, double alternate triangular snake is obtained from a path $u_1, u_2, \dots, u_n$ by joining u_i and u_{i+1} to new vertices and wi[4].

III. RESULTS

Theorem 3.1

Splitting graph of Star graph admits a Lucky edge labeling.

Proof:

Let $v_1, v_2, \dots$ be the vertices of star graph $K_{1,n}$ and $v_1', v_2', \dots$ be the newly added vertices with K to form G [5] .

We define $f: v \to \{1,2,3,\dots$ by

i) $f(v) =$

ii) $f(v') =$

iii) $f(v_i) = i, 1 \le i \le$

when n = 2, then $f(v') = j + 2$, for $i = 1, j = 1$ and for $i = 2, j =$

Similarly, for n = 3,4, 5, … the vertex labeling has tabulated below

n	i	j
2	1,2	1,2
3	1,2,3	2,3,4
4	1,2,3,4	3,4,5,6
5	1,2,3,4	4,5,6,7,8

We define Edge labeling as $f^*: E(G) \to \{2,3,4 \dots 2n +$ labeling of the Edges has tabulated below where n denotes the number of splitting

	$f^*(V_i, V')$	$f^*(V_i, V)$	$f^*(V, V_i')$
When n = 2	3,4	2,3	4,5

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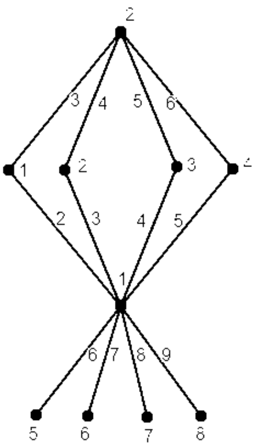
When n = 3	3,4,5	2,3,4	5,6,7
When n = 4	3,4,5,6	2,3,4,5	6,7,8,9
When n = 5	3,4,5,6,7	2,3,4,5,6	7,8,9,10,1 1

Illustration:

Lucky Edge labeling of S (^{l} is shown below



The Lucky edge number of splitting graph of star graph is 2n+1, when n = 2,3,4,… respectively.

Theorem 3.2

Alternate triangular snake $A($ graph admits Lucky edge labeling.

Proof:

Let $A($ be alternate triangular snake obtained from a path $u_1, u_2, \cdots$ by joining u_i *and* u alternatively to new vertex . [4] Number of vertices is equal to the twice of . $A($ has 3i+1 vertex, and 4i+3 edges, where $i = 1,2,3,\ldots$

Let $f: v(G) \to \{1,2$ and $f *: E(G) \to \{3,4$ defined by,

Case(i) When n is an odd

i) $f(u_1) =$

ii) $f(u_n) =$

iii) $f(v_i) =$

iii) Rest of the vertices (u_1, u_n) pa can be labeled as pair of 1's and pair of 2's alternatively

Edge labeling must be given as follows

iv) $f * (u_i, u_{i+1}) = 3, when\ i\ =\ 1,3\ ,5, ..$

v) $f * (u_i, u_{i+1}) = 2, when\ i\ =\ 2,4,6, \ldots (n -$

vi) $f * (u_{4i-3}, v_j) = 5\ when \begin{cases} i = 1,2,3, \\ j = 1,3,5, \end{cases}$ respectively

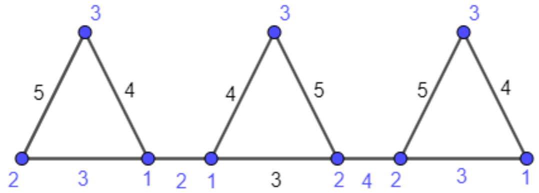
vii) $f * (u_{4i}, v_j) = 5\ when \begin{cases} i = 1,2,3, \\ j = 2,4,6, \end{cases}$ respectively

viii) $f * (u_{4i-2}, v_j) = 4 when \begin{cases} i = 1,2,3, \\ j = 1,3,5, \end{cases}$ respectively

ix) $f * (u_{4i-1}, v_j) = 4\ when \begin{cases} i = 1,2,3, \\ j = 2,4,6, \end{cases}$ respectively

Illustrations:

Lucky edge labeling of graph $A($ is shown in the following figure



Lucky number of $A($ is 5.

Case(ii) When n is an even

i) $f(u_1) =$

ii) $f(u_n) =$

iii) $f(v_i) =$

iv) Rest of the vertices can be labeled as pair of 1's and pair of 2's alternatively.

Edge labeling must be given by

iv) $f * (u_i, u_{i+1}) = 3, when\ i\ =\ 1,3\ ,5, ..$

v) $f * (u_i, u_{i+1}) = 2, when\ i\ =\ 2,4,6, \ldots (n -$

vi) $f * (u_{4i-3}, v_j) = 5\ when \begin{cases} i = 1,2,3, \\ j = 1,3,5, \end{cases}$ respectively

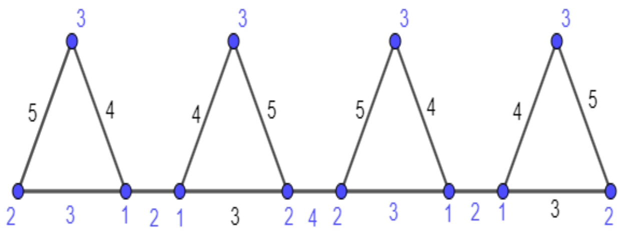
vii) $f * (u_{4i}, v_j) = 5\ when \begin{cases} i = 1,2,3, \\ j = 2,4,6, \end{cases}$ respectively

viii) $f * (u_{4i-2}, v_j) = 4 when \begin{cases} i = 1,2,3, \\ j = 1,3,5, \end{cases}$ respectively

ix) $f * (u_{4i-1}, v_j) = 4\ when \begin{cases} i = 1,2,3, \\ j = 2,4,6, \end{cases}$ respectively

Illustration

Lucky edge labeling of the graph $A($ is shown in the following figure



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Theorem 3.3

An alternate quadrilateral snake admits Lucky edge labeling.

Proof:

Let $A(Q$ is obtained from a path $u_1, u_2, \cdots$ by joining u_i, u to new vertices $v_i,$ respectively and then joining and . [4]

$A(Q$ has 4i+4 vertices and 5i+4 edges, where i = 1,2,3…n

Let the function $f: v(G) \rightarrow \{1,2$ and $f *: E(G) \rightarrow \{3,4,5$

We consider the following two cases, for the given labeling

Case (i): When n is an odd

i) $f(u_1) =$

ii) $f(u_n) =$

iii) $f(v_i) = 3, for\ a$

iv) $f(w_i) = 3, for\ a$

v)Rest of the vertices in can be labeled as pair of 1's and pair of 2's alternatively

Edge labeling must be defined as,

i) $f(u_i, u_{i+1}) = 3\ for\ i\ =\ 1,3,5, ..$

ii) $f(u_{4i-2}, u_{4i-1}) = 2\ for\ i\ =\ 1,2,3..$

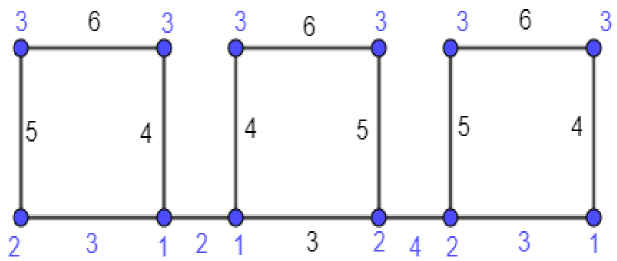
iii) $f(u_{4i+1}, u_{4i}) = 4\ for\ i\ =\ 1,2,3..$

iv) $f(u_i, v_i) = 5\ for\ i\ =\ 1,4,5,8,9,12,13$

v) $f(u_i, v_i) = 4\ for\ i\ =\ 2,3,6,7,10,11$

vi) $f(v_i, w_i) = 6\ for\ i\ =$,2,3, 4,…n

Illustration:



Lucky edge labeling of $A(Q$ is 6.

Case (ii): When n is an even

i) $f(u_1) =$

ii) $f(u_n) =$

iii) $f(v_i) = 3, for\ i\ =\ 1,2,3, ..$

iv) $f(w_i) = 3, for\ i\ =\ 1,2,3, ..$

v)Rest of the vertices in can be labeled as pair of 1's and pair of 2's alternatively

Edge labeling must be defined as,

i) $f(u_i, u_{i+1}) = 3\ for\ i\ =\ 1,3,5, ..$

ii) $f(u_{4i-2}, u_{4i-1}) = 2\ for\ i\ =\ 1,2,3..$

iii) $f(u_{4i+1}, u_{4i}) = 4\ for\ i\ =\ 1,2,3..$

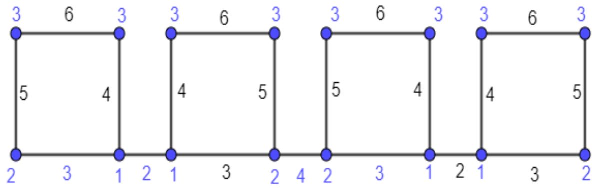
iv) $f(u_i, v_i) = 5\ for\ i\ =\ 1,4,5,8,9,12,13$

v) $f(u_i, v_i) = 4\ for\ i\ =\ 2,3,6,7,10,11$

vi) $f(v_i, w_i) = 6\ for\ i\ =$,2,3, 4, …n

Illustration:

Lucky edge labeling of $A(Q$ is shown below



THEOREM 3.4

A double alternate triangular snake $DA($ which admits a Lucky edge labeling, where 'n' represents number of triangles.

Proof

Let $DA($ is obtained from a path $u_1, u_2, \cdots$ by joining and u to new vertices $v_i and$[4]

$DA($ has 4i+4 vertices, and 6i+5 edges, where $i\ =\ 1,2,3$

$Let f: v(G) \rightarrow \{1,2,3,4\} and f^* E(G) \rightarrow \{2,3,4,5,6,7\}$

We consider the following cases for labeling

Case(i) When $n\ =\ 4k + 2, wherek\ =\ 1,2,3$

i) $f(u_i) =$

ii) $f(u_n) =$

iii) Rest of the vertices in $u_1 -$ path can be labeled as pair of 1's and pair of 2's alternatively

iv) $f(v_i) = 4, for$

v) $f(w_i)\ = 2, for$

Edge labeling must be given

i) $f(u_i, u_{i+1}) = 4, for i =\ 1,3,5, ..$

ii) $f(u_i, u_{i+1}) = 2, for i =\ 2,4, \dots (n -$

iii) $f(u_i, v_i) = 7, for i =\ 1,3,5, ..$

iv) $f(u_n, v_n) = 5, Here\ 'n'\ bethelastvert$

Number of vertices in is equal to twice the vertices in

Rest of the edges of uivi can be labeled as pair of 5's and pair of 7's alternatively

v) $f(u_i, w_i) =$

vi) $f(u_n, W_n) = 3\ Here\ 'n'\ bethelastvert$

Rest of the edges of uiwican be labeled as pair of 3's and pair of 5's alternatively

Illustrations:

Double Alternate Triangular Snake $DA($.

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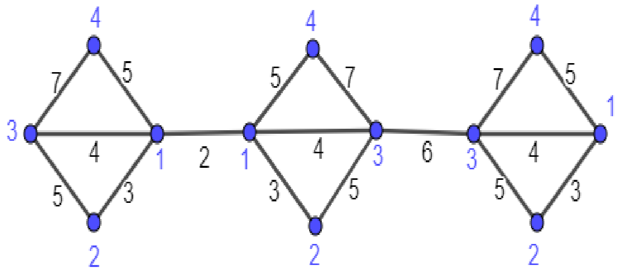


Lucky number of the above graph is 7.

Case(ii): When $n = 4k, where k = 1,2,3,..$

i) $f(u_1) =$

ii) $f(u_n) =$

iii) $f(v_i) = 4$ for a

iv) $f(w_i) = 4$ for a

v)Rest of the vertices in $u_1 -$ path can be labeled as pair of 1's and 3's alternatively.

Edge labeling must be given

i) $f(u_i, u_{i+1}) = 4, for\, i = 1,3,5,..$

ii) $f(u_i, u_{i+1}) = 2, for\, i = 2,4,...(n -$

iii) $f(u_i, v_i) = 7, for\, i = 1\ and\, i =$

Here 'n' be the last vertex

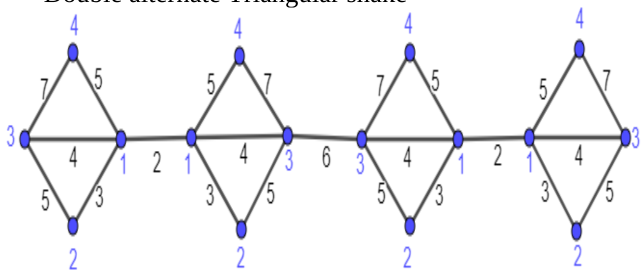
Rest of the edges of uivican be labeled as pair of 5's and pair of 7's alternatively

iv) $f(u_i, w_i) = 5, for\, i = 1\ and\, i =$

Rest of the edges of uiwican be labeled as pair of 3's and pair of 5's alternatively.

Illustration:

Double alternate Triangular snake $DA($



Lucky number of the above graph is 7.

IV. FINDINGS

Here we proved that the lucky edge number of splitting graph of star graph is 2n+1, when n = 2,3, 4,… respectively.

Lucky edge number of alternate triangular snake is 5.

Lucky edge number of Alternate Quadrilateral Snake is 6.

Lucky edge number of Double alternate triangular snake is 7.

V. CONCLUSION

In this paper, it is proved that Lucky edge labelling of certain type of graphs. It is of interest to extend these polygons and regular graphs.And also find the existence of proper lucky edge labeling of corona operations on some graphs.

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