Design of Fractional Order Fuzzy Logic Controller for BLDC motor

G. Venu, S. Tara Kalyani

Abstract: In this paper a Fractional Order Fuzzy Logic Controller (FOFLC) for control of BLDC motor. The proposed controller using fractional order integer calculus. The gains of Fractional Order P.I tuned with FLC is known as FOFLC. The tuning of proportional, integral gains of the controller with Fuzzy Logic Controller using fuzzy linguistic rules. This controller proposed to BLDC motor dynamic model using matlab-simulink under various operating modes such as constant speed mode and constant torque mode as a motor and generator. The performance of the BLDC motor with FOFLC compared to FOP.I and conventional P.I controller by observing torque, speed characteristics in terms of dynamic parameters such as rise time, peak time and overshoot.

Index Terms: BLDC, FOFLC, FUZZY LOGIC, FOP.I, P.I

I. INTRODUCTION

Conventional DC (CDC) motors, magnetic flux produced by exciting field winding of the motor with direct current. The construction of BLDC motor similar to ordinary synchronous motor. The rotor of BLDC motor made up of permanent magnetic. The shape of the back EMF waveform is trapezoidal in BLDC motor. The analysis of BLDC motor is difficult due to the complexity of dynamic model. BLDC motors used in several applications such as auto mobile, aero space. presented a neuro fuzzy controller for BLDC motor, a sliding mode observer based controllers proposed in [1,2]. A fuzzy reconfigurable control methodology presented[3]. The superiority of sliding mode fuzzy controller tuning of P.I.D controller demonstrated over normal P.I.D controller. An adaptive fuzzy P.I/PD controller proposed to obtain stable dynamic performance of BLDC. [4] a linear quadratic observer proposed to estimate the measurable st...
Where R is designated as the stator resistance on each phase

\[ R = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \]

L is designated as the matrix of inductance for both self and mutual inductance

\[ L = \begin{bmatrix} L_s & -M \\ -M & L_m \\ -M & 0 \end{bmatrix} \]

\[ e = [e_a \ e_b \ e_c]^T \]

is the vector of back EMF which is trapezoidal.

\[ \frac{d^2 e_a}{dt^2} = L_s \left( \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right) - R \left( \begin{array}{c} e_a \\ e_b \\ e_c \end{array} \right) \]

\[ \frac{d^2 e_b}{dt^2} = L_s \left( \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right) - R \left( \begin{array}{c} e_a \\ e_b \\ e_c \end{array} \right) \]

\[ \frac{d^2 e_c}{dt^2} = L_s \left( \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right) - R \left( \begin{array}{c} e_a \\ e_b \\ e_c \end{array} \right) \]

(7)

\[ L_s = L_{s,M} \]

The equation of motion can be denoted as

\[ \frac{d\omega}{dt} = \frac{1}{J}(T_{em} - T_L - f(\omega_r)) \]

(8)

\[ \omega_r \text{- mechanical angular speed [rad/s], } T_L \text{- load torque in [N m], } J \text{- load inertias of motor shaft [kg m}\^2\text{], } F \text{- coefficient of damping friction in [N m s/rad m], } T_{em} \text{- electromagnetic torque in [N}. \]

Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Speed (rpm)</th>
<th>Power (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>10</td>
<td>8.6</td>
</tr>
<tr>
<td>y</td>
<td>15</td>
<td>12.4</td>
</tr>
<tr>
<td>z</td>
<td>20</td>
<td>15.3</td>
</tr>
</tbody>
</table>

Figure 1. Block diagram model of BLDC motor speed control using FOP.I controller.

**Design Methodology for Fractional Order Fuzzy Logic (Foplc) Controller for Blc Motor**

Fractional Order P.I controller has been designed and presented with tuning of gains \( K_p, K_i \) with fuzzy controller have been proposed to obtain good performance characteristics of BLDC motor by adjusting \( K_p, K_i \) with fuzzy linguistic rules.

The fractional order calculus derived from ordinary calculus. According to Riemann-Liouville (R-L) the fractional derivative is given by[18]

\[ D^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t (t-s)^{\alpha-1}f(s)ds \]

(10)

Where \( n-1 < \alpha < n \), \( n \) being an integer value and \( \Gamma \) is designated as Euler’s gamma function. According to R-L the other definition [18] is

\[ D^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_0^t (t-s)^{n-\alpha-1}f(s)ds \]

(11)

Where \( D^\alpha \) is fractional operator. According to Riemann-Liouville definition[10] Laplace transformation of fractional derivative is

\[ L_a[D^\alpha f(t)] = \frac{1}{s^{\alpha+1}} - \frac{1}{s^{\alpha}} f(0) \]

(12)

For \( n-1 < \alpha < n \) where \( L \) f(t) indicates the normal Laplace transformation.

The representation of fractional order transfer function involves infinite number of poles and zeros. Refined Oustaloup filter[18] used in the design of FOP.I controller.

The fractional order derivative represented in terms of recursive distribution of transfer function in pole-zero notation.

\[ s^\alpha = K\prod_{n=1}^{\infty} \left( 1 + \frac{1}{\omega_n^{2\alpha}} + \frac{1}{\omega_n^{2\alpha}} \right) \]

(13)

‘K’ is an adjusted gain and frequency of poles and zeros are given by the equation.

\[ \omega_n = \omega_n^\alpha \]

(14)

\[ \omega_{n+1} / \omega_n = \frac{1}{n} \]

(15)

\[ \omega_{n+1} = \omega_n^\alpha \]

(16)

\[ \omega = \omega_n / \omega_n^\alpha \]

(17)

The commonly used form of FOP.I is the \( p\alpha \), where \( \alpha \) is a real integer. The transfer function model of FOP.I is given by

\[ G(s) = Kp + \frac{Kp}{s^\alpha} \]

(19)

The differential equation of fractional order P.I controller is described in equation (20)

\[ u(t) = Kp \int e(t) + Ki \int D^\alpha e(t) \]

(20)

**a. Design of Oustaloup Filter**

The fractional-order operator \( s^\alpha \) can be approximated by the fractional-order transfer function as

\[ K(s) = \left( \frac{1}{\omega_n^{2\alpha}} \right) \frac{1}{s^\alpha} \]

(21)

Where \( 0 < \alpha < 1 \), \( s = j\omega, b > 0, d > 0 \), and

\[ K(s) = \frac{1}{\omega_n^{2\alpha}} (1 + \frac{d^2 + d^\alpha}{s^\alpha}) \]

(22)

In the frequency range \( \omega_b < \omega < \omega_h \) by using a Taylor series expansion, obtain

\[ K(s) = \frac{1}{\omega_n^{2\alpha}} \left( 1 + \frac{\lambda(s)}{s^\alpha} \right) + \ldots \]

(23)

\[ p(s) = \frac{1}{\omega_n^{2\alpha}} (1 + \frac{d^2 + d^\alpha}{s^\alpha}) \]

(24)

It is then found that

\[ s^\alpha \]

approximately written as.
Thus, the fractional-order differentiator is defined as
\[ s^\lambda \approx \left( \frac{d\omega_r^2}{d^2 + \omega_d^2} \right) \left( \frac{1}{s^2 + \omega_d^2} \right)^\lambda \] (25)

Expression (26) is stable if and only if all the poles are on the left-hand side of the complex s-plane. It is easy to check that expression (26) has three poles:
• One of the poles is located at \(-\frac{b}{a}\) which is a negative real pole since \(a > 0, b > 0, d > 0\);
• The two other poles are the roots of the equation
\[ d(1 - \lambda)s^2 + a\omega_h s + d\lambda = 0 \] (27)
Whose real parts are negative since \(0 < \lambda < 1\). Thus, the above transfer function model is stable with in band of frequencies \((\omega_p, \omega_h)\). The fractional-order part of expression approximated as
\[ s^\lambda \approx \left( \frac{d\omega_r^2}{d^2 + \omega_d^2} \right) \left( \frac{1}{s^2 + \omega_d^2} \right)^\lambda \] (26)

Thus, the continuo us rational transfer function model can be obtained as
\[ s^\lambda \approx \left( \frac{d\omega_r^2}{d^2 + \omega_d^2} \right) \left( \frac{1}{s^2 + \omega_d^2} \right)^\lambda \] realized in terms of transfer function model of Oustaloup Filter.

The proportional gain(KP) is adjusted by the linguistic rules [24] with order of rule table 5X5. The integral gains can be adjusted with another rule table given below with same order.

Table-I: Rule table for adjustment of proportional gains (KP).

<table>
<thead>
<tr>
<th>Δe</th>
<th>e</th>
<th>NB</th>
<th>NM</th>
<th>ZO</th>
<th>PM</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NM</td>
<td>ZO</td>
<td>ZO</td>
</tr>
<tr>
<td>NM</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NM</td>
<td>ZO</td>
<td>ZO</td>
</tr>
<tr>
<td>ZO</td>
<td>NM</td>
<td>NM</td>
<td>ZO</td>
<td>PM</td>
<td>PM</td>
<td>PM</td>
</tr>
<tr>
<td>PM</td>
<td>ZO</td>
<td>ZO</td>
<td>NM</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
</tr>
<tr>
<td>PB</td>
<td>ZO</td>
<td>ZO</td>
<td>NM</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
</tr>
</tbody>
</table>

Table-II: Rule table for adjustment of proportional gains (KI).

The linguistic labels used to describe the Fuzzy sets were ‘Negative Big’ (NB), ‘Negative Medium’ (NM), ‘Negative Small’ (NS), ‘Zero’ (ZO), ‘Positive Small’ (PS), ‘Positive Medium’ (PM), ‘Positive Big’ (PB). It is possible to assign the set of decision rules as shown in Table I&II. The fuzzy rules are extracted from fundamental knowledge and human experience about the process. These rules contain the input/output relationships that define the control strategy. Each control input has five fuzzy sets so that there are atmost 25 fuzzy rule.

III. RESULTS AND ANALYSIS

In this proposed work simulations are performed under various operating modes such as constant speed mode, constant torque mode as motor, generator and reverse motoring mode.

3.1. Constant Speed Mode:

In this condition motor is kept at constant speed and speed increased to 1000 RPM and applying different loads.

Case 1: [0 sec to 0.5 sec]

Motor is initially at no load and zero speed and speed command is raised to 1000 RPM by accelerating rotor speed. Hence controller increases the reference torque to maximum to accelerate the motor. Once the speed is approaching to speed command, controller decreased the torque and stabilized rotor speed.
Design of Fractional Order Fuzzy Logic Controller for BLDC motor

Table 3: Comparison of dynamic parameters at the time of starting with P.I, FOP.I and FOFLC.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>P.I Control</th>
<th>FOP.I</th>
<th>FOFLC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>0.0444 sec</td>
<td>0.0284 sec</td>
<td>0.0284 sec</td>
</tr>
<tr>
<td>Settling Time</td>
<td>0.228 sec</td>
<td>0.2078 sec</td>
<td>0.1808 sec</td>
</tr>
<tr>
<td>Peak Overshoot</td>
<td>14.664 %</td>
<td>11.99 %</td>
<td>9.51 %</td>
</tr>
</tbody>
</table>

Case 2: [0.5 sec to 1.0 sec]
Motor is at a speed of 1000 rpm and a full load applied to the BLDC motor.

Figure 4: Torque, speed responses of BLDC motor Motoring mode.

Table 4: Comparison of dynamic parameters with P.I, FOP.I and FOFLC at the time of running as a generator.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>P.I Control</th>
<th>FOP.I</th>
<th>FOFLC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>0.0444 sec</td>
<td>0.0284 sec</td>
<td>0.0284 sec</td>
</tr>
<tr>
<td>Settling Time</td>
<td>0.1981 sec</td>
<td>0.4092 sec</td>
<td>0.2091 sec</td>
</tr>
<tr>
<td>Peak Undershoot</td>
<td>57.96 %</td>
<td>14.65 %</td>
<td>9.5131 %</td>
</tr>
</tbody>
</table>

Table 5: Comparison of dynamic parameters with speed command drop from 1000 RPM to 500 RPM in constant torque mode with P.I, FOP.I, FOFLC.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>P.I Control</th>
<th>FOP.I</th>
<th>FOFLC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>0.0166 sec</td>
<td>0.0142 sec</td>
<td>0.0142 sec</td>
</tr>
<tr>
<td>Settling Time</td>
<td>0.2214 sec</td>
<td>0.0900 sec</td>
<td>0.0408 sec</td>
</tr>
<tr>
<td>Peak Undershoot</td>
<td>18.20 %</td>
<td>3.92 %</td>
<td>2.65 %</td>
</tr>
</tbody>
</table>

Table 6: Comparison of parameters with speed command drop from 1000 RPM to 500 RPM in constant torque mode with P.I, FOP.I, FOFLC.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>P.I Control</th>
<th>FOP.I</th>
<th>FOFLC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>0.0448 sec</td>
<td>0.0142 sec</td>
<td>0.0142 sec</td>
</tr>
<tr>
<td>Settling Time</td>
<td>0.2056 sec</td>
<td>0.0763 sec</td>
<td>0.0552 sec</td>
</tr>
<tr>
<td>Peak Undershoot</td>
<td>18.20 %</td>
<td>3.92 %</td>
<td>2.65 %</td>
</tr>
</tbody>
</table>

Case 3: [1.0 sec to 1.5 sec]
Motor is at a speed of 1000 rpm and changing from motoring mode to generating mode by applying -6 Nm.

Figure 5: Torque, speed responses of BLDC motor with P.I, FOP.I, FOFLC at the time of running as a generator.

b. Constant Torque Mode: In this condition motor torque kept constant and speed command has given at 1000 RPM. In this work two cases are considered to test the performance and effectiveness characteristics of BLDC motor with P.I, FOP.I and FOFLC. The dynamic model of BLDC motor simulated using MATLAB-Simulink under possible two cases given below. In case-1, two conditions are studied (i.e speed command drop from 1000 RPM to 500 RPM, Speed command again raised to 500 RPM to 1000 RPM).

Case – 1: [0.5 sec – 1.2 sec]: Comparison of parameters with speed command drop from 1000 RPM to 500 RPM.

Figure 6: Torque value and speed characteristics of a BLDC motor with P.I, FOP.I, FOFLC at the time of starting.

Figure 7: Torque value and speed characteristics of a BLDC motor with P.I, FOP.I, FOFLC at the time of running as a generator.
The torque, speed responses are faster and smoother compared to the fractional P.I Controller and conventional P.I controller. The performance of the controller tested under different operating conditions and sudden load disturbance. The BLDC motor control under various operating conditions.

Figure 6: Torque-speed increase characteristics of a BLDC motor with P.I, FOP.I, FOFLC in constant torque mode in reverse motoring mode.

Table 7: Comparison of parameters in constant torque mode with P.I, FOP.I, FOFLC when Speed command changes from 1000 RPM to -600 RPM (Reverse Motoring).

IV. CONCLUSION

In this paper, a Fractional intelligent controller popularly known as Fractional Order Fuzzy P.I controller proposed for BLDC motor control under various operating conditions. The performance of the controller tested under different operating conditions and sudden load disturbance. The performance of fractional order fuzzy controller compared to fractional P.I Controller and conventional P.I controller. The torque, speed responses are faster and smoother compared to FOP.I and P.I controllers in terms of overshoot and settling time.

ACKNOWLEDGEMENTS

APPENDIX

The BLDC motor drive parameters are as follows:
- Resistance of Armature = Ra = 0.5 Ω
- Inductance of Armature = L = 8 mH
- Back constant of e.m.f = Ke = 0.55 V/rad/s
- Mechanical Inertial value = J = 0.0465 kg.m

REFERENCES