Time Truncated Two Sided Modified Chain Sampling Plans for Exponential Distribution

Sharifah NajlaaHanini Syed Abdullah, Nazrina Aziz, Mohd AzriPawan Teh

Abstract: In this paper, Two Sided Modified Chain Sampling Plans (TSMChSP) for Exponential distribution is presented. The decision of acceptance lot can be made by ensuring no defects in both preceding and succeeding samples. The design parameters such as the minimum sample size and operating characteristic values are calculated to ensure the consumer's risk at a specified quality level. The main purpose of this article is to produce the TSMChSP for Exponential distributions. An example is provided for illustrative purpose.Then, the article moving on further to compare the performances of TSMChSP and TSChSP, based on two criteria, which are the number of minimum sample size, n and the probability of lot acceptance, $L(p)$. The article concluded that, the TSMChSP has a better performance compared to the TSChSP in both criteria.

Keywords: Two sided Modified Chain Sampling Plan (TSMChSP), Consumer's risk, Operating characteristic values, Exponential distribution, Minimum size

I. INTRODUCTION

Acceptance sampling is a quality control method used to accept or reject a lot after testing a random sample of a product. Many re-searchers have given distinctive definitions for acceptance sampling but still all definitions have a similar essential thought of what acceptance sampling is. Based on all the definitions that have been stated before, it is clearly prove that acceptance sampling is an extremely valuable technique when a lot is so large, but it is not applicable to inspect each item in the lot because the cost will be higher and it will take more time. Besides, there is a possibility to neglect some defective products and may pass the inspection point. The acceptance sampling plans can be classify into various distinctive ways. However, there are two major type of sampling plan namely attribute sampling plans and variable sampling plans. The attribute sampling plan are measured based on "go, no-go" basis meanwhile the variable sampling plans are measured based on a numerical scale such as height and weight. It is available with smaller sample size and gives more information about the lot than the attribute sampling plan.

For attribute sampling plans there are few type of sampling plan such as single acceptance sampling plan (SSP), chain acceptance sampling plan (CHSP-1), two-sided complete chain sampling plan CChSP(0,1), two sided-modified complete chain sampling plan MCChSP(c_1, c_2, i, j), two sided chain sampling plan (TSChSP) and two sided modified chain sampling plan (TSMChSP). For this article, we will focus on TSMChSP as we found there is so much opportunity to get better of the arrangement by concentrating on minimum number of sample size, n and the probability of lot acceptance, $L(p)$.

The first attribute sampling plan is the SSP developed by Epstein (1954).The SSP is the most widely recognized and simplest arrangement to utilize. However, it is inefficient when dealing with high quality products. In order to overcome the problem, Dodge (1955) proposed ChSP-1 to reduce the incompetence and less discriminatory power of the SSP when the acceptance number is equivalent to zero. However, the consumers are not happy with ChSP-1 because it only considered preceding information only.

The CChSP(0,1) was further developed by Devaarul.S and Edna.K (2011) to ensure both preceding and succeeding samples are inspected. It gives more protection to the consumer while retaining the same amount of protection to the producer compare to ChSP-1. Later in 2018, Devaru and Vijila introduced MCChSP($c_1, c_2. i, j$)to protect producer by allowing the product lot to have acceptance number more than zero.

The TSChSP was initiated by Mughal, Zain and Aziz (2015). The operating procedure for TSChSP consider only one defective in both preceding and succeeding samples. However, the plan still has drawback where it still considering at least one defect in both 'i' preceding and 'j' succeeding sample. To solve the drawback in the TSChSP, Mughal, Zain and Aziz (2015) proposed TSMChSP for Pareto Distribution of the 2nd kind. By applying the TSMChSP, it can ensure no defects in both preceding and succeeding samples. All sampling plan that have been discuss earlier have been develop for various lifetime distributions. In industry there are various product that follow different lifetime distribution. For instance, capacitors and integrated circuit follow Exponential distribution. Past studies on the Single, Chain,

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Group and Modified Group Chain Acceptance Sampling Plan that follows exponential distribution were first discussed by Epstein (1954), Ramaswamy and Jayasri (2012), Kundu, Jun and Ahmad (2011), Teh, Aziz and Zain (2018), Jamaludin, Zain and Aziz (2017).However, to our knowledge TSMChSP only have been develop for Pareto Distribution of the 2nd kind. Therefore, in this study, TSMChSP for exponential distribution is developed.

II. GLOSSARY OF SYMBOLS

$\boldsymbol{n}$: Sample size
$\boldsymbol{d}$: Number of defective products
$\boldsymbol{i}, \boldsymbol{j}$: Allowable preceding and succeeding lots
$\boldsymbol{\alpha}$: Producer's risk (Probability of rejecting a good lot)
$\boldsymbol{\beta}$: Consumer's risk (Probability of accepting a bad lot)
$\boldsymbol{\mu}/\boldsymbol{\mu_0}$: Mean ratio
$\boldsymbol{L(p)}$: Probability of lot acceptance
$\boldsymbol{t_0}$: Test termination time

III. EXPONENTIAL DISTRIBUTION

Exponential distribution is a simple distribution with just a single parameter and is ordinarily used to model reliability data. Not only that, it is also gives a decent model for the phase of a product's life when it is just as likely to fail at any time.

According to Epstein (1954) the Cumulative Distribution Function (CDF) of an exponential distribution can be written as follow:

$$F(t;\, \sigma) = 1 - exp(-\frac{t}{\alpha}), \quad t > 0 \qquad (1)$$

Given $\boldsymbol{\alpha}$ is the scale parameter. Eq.2 shows probability of failure, $\boldsymbol{p}$. It is a probability that a product will be fail at a specified time and can be obtained as:

$$p = 1 - exp[-a(\mu/\mu_0)]. \qquad (2)$$

There are two risk in acceptance sampling which is producer's and consumer's risk. The probability of rejecting a good lot is known as producer's risk and the chance of accepting bad lot is known as consumer's risk. Consumer's risk is used when selecting the design parameters for the TSMChSP and it is presented as, $\boldsymbol{\beta = 1 - p*}$. Then the minimum sample size for the TSMChSP is calculated by using:

$$L(p) \le \beta \qquad (3)$$

The operating characteristic (OC) function of TSMChSP is given by:

$$L(p) = (P_0)^{2i+1} + P_1(P_0)^{2i} \qquad (4)$$

Where, $\boldsymbol{P_0}$ is the probability of finding zero defective in a sample and $\boldsymbol{P_1}$ is the probability of finding one defective in a sample.The operating characteristic (OC) function of TSMChSP is given by:

$$L(p) = (1-p)^{n(2i+1)} + np(1-p)^{2ni+n-1} \qquad (5)$$

Figure 1 displays all the possible outcomes of TSMChSP based on preceding and succeeding lots.

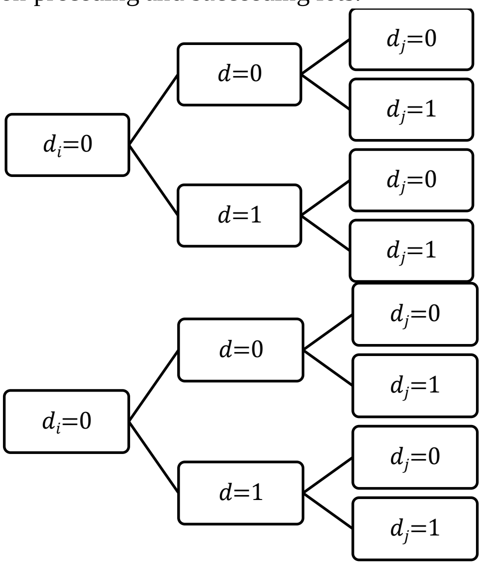

Fig. 1 Tree Diagram of TSMChSP

The operating steps are make based on tree diagram above. For instance, if the number of preceding and succeeding is equal to zero, then accept the lot but if there is a defective either in preceding and succeeding lot, then reject the lot.

IV. OPERATING STEPS

TSMChSP requires three design parameters which are the sample size, allowable preceding and succeeding lots, denoted by $\boldsymbol{n}$, $\boldsymbol{i}$ and $\boldsymbol{j}$, respectively. The operating steps are as follow:

i. For each lot, find the minimum sample size and test each unit for conformance to the specified requirements.

ii. Accept the lot, if $\boldsymbol{d = 0}$ provided that '$\boldsymbol{i}$' preceding and '$\boldsymbol{j}$'succeeding samples have no defects, if$\boldsymbol{d} > 1$, reject the lot.

iii. Accept the lot if the number of defectives, $\boldsymbol{d = 1}$, provided that have no defectives are found in both '$\boldsymbol{i}$'preceding and '$\boldsymbol{j}$'succeeding samples.

The performances of the TSMChSP are measured based on two criteria which are the number of minimum sample size, $\boldsymbol{n}$ and the probability of lot acceptance, $\boldsymbol{L(p)}$.Table 1 shows the number of minimum sample size, $\boldsymbol{n}$ for TSMChSP for Exponential distribution at different values of design parameters: specified constant, $\boldsymbol{a}$, consumer's risk, $\boldsymbol{\beta}$, number of allowable preceding and succeeding lots $\boldsymbol{i}, \boldsymbol{j}$.

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V. RESULT

Table. 1 Number of minimum sample size,$n$for Exponential Distribution

β	i,j	a							
		0.25	0.5	0.75	1	1.25	1.5	1.75	2
0.01	1	8	4	3	3	2	2	2	2
	2	5	3	2	2	1	1	1	1
	3	3	2	2	1	1	1	1	1
	4	3	2	1	1	1	1	1	1
	5	2	1	1	1	1	1	1	1
0.05	1	6	3	2	2	2	1	1	1
	2	3	2	1	1	1	1	1	1
	3	2	1	1	1	1	1	1	1
	4	2	1	1	1	1	1	1	1
	5	2	1	1	1	1	1	1	1
0.10	1	5	2	2	2	1	1	1	1
	2	3	2	1	1	1	1	1	1
	3	2	1	1	1	1	1	1	1
	4	2	1	1	1	1	1	1	1
	5	1	1	1	1	1	1	1	1

As shown in Table 1, the number of minimum sample size, n for the TSMChSP following Exponential distribution at different values of design parameters is presented satisfying Eq.4.

Based on Table 1, the number of minimum sample size, n decreases as the consumer's risk, β, the specified constant, a and the number of allowable preceding and succeeding lots, i,j increases. For in-stance, if the consumer's risk, β is 0.10, the number of allowable preceding and succeeding lots, i,j is 1 and the specified constant, a is 0.25, then the

number of minimum sample size, n is 5. It shows that, the number of minimum sample size, n is inversely proportional to the design parameters.

Once the minimum sample size, n is obtained, one may interested to find the probability of a lot acceptance, $L(p)$. The probability of lot acceptance, $L(p)$ increases as the value of mean ratio, $^{\mu}/_{\mu_0}$increases. Consider $\beta = 0{,}10, n = 5$, and $a = 0.25$, the probability of lot acceptance, $L(p)$ is 0.0569. The probability of lot acceptance, $L(p)$ increases from 0.0569 to 0.8086 when the value of mean ratio, $^{\mu}/_{\mu_0}$increases from 1 to 12 as shown in Figure 2. It means that the probability of lot acceptance, $L(p)$ increases from 5.69% to 80.86% when the true mean lifetime of a product is 12 times higher than the specified mean lifetime.

Table. 2 Probability of lot acceptance for exponential distribion

			$^{\mu}/_{\mu_0}$						
β	n	α	1	2	4	6	8	10	12
0.01	8	0.25	0.0081	0.1028	0.3383	0.4931	0.5923	0.6600	0.7087
	4	0.50	0.0061	0.0798	0.2844	0.4332	0.5342	0.6058	0.6587
	3	0.75	0.0047	0.1369	0.7399	1.2986	1.7204	2.0366	2.2791
	3	1.00	0.0008	0.0327	0.1952	0.3445	0.4543	0.5348	0.5955
	2	1.25	0.0033	0.0644	0.2659	0.4192	0.5241	0.5982	0.6528
	2	1.50	0.001	0.0359	0.2013	0.3499	0.4586	0.5382	0.5982
	2	1.75	0.0003	0.0199	0.1520	0.2915	0.4008	0.4838	0.5478
	2	2.00	0.0001	0.0110	0.1144	0.2424	0.3499	0.4346	0.5013
0.05	6	0.25	0.0300	0.1896	0.4503	0.5930	0.6783	0.7345	0.7741
	3	0.50	0.0233	0.1531	0.3915	0.5352	0.6258	0.6873	0.7316
	2	0.75	0.0333	0.3162	0.9740	1.4171	1.7093	1.9129	2.0619
	2	1.00	0.0110	0.1144	0.3499	0.5013	0.5982	0.6642	0.7120
	2	1.25	0.0033	0.0644	0.2659	0.4192	0.5241	0.5982	0.6528
	1	1.50	0.0498	0.2231	0.4724	0.6065	0.6873	0.7408	0.7788
	1	1.75	0.0302	0.1738	0.4169	0.5580	0.6456	0.7047	0.7470
	1	2.00	0.0183	0.1353	0.3679	0.5134	0.6065	0.6703	0.7165
0.1	5	0.25	0.0569	0.2554	0.5179	0.6491	0.7251	0.7743	0.8086
	2	0.50	0.0861	0.2905	0.5372	0.6603	0.7323	0.7793	0.8123
	2	0.75	0.0333	0.3162	0.9740	1.4171	1.7093	1.9129	2.0619
	2	1.00	0.0110	0.1144	0.3499	0.5013	0.5982	0.6642	0.7120
	1	1.25	0.0821	0.2865	0.5353	0.6592	0.7316	0.7788	0.8119
	1	1.50	0.0498	0.2231	0.4724	0.6065	0.6873	0.7408	0.7788
	1	1.75	0.0302	0.1738	0.4169	0.5580	0.6456	0.7047	0.7470
	1	2.00	0.0183	0.1353	0.3679	0.5134	0.6065	0.6703	0.7165

Suppose that u and u_0 are true and specified mean life of a product respectively. A lot is considered good if the true mean life, $u > u_0$. For more explanation, one may refer to Jamaludin et al. (2017). Assume that the lifetime distribution of the product follows Exponential distribution with the level of consumer's confidence is 0.99. The product is consider as good only when its true mean life is at least 1000 hours. If, during inspection hours have not more than one defects with zero defects in both preceding and succeeding samples then the lot will be accepted. Otherwise, the lot will be rejected.

Then, the performances of the TSMChSP and TSChSP are compared. The performances of these two sampling plans are explained based on two criteria, which are the number of minimum sample size, n and the probability of lot acceptance, $L(p)$.

Table 3 shows the number of minimumsample size, n for the TSMChSP and the TSChSP for Exponential distribution when consumer's risk is 0.01.

Table. 3 Number of minimumsample size, n for Exponential distribution for different sampling plans.

β	α	TSMChSP	TSChSP
		n	
0.01	0.25	8	9
	0.5	4	5
	0.75	3	3
	1	3	3
	1.25	2	2
	1.5	2	2
	1.75	2	2
	2	2	2

As shown in Table3, for most values of the specified constant, a, the number of minimum sample size, n for TSMChSP is smaller compared to TSChSP. For instance, TSMChSP requires 8 minimum sample size meanwhile the TSChSP needs 9 minimum sample size when the design parameters for both plan are $(\beta, \alpha) = (0.01, 0.25)$. The smaller sample size, $n$will reduce the cost and inspection time.

Table 4 shows the probability of lot acceptance, $L(p)$ for the TSMChSP and TSChSP for Exponential distribution. It presents that TSMChSP is smaller compared to TSChSP, for most value of specified constant, a. For instance, the probability of lot acceptance, $L(p)$ for the TSMChSP is 0.0081 meanwhile for the TSChSP is 0.0072 when the design parameters for both plan are $(\beta, {}^{\mu}/_{\mu_0}) = (0.01, 1)$.

Besides, the probability of lot acceptance, $L(p)$ for both sampling plans increases as the value of mean ratio, ${}^{\mu}/_{\mu_0}$increases. For instance, the probability of lot acceptance, $L(p)$ with $\beta = 0.01$, for TSMChSP and TSChSP are 0.0081 and 0.0072, respectively when the value of mean ratio is ${}^{\mu}/_{\mu_0} = 1$.

Table. 4 Probability of lot acceptance for Exponential distribution for the TSMChSP and TSChSP.

β	$\frac{\mu}{\mu_0}$	TSMChSP	TSChSP
0.01	1	0.0081	0.0072
	2	0.1028	0.1162
	4	0.3383	0.3997
	6	0.4931	0.5733
	8	0.5923	0.6758
	10	0.6600	0.7412
	12	0.7087	0.7857

The probability of lot acceptance, $L(p)$ for both sampling plans increases to 0.7087 and 0.7857, respectively when the value of mean ratio, ${}^{\mu}/_{\mu_0} = 12$. It is found that, the probability of lot acceptance, $L(p)$ for TSMChSP increases from 0.81% to 70.87%. Meanwhile, TSChSP increases from 0.72% to 78.57% when true mean lifetime of a product is twelve times higher than the specified mean lifetimefor both sampling plans. Figure 2 described the effect of different values of mean ratio, ${}^{\mu}/_{\mu_0}$ on the probability of lot acceptance, $L(p)$.

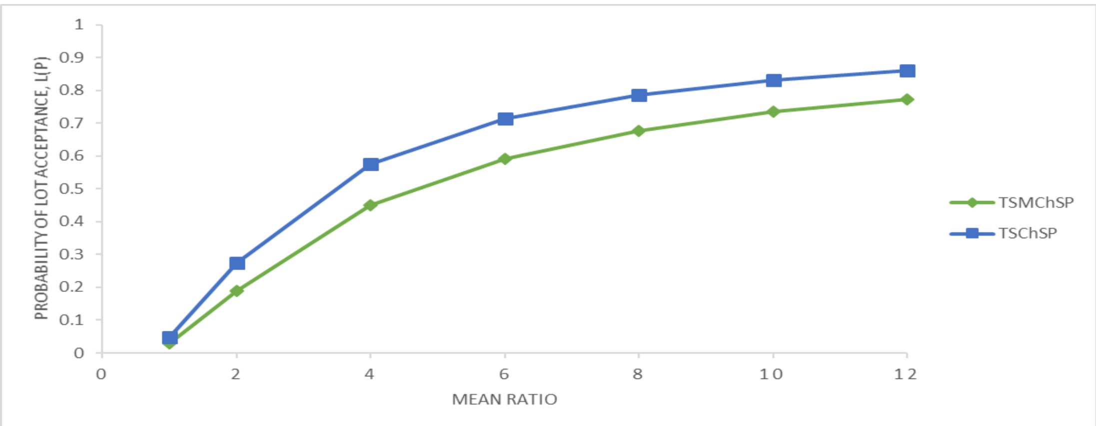

Fig. 2 Probability of lot acceptance versus mean ratio for Exponential distribution for two different sampling plans

Figure 2 indicates that the lot of acceptance,$L(p)$ for exponential distribution for the TSMChSP and TSChSP when the design parameters for both sampling plans are $(\beta, \alpha) = (0.05, 0.25)$.

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As can be seen in Figure 2, the value of mean ratio' μ/μ_0 and the probability of lot acceptance, $L(p)$ for TSMChSP are always lower compared to TSChSP. For details explanation, we may referred to the probability of lot acceptance, $L(p)$ for Exponential distribution when $\mu/\mu_0 = 1$ and $\beta = 0.05$ is 0.0081 for TSMChSP and 0.0072 for TSChSP. The finding shows that the performance of the TSMChSP is better than TSChSP in terms of the number of minimum sample size, n and the probability of lot acceptance, $L(p)$.

VI. CONCLUSION

In this study, TSMChSP for exponential distribution is developed with the main purpose of minimizing the consumer's risk, β. The design parameters used in this article are specified constant, a, consumer's risk, β, number of allowable preceding and succeeding lots i, j. The performances of the TSMChSP are measured based on two criteria, which are minimum sample size, n and the probability of lot acceptance, $L(p)$.

The first criterion discussed that, the number of minimum sample sizes, n decreases as the value of specified constant, a increases. Besides, the number of minimum sample size, n also decreases as the number of allowable preceding and succeeding lots, i, j increase. Finally, the number of minimum sample sizes, n also decreases as the value of consumer's risk, β increases. For the second criterion, it can be explained that, the probability of lot acceptance, $L(p)$ increases as the value of mean ratio, μ/μ_0 increases. This result is comparable with past studies.

The performances of the TSMChSP are then compared with the TSChSP based on two criteria, which are the number of minimum sample size, n and the probability of lot acceptance, $L(p)$. Based on these two criterion the performance of the TSMChSP is better than TSChSP. The result has shown that the TSMChSP has a smaller number of minimum sample size, n compared to TSChSP. Besides, the probability of lot acceptance, $L(p)$ for TSMChSP is smaller compared to TSChSP. For future researches, TSMChSP can be further developed for other lifetime distributions. By having TSMChSP with various lifetime distribution in future, it can allows producer in industry to select the best sampling plan for their production.

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