The New Two-sided Group Chain Sampling Plan for Pareto Distribution of the 2nd Kind

Muhammad Farouk, Nazrina Aziz, Zakiyah Zain

Abstract: The new two-sided complete group chain acceptance sampling plan was first introduced in 2015. The advantage of the plan is that it enables simultaneous multiple inspection of products and minimized the decrease of probability of lot acceptance for zero-acceptance-number, while improving both producer's and consumer's risks. This study proposes a new approach called the new two-sided group chain sampling plan, which emphasized on consumer protection. The number of optimal groups is determined from observing various consumers' risks, test termination time and mean ratio. A time-truncated life test is performed with pre-specified parameters and the operating characteristics values of the plan under various parameters are measured.

Keywords: Probability of Lot Acceptance; Pareto Distribution of the 2nd Kind; Truncated Life Test; The New Two-Sided Group Chain Sampling Plan

I. INTRODUCTION

Quality inspection is a process that takes place in any mass production. Ideally, to ensure that the products being produced are of the highest quality and conforms to all the producer's requirements, a 100% inspection should be done on the behalf of the producer. Realistically, however, it is not achievable due to a number of constraints being considered such as time, cost, and in some cases the destructive nature of the product being inspected, such as food.

Thus, the idea of acceptance sampling is created to provide an alternative to 100% inspection. The simple idea behind acceptance sampling is to take a number of samples from a production lot and inspect them, then sentencing the acceptance of the lot based on the inspection results. Ever since the inception of acceptance sampling, many types of sampling plans have been proposed, and newer ones are normally improvements or evolutions from the older.

One example of the acceptance sampling plan is the chain sampling plan by [1], or commonly known as ChSP-1. This plan takes into account not only the inspection result from the current production lot, but also from the preceding lots. However, ChSP-1 has a major challenge from being implemented in the mass production industry, which inhibits multiple inspections from taking place. Thus, another plan called the group sampling plan (GSP) was proposed by [2]. Since then, the GSP has been explored in a few other occasions, such as [3, 4]. The GSP involves grouping the samples into smaller groups to be inspected by different testers simultaneously. Therefore, the GSP proves to be more cost and time effective to the producer, while possessing similar discriminating power for the quality of the lot.

More recently, the group chain sampling plan (GCSP) was proposed by [5] as the solution to the shortcomings of both ChSP-1 and GSP. Like the name suggests, GCSP is simply the combination of ChSP-1 and GSP, in which it considers the producer's historical performance from the information obtained from the inspection of preceding lots. Not only that, like the GSP, the GCSP involves grouping the samples into smaller groups to be inspected by a number of testers at the same time. The study by [5] proves that the GCSP has better discriminating power compared to ChSP-1 while allowing multiple inspections to take place. Since then, the behavior of the GCSP had been tested with a few other lifetime distributions, namely the Rayleigh distribution by [6], Exponential distribution by [7], the Inverse Rayleigh distribution by [8], and the Log-logistic distribution by [9].

In 2012, a study by [10] suggested a plan that improves the protection towards the consumer while maintaining the protection towards the producer. The plan is named as two-sided complete chain sampling plan, and is similar to the ChSP-1, except that it accounts for the information from both preceding and succeeding lots. This plan highlights the importance of a continuous level of high performance from the producer, due to the fact that any irregularities in the production outputs greatly affect the lot sentencing. However, the problem of multiple inspections reoccurred.

Besides GCSP and the two-sided complete chain sampling plan, there is another extension to group and chain acceptance sampling plans, which is the modified group chain sampling plan by [11, 12]. The modified group chain sampling plan is proven to perform better than the legacy

Revised Manuscript Received on March 08, 2019

Muhammad Farouk, School of Quantitative Sciences, College of Arts and Sciences, Universiti Utara Malaysia, 06010 Sintok, Kedah, Malaysia

Nazrina Aziz, School of Quantitative Sciences, College of Arts and Sciences, Universiti Utara Malaysia, 06010 Sintok, Kedah, Malaysia
Institute of Strategic Industrial Decision Modelling (ISIDM), School of Quantitative Sciences (SQS), College of Arts and Sciences, Universiti Utara Malaysia, 06010 Sintok, Kedah, Malaysia

Zakiyah Zain, Centre for Testing Measurement and Appraisal (CeTMA), Universiti Utara Malaysia, 06010 Sintok, Kedah, Malaysia.

sampling plans, however it is evident in [13] their proposed new two-sided complete group chain sampling plan (NTSCoGCh) provides higher probability of lot acceptance. This in turn means that the plan provides more freedom to the producer. In their study, [13] utilized the Pareto distribution of the 2ndkind as the product lifetime distribution.

The term ‘complete’ in the name of the sampling plans by [10] and [13] brings into focus the technical aspect of acceptance criteria in the studies. Both of the plans have five acceptable criteria which results in lot sentencing. This is done to promote the probability of lot acceptance that the plans yield. However, it may also result in type-II error during lot sentencing. Thus, this study intends to propose a new consumer-focused sampling plan called the new two sided group chain sampling plan (NTSGCh), which will operate based on four acceptance criteria. A truncated life test on auto-generated product lifetime data with Pareto distribution of the 2nd Kind is conducted to measure the performance of the plan as compared to its predecessor, the NTSCoGCh as proposed by [12].

II. GLOSSARY OF PARAMETERS

This subsection lists down the parameters involved in the study, along with the symbols used to represent them in calculations and further discussions.

g : Minimum number of groups required
r : Number of testers within a group
n : Sample size taken from a production lot
d : Number of defectives found in a tested sample
β : Consumer’s risk (probability of accepting a bad lot)
δ : Scale parameter
λ : Shape parameter
t_0 : Test termination time
μ_0 : Specified average lifetime of a product
μ : True average lifetime of a product
P(a) : Probability of lot acceptance
μ/μ_0 : Mean ratio
b : Test termination time multiplier

III. METHODOLOGY

Time Truncated Life Test

A time truncated life test is applied in situations where the product being tested have long life duration. Instead of putting the product on test until it stops functioning, a time truncated life test makes decision on lot sentencing based on findings obtained after a specified termination time, t_0, of the life test. The number of defectives, d, after the termination time is counted and compared to the pre-specified acceptance number, c.Lot sentencing decision is made based on whether d is less than or equal to c. In this study, t_0 is set as the multiple of the product’s specified average lifetime, μ_0 and a constant, b.

Operational Procedure

The operational procedure of the NTSGCh is as follows:

I. A sample of size is drawn from the production lot. Divide the sample into$g$ groups, with $r$testers (units) in a group before starting the life test.

The test is stopped at $t = t_0$. Inspect all units simultaneously.

III. If number of defectives, $d > 1$, reject the production lot.

V If$d = 0$, accept the production lot given that the preceding and succeeding lots have at most 1 defective unit, $d_i + d_j \leq 1$.

III. If $d = 1$, accept the lot if and only if the cumulative number of defectives in the preceding and succeeding lots is zero, $d_i + d_j = 0$.

Probability of Defective

The probability of defective, denoted as p, can be obtained from the cumulative distribution function (CDF) of the Pareto distribution of the 2nd Kind. According to [9], the CDF of the Pareto distribution of the 2nd Kind is given as:

$$F(t;\delta) = 1 - \left[1 + \frac{t}{\delta}\right]^{-\lambda}, t > 0, \delta > 0, \lambda > 0 \qquad (1)$$

and the mean is given by:

$$\mu = \frac{\delta}{\lambda - 1} \qquad (2)$$

The pre-specified termination time is introduced as:

$$t_0 = b\mu_0 \qquad (3)$$

Substituting equations (2) and (3) into equation (1) and simplified further, the probability of defective is:

$$p = 1 - \left[1 + \frac{b}{(\lambda-1)\left({}^{\mu}/_{\mu_0}\right)}\right]^{-\lambda}, \left({}^{\mu}/_{\mu_0}\right) > 0, \lambda > 1 \qquad (4)$$

Probability of Lot Acceptance

Suppose that P_0 and P_1 are the probabilities of finding zero and one defectives in the current sample respectively, and P_0^i, P_1^i, P_0^j, and P_1^j are the probabilities of finding zero and one defectives in the preceding i and succeeding j samples. With four acceptance criteria, and the assumption that the number of preceding and succeeding lots are the same $(i = j)$, the probability of lot acceptance can be written as:

$$P(a) = P_0^i\,[P_0^{i+1} + 2P_0P_1^i + P_0^iP_1] \qquad (5)$$

The Binomial model can be applied due to the dual nature of lot sentencing. Thus, equation (5) can be rewritten as:

$$P(a) = [(1-p)^{(gr)}]^i * \Big[[(1-p)^{(gr)}]^{(i+1)} + 2(1-p)^{gr} *$$
$$grp1{-}pgr{-}1i{+}[(1{-}p)gr]i{*}grp1{-}pgr{-}1 \qquad (6)$$

IV. RESULTS AND DISCUSSIONS

In this study, the parameter values are pre-specified, where

$r = \{2, 3, 4, 5\}, b =$

$\{0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0\}$, $i = j = \{1, 2, 3, 4\}$, $\beta = \{0.25, 0.1, 0.05, 0.01\}$,and $^{\mu}/_{\mu_0} = \{1, 2, 4, 6, 8, 10, 12\}$.The reason behind these pre-specified values are to observe the behaviour of the sampling plan under various design parameters. Then, the minimum number of groups, $g$is calculated for various sets of design parameters with respect to the consumer's risk, β. Table 1 shows the minimum number of groups under the constraint of various design parameters.

Table 1: The minimum number of groups required for Pareto distribution of the 2nd Kind with$\lambda = 2$

			b							
β	r	i	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
0.1	2	1	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1	1	1
	4	3	1	1	1	1	1	1	1	1
	5	4	1	1	1	1	1	1	1	1
0.05	2	1	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1	1	1
	4	3	1	1	1	1	1	1	1	1
	5	4	1	1	1	1	1	1	1	1
0.01	2	1	3	1	1	1	1	1	1	1
	3	2	2	1	1	1	1	1	1	1
	4	3	1	1	1	1	1	1	1	1
	5	4	1	1	1	1	1	1	1	1

It is important to note that the sample size to be drawn from the production lot is the product of the minimum number of groups, g, and the number of testers, r. For instance, if a company wants to apply the plan with 10% consumer's risk, with 2 testers in a group, for the duration of $b = 0.25$, then the number of products sampled should be 4 products which are divided into two groups of two products.

Another simulation was conducted by using [9]'s NTSCoGCh probability of lot acceptance equation, through the use of similar set of design parameters. In comparison, the minimum number of groups yielded in the NTSGCh is exactly the same as the ones yielded from the NTSCoGCh. Hence, it can be said that the proposed plan operates on the same level of effectiveness in terms of the number of samples to be drawn from the lot.

Table 2 below shows the resulting probability of lot acceptance for the NTSGCh operating with a 10% consumer's risk. The table includes the probability of lot acceptance for production lot from different mean ratios, inspected for various duration of life test. The mean ratio is the ratio of the true mean lifetime of a product to its specified mean lifetime. A mean ratio of less than one means that on average, all products in the production lot does not function as long as it is expected of. Thus, it can also be said that a higher value of mean ratio generally means that the product comes from a relatively good quality background.

Hence, it is evident in Table 2 that higher mean ratio results in a higher probability of lot acceptance. This can be further explained that as the life test is stopped at t_0, it is less likely to find a defective product in a sample that comes from the production lot with high mean ratio. This is ultimately true when the duration of life test is very short in comparison to the specified mean lifetime of the product. For instance, imagine a sample of products with a specified mean lifetime of 100 hours. It is known to the producer from production history that the production lots have a mean ratio of 12, which means in actual that the product may have a true average lifetime of 1200 hours. Thus, when the life test is conducted for 25 hours ($b = 0.25$), there is a 91.766% chance that inspected lot will fulfil either one of the four acceptance criteria, hence it is accepted. Now, imagine the same product that comes from a production lot with a mean ratio of 1, which means that the true average lifetime is 100 hours. If the same life test duration is conducted, there is only a 3.66% that the lot under inspection will be accepted.

The probability of lot acceptance for the NTSGCh is mapped against the probability of lot acceptance for the NTSCoGCh. The comparison between the two sampling plans can be found in Table 3. It can be observed that the NTSGCh yields a lower probability of lot acceptance compared to the NTSCoGCh. This finding is expected, since the objective of proposing NTSGCh is to provide better protection to the consumer. Therefore, if the lot under inspection comes from a low mean ratio background, it is less likely that it will be accepted by the NTSGCh compared to the NTSCoGCh. However, it is also worth noting that the decrease in probability of lot acceptance also occur in inspection of high mean ratio production lots.

Table. 2 The probability of lot acceptance for the NTSGCh with Pareto distribution of the 2nd Kind with $\lambda = 2$

			$\mu/\mu0$						
β	g	b	1	2	4	6	8	10	12
0.1	2	0.25	**0.03660**	0.24791	0.59445	0.75865	0.84179	0.88875	**0.91766**
	1	0.50	0.00004	0.00173	0.02435	0.07714	0.14283	0.20866	0.26954
	1	0.75	0.00000	0.00024	0.00561	0.02435	0.05719	0.09844	0.14283
	1	1.00	0.00000	0.00004	0.00173	0.00882	0.02435	0.04795	0.07714
	1	1.25	0.00000	0.00001	0.00063	0.00369	0.01121	0.02435	0.04269
	1	1.50	0.00000	0.00000	0.00024	0.00173	0.00561	0.01300	0.02435
	1	1.75	0.00000	0.00000	0.00009	0.00087	0.00302	0.00732	0.01438
	1	2.00	0.00000	0.00000	0.00004	0.00045	0.00173	0.00434	0.00882

Table. 3 The comparison of probability of lot acceptance for NTSGCh and NTSCoGh when$r = 2, i = 1$.

			NTSGCh		NTSCoGh	
			μ/μ_0			
β	g	b	1	12	1	12
0.1	2	0.25	0.03660	0.91766	0.06051	0.93495
	1	0.50	0.00004	0.26954	0.00074	0.26954
	1	0.75	0.00000	0.14283	0.00015	0.14283
	1	1.00	0.00000	0.07714	0.00002	0.07715
	1	1.25	0.00000	0.04269	0.00000	0.04273
	1	1.50	0.00000	0.02435	0.00000	0.02444
	1	1.75	0.00000	0.01438	0.00000	0.01456
	1	2.00	0.00000	0.00882	0.00000	0.00910

V. CONCLUSION

This article proposes a new derived two-sided group chain sampling plan. By implementing the Pareto distribution of the 2nd Kind, the proposed NTSGCh has been proven to be able to provide better protection to the consumer while operating on similar resources as compared to the NTSCoGCh.

The NTSGCh provides the producers with another option of sampling plan to implement in the industry. However, with so many lifetime distributions involved in the manufacturing industry, it will be interesting to see how the NTSGCh will behave under the implementation of other lifetime distributions.

Acknowledgement

Firstly, I would like to thank my mother, sister and wife for their continuous support for the duration of this study. Secondly, to the ever-present guidance and wisdom from both my supervisors. Finally, I would also like to thank my friends for their insightful knowledge of Microsoft Office.

REFERENCES

1. Dodge, H. F. (1955). Chain sampling inspection plan. Industry Quality Control, 11(4), pp. 10-13.
2. Aslam, M., & Jun, C. H. (2009). A group acceptance sampling plan for truncated life test having Weibull distribution. Journal of Applied Statistics, 36(9), pp. 1021-1027.

Aslam, M., Kundu, D., Jun, C. H., & Ahmad, M. (2011). Time Truncated Group Acceptance Sampling Plans for Generalized Exponential Distribution. Journal of Testing and Evaluation, 39(4).

3. Ramaswamy, A. R. S., &Sutharani, R. (2013). Designing Group Acceptance Sampling Plans for the Weibull Distribution and Gamma Distribution Using Minimum Angle Method. International Journal of Mathematics and Statistics Studies, 1(4), pp. 23-36.
4. Mughal, A. R., Zain, Z., & Aziz, N. (2015a). Time truncated group chain sampling strategy for Pareto distribution of the 2nd kind. Research Journal of Applied Sciences, Engineering and Technology, 10(4), pp. 471-474.
5. Teh, M. A. P., Aziz, N., & Zain, Z. (2016). Time truncated group chain sampling plans for Rayleigh distribution. Global Journal of Pure and Applied Mathematics, 12 (4), pp. 3693-3699.
6. Teh, M. A. P., Aziz, N., & Zain, Z. (2018). Group chain sampling plans based on truncated life test for Exponential distribution. International Journal of Pure and Applied Mathematics, 119 (3), pp. 491-500.
7. Teh, M. A. P., Aziz, N., & Zain, Z. (2016). Group chain sampling plans based on truncated life test for Inverse Rayleigh distribution. Research Journal of Applied Sciences, 11 (11), pp. 1432-1435.
8. Teh, M. A. P., Aziz, N., & Zain, Z. (2016). Group chain sampling plans based on truncated life tests for Log-Logistic distribution. International Journal of Applied Engineering Research, 11 (16), pp. 8971-8974.
9. Deva Arul, S. & Rebecca, I.E.K. (2012). Two-sided complete chain sampling plans for attribute quality characteristics (CChSP-0,1). Karunya Journal of Research. 3(1), pp. 8-16.
10. Mughal, A. R., Zain, Z., & Aziz, N. (2015b). Modified Group Chain Sampling Strategy for Pareto Distribution of the 2nd Kind. Research Journal of Applied Sciences, Engineering and Technology.
11. Jamaludin, A. J., Zain, Z. & Aziz, N. (2016). A modified group chain sampling plans for lifetimes following a Rayleigh distribution. Global Journal of Pure and Applied Mathematics, 12 (5), pp. 3941-3947.
12. Mughal, A.R., Zain, Z., & Aziz, N. (2015c). New Two-Sided Complete Group Chain Sampling Plan for Pareto Distribution of the 2nd Kind. Research India Publication. 10 (12), pp. 31855-31860.