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Constructing Group Chain Acceptance Sampling Plans (GChSP) for Gamma Distribution

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***Abstract:** This article develops group chain acceptance sampling plans (GChSP) for Gamma distribution when the life test is truncated at a pre-specified time. The Gamma distribution is chosen as most electronic products such as carbon-film resistors, light-emitting diodes and integrated logic family follow this distribution. The design parameters such as the total of minimum groups, g and probability of lot acceptance, $L(p)$ are calculated by minimizing the consumer's risk, β at a certain specified design parameter. Quality parameter is describe in terms of mean with assumption that the test termination time, t_0, the specified constant, a, the number of allowable preceding lots, iand the number of products, r are pre-fixed. An example is given for determination purpose for the GChSP. The article continues with performances comparison between the GChSP and the group acceptance sampling plan (GSP). The article concludes that the GChSP has better performances compared to the GSP in terms of the number of minimum groups, g, the probability of lot acceptance, $L(p)$, the cost and the inspection time.*

***Keywords:** Group chain acceptance sampling plan (GOhSP); Consumer's risk; Gamma distribution; probability of lot acceptance; number of minimum groups*

I. INTRODUCTION

Acceptance sampling is defined as a set of procedures, used to accept or dismiss a submitted lot based on a random sample selected from the lot [1-2]. Based on the definition itself, it is clearly stated that the purpose of conducting acceptance sampling is either to accept or reject a submitted lot. Most people would have an idea that acceptance sampling measures the quality of the product. It turns out that, it does not. There are two types of sampling plan. The first type is attribute sampling plan meanwhile the second type is variable sampling plan. The attribute sampling plan is measured based on binary outcomes meanwhile the variable sampling plan is measured based on numerical value.

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For both types, they have their own sampling plans such as single acceptance sampling plan (SSP), double acceptance sampling plan (DSP) and group acceptance sampling plan (GSP). For this article, we only focus on the attribute sampling plan; the group chain acceptance sampling plan (GChSP).

Many authors have applied Gamma distribution in the acceptance sampling area. Gupta and Groll used the Gamma distribution for the SSP [3]. Aslam, Jun and Ahmad used the Gamma distribution for the GSP [4]. Anburajan and Ramaswamy extended the idea of a two stage group acceptance sampling plan for Gamma distribution items [5]. For the GChSP, none has constructed the plan for Gamma distribution. However, the GChSP has been developed by several researchers using different distributions [6-9] [13].

Therefore, the purpose of this article is to produce the GChSP when the lifetime of a product follows Gamma distribution.

Tables are constructed for the number of minimum groups, g and probability of lot acceptance, $L(p)$ for different values of the design parameters.

II. GAMMA DISTRIBUTION

Gamma distribution has many applications in other fields. Husak, Michaelsen and Funk applied the Gamma distribution to observe the monthly rainfall in Africa [10]. Rohan, Fairweather and Grainger used Gamma distribution to ascertain half-life of rotenone in the freshwater [11]. Gupta and Groll extended the application of the Gamma distribution to the SSP [3], meanwhile Ramaswamy and Sutharani developed the GSP using the same distribution [12].

For Gamma distribution, the cumulative distribution function (CDF) is given by

$$\boldsymbol{F(t;\,\sigma,\gamma) = 1 - \sum_{c=0}^{\gamma-1} exp\left(-\frac{t}{\sigma}\right)\frac{\left(\frac{t}{\sigma}\right)^{c}}{c!}}\ , \boldsymbol{t} > 0 \tag{1}$$

where σ and γ are the scale and shape parameters respectively. The mean of the distribution is written as

$$\boldsymbol{\mu = \gamma\sigma}. \tag{2}$$

The test termination time, $\boldsymbol{t_0}$ is given by equation (3), where it is a different of the specified mean life, $\boldsymbol{\mu_0}$ and specified constant, $\boldsymbol{a}$, written as

$$\boldsymbol{t_0 = a\mu_0}.$$

The probability of failure, $\boldsymbol{p}$ can be estimated by

$$\boldsymbol{p = 1 - \sum_{c=0}^{\gamma-1} exp\left[-a\gamma\left(\frac{\mu_0}{\mu}\right)\right]\frac{\left[a\gamma\left(\frac{\mu_0}{\mu}\right)\right]^{c}}{c!}}.$$

III. OPERATING STEPS

The GChSP is applied with the following steps [6-9]:

1. For each lot, number of minimum groups, $\boldsymbol{g}$is obtained and allocate $\boldsymbol{r}$ products to each group. The sample size is then given by $\boldsymbol{n} = \boldsymbol{g} * \boldsymbol{r}$.
2. A lot is accepted when $d = 0$ and the lot is rejected if $d > 1$.
3. A lot is accepted if $\boldsymbol{d} = \mathbf{1}$ and the inspection is continued if no deficient are found in the preceding $\boldsymbol{i}$ lots.
4. The number of minimum groups, $\boldsymbol{g}$, in the GChSP is determined by solving

$$\boldsymbol{L}(\boldsymbol{p}) \leq \boldsymbol{\beta}. \tag{3}$$

The probability of lot acceptance, $\boldsymbol{L}(\boldsymbol{p})$ for the GChSP is given by

$$\boldsymbol{L}(\boldsymbol{p}) = (\mathbf{1} - \boldsymbol{p})^{g*r} + \boldsymbol{g} * \boldsymbol{r} * \boldsymbol{p}(\mathbf{1} - \boldsymbol{p})^{(g*r-1)}(\mathbf{1} - \boldsymbol{p})^{g*r*i}. \tag{4}$$

In Table 1, the number of minimum groups, $\boldsymbol{g}$ is presented satisfying equation (5) when $\boldsymbol{\beta} = \mathbf{0.25}, \mathbf{0.10}, \mathbf{0.05}, \mathbf{0.01}$; $\boldsymbol{a} = \mathbf{0.7}, \mathbf{0.8}, \mathbf{1.0}, \mathbf{1.2}, \mathbf{1.5}, \mathbf{2.0}$; $\boldsymbol{r} = \mathbf{2(1)5}$ and $\boldsymbol{i} = \mathbf{1(1)4}$. These design parameters are consistent with the previous literatures [6-9][13] for producing the GChSP using Gamma distribution. Once the total of minimum groups, $\boldsymbol{g}$ is calculated, the expectation of lot acceptance, $\boldsymbol{L}(\boldsymbol{p})$ can be obtained by using various values of design parameters. For a fixed $\boldsymbol{r}$ and $\boldsymbol{i}$, the probability of lot acceptance, $\boldsymbol{L}(\boldsymbol{p})$ as a function of the mean ratio, $\frac{\boldsymbol{\mu}}{\boldsymbol{\mu_0}}$ are shown in Table 2.

IV. RESULT

Table 1 represents the total of minimum groups, g for the GChSP for Gamma distribution at different values of design parameters.

Table. 1: Number of minimum groups for gamma distribution when$\boldsymbol{\gamma} = \mathbf{2}$.

			a					
β	r	i	0.7	0.8	1.0	1.2	1.5	2.0
0.01	2	1	5	4	3	2	2	1
	3	2	3	3	2	2	1	1
	4	3	3	2	2	1	1	1
	5	4	2	2	2	1	1	1
0.05	2	1	4	3	2	2	2	1
	3	2	2	2	2	1	1	1
	4	3	2	2	1	1	1	1
	5	4	2	1	1	1	1	1
0.10	2	1	3	2	2	2	1	1
	3	2	2	2	1	1	1	1
	4	3	2	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.25	2	1	2	2	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1

Based on Table 1, it shows that the number of minimum groups, g decreases as the consumer's risk, β, the specified constant, a, the number of products, r and the number of allowable preceding lots, i increase. For example, the number of minimum groups, g is 5 when the design parameters are $(\gamma, \beta, r, i, a) = (2, 0.01, 2, 1, 0.07)$. The number of minimum groups, g decreases to 1 when the design parameters are $(\gamma, \beta, r, i, a) = (2, 0.25, 5, 4, 2.0)$.

Table 2 shows the probability of lot acceptance, $L(p)$ when the number of products, r is 2 and the number of allowable preceding lots, i is 1 for different value of shape parameters for Gamma distribution. The expectation of lot acceptance, $L(p)$increases as the mean ratio, $\frac{\mu}{\mu_0}$ increases. For instance, the probability of lot acceptance, $L(p)$ is 0.0055 when the design parameters are $\left(\gamma, \beta, r, i, a, g, \frac{\mu}{\mu_0}\right) = (2, 0.01, 2, 1, 0.7, 5, 1)$. The probability of lot acceptance, $L(p)$ increases to 0.9946 when the design parameters are $\left(\gamma, \beta, r, i, a, g, \frac{\mu}{\mu_0}\right) = (2, 0.01, 2, 1, 0.7, 5, 12)$.

Table. 2: **Probability of lot acceptance for gamma distribution when γ=2**

			$\frac{\mu}{\mu_0}$						
β	g	a	1	2	4	6	8	10	12
0.01	5	0.7	0.0055	0.2462	0.7958	0.9387	0.9768	0.9895	0.9946
	4	0.8	0.0060	0.2465	0.7914	0.9364	0.9756	0.9889	0.9943
	3	1.0	0.0047	0.2129	0.7584	0.9224	0.9695	0.9859	0.9927
	2	1.2	0.0098	0.2685	0.7908	0.9334	0.9738	0.9878	0.9937
	2	1.5	0.0016	0.1266	0.6499	0.8713	0.9456	0.9738	0.9860
	1	2.0	0.0098	0.2443	0.7518	0.9130	0.9636	0.9824	0.9906
0.05	4	0.7	0.0163	0.3562	0.8551	0.9589	0.9848	0.9932	0.9965
	3	0.8	0.0233	0.3911	0.8667	0.9621	0.9859	0.9937	0.9968
	2	1.0	0.0315	0.4164	0.8713	0.9628	0.9860	0.9937	0.9967
	2	1.2	0.0098	0.2685	0.7908	0.9334	0.9738	0.9878	0.9937
	2	1.5	0.0016	0.1266	0.6499	0.8713	0.9456	0.9738	0.9860
	1	2.0	0.0098	0.2443	0.7518	0.9130	0.9636	0.9824	0.9906
0.10	3	0.7	0.0506	0.5070	0.9100	0.9759	0.9913	0.9961	0.9981
	2	0.8	0.0968	0.6011	0.9334	0.9824	0.9937	0.9972	0.9986
	2	1.0	0.0315	0.4164	0.8713	0.9628	0.9860	0.9937	0.9967
	2	1.2	0.0098	0.2685	0.7908	0.9334	0.9738	0.9878	0.9937
	1	1.5	0.0523	0.4647	0.8792	0.9636	0.9859	0.9935	0.9966
	1	2.0	0.0098	0.2443	0.7518	0.9130	0.9636	0.9824	0.9906
0.25	2	0.7	0.1642	0.6983	0.9564	0.9890	0.9961	0.9983	0.9991
	2	0.8	0.0968	0.6011	0.9334	0.9824	0.9937	0.9972	0.9986
	1	1.0	0.2443	0.7518	0.9636	0.9906	0.9966	0.9985	0.9992
	1	1.2	0.1357	0.6354	0.9361	0.9824	0.9935	0.9971	0.9985
	1	1.5	0.0523	0.4647	0.8792	0.9636	0.9859	0.9935	0.9966
	1	2.0	0.0098	0.2443	0.7518	0.9130	0.9636	0.9824	0.9906

Suppose that an experimenter is interested that the actual mean life of a product is at least 1000 hours. The inspection activity is designed such that it will be terminated at 700 hours with the consumer's risk of 0.05. Based on the consumer's risk values and the test termination time, the number of minimum groups, g is determined by using the GChSP based on truncated life test. Assume that the lifetime of a product follows Gamma distribution with 2 as the shape parameter. If the experimenter designs the inspection based on the number of products, $r = 2$ and the number of allowable preceding lots, $i = 1$, then the number of minimum groups, g is 4, as presented in Table 1. Therefore, the design parameters for the GChSP are $(a, r, i, g) = (0.7, 2, 1, 4)$. It means the designer has to choose a sample of size 8 from the submitted lot and put 2 products to each of the 4 groups. The lot is confirmed, likewise one defective is observed within 700 hours and no defective products are recognized in the 1 consequent sample. If the practitioner has the equal arrangement parameters, the expectation of lot acceptance, $L(p)$ expansion from 0.0163 to 0.9965 when the mean ratio, $\frac{\mu}{\mu_0}$ increases from 1 to 12, as shown in Table 2.

Performances of the GChSP and GSP are compared for Gamma distribution. For both sampling plans, the performances are measured based on the number of minimum groups, g and the probability of lot acceptance, $L(p)$.

Table 3 shows the number of minimum groups, g for the GChSP and the GSP for Gamma distribution when the value of shape parameter is 2.

Table. 3: **Number of minimum groups for gamma distribution for different sampling plans when γ=2.**

			GChSP	**GSP with $c = 1$**
β	r	a	g	
0.25	2	0.7	2	3
		0.8	2	3
		1	1	2
		1.2	1	2
		1.5	1	2
		2	1	1

Based on Table 3, it shows that the GChSP has lower number of minimum groups, g compared to the GSP. For instance, the GChSP requires 2 minimum groups meanwhile the GSP needs 3 minimum groups when the design parameters for both plans are $(\gamma, \beta, r, i, a) = (2, 0.25, 2, 1, 0.7)$.

The finding is crucial as the low number of minimum groups, g leads in reducing the cost and inspection time. Since the GChSP has lower number of minimum groups, g compared to the GSP, therefore the GChSP reduces the cost and inspection time.

Table 4 presents the probability of lot acceptance, $L(p)$ for the GChSP and the GSP for Gamma distribution when the design parameters for both plans are $(\beta, r, a) = (0.25, 2, 0.7)$.

Table. 4: Probability of lot acceptance for gamma distribution for two different sampling plans when γ=2

$\frac{\mu}{\mu_0}$	GChSP	GSP with $c = 1$
1	0.1642	0.2208
2	0.6983	0.7628
4	0.9564	0.9688
6	0.9890	0.9923
8	0.9961	0.9973
10	0.9983	0.9988
12	0.9991	0.9994

Based on Table 4, the probability of lot acceptance, $L(p)$ for the GChSP is always lower compared to the GSP. For example, the expectation of lot acceptance, $L(p)$ is 0.1642 for the GChSPwhile, 0.2208 for the GSP when the design parameters for both plans are $(\beta, r, a) = (0.25, 2, 0.7)$. The finding is common as the article focuses on minimizing the consumer's risk, β, therefore it is expected that the GChSP has lower probability of lot acceptance, $L(p)$ compared to the GSP. The OC curves for both sampling plans are clearly illustrated in Figure 1.

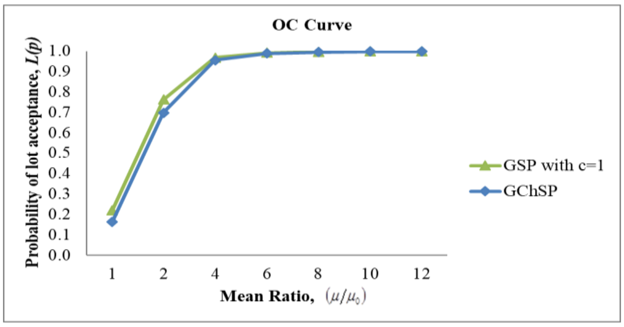

Fig. 1: Probability of lot acceptance versus mean ratio for Gamma distribution for two different sampling plans when γ=

Based on Fig.1, it shows that the probability of lot acceptance, L(p) for the GChSP is lower compared to the GSP for all values of the mean ratio, $\frac{\mu}{\mu_0}$. The finding is significant as the low probability of lot acceptance, L(p) reduces the risk for consumers to receive a defective product. Therefore, consumer would prefer the GChSP as the plan will benefit them in term of reducing the risk for them to have a defective product, compared to the GSP.

V. CONCLUSION

This article develops GChSP for Gamma distribution at different design parameters. The design parameters used in this article are consumer's risk, β, specified constant, a, number of products, r, number of allowable preceding lots, i, mean ratio, $\frac{\mu}{\mu_0}$ and shape parameter, γ.

The performances of the GChSP is measured based on two criteria. The first criterion is the number of minimum groups, g and the second criterion is the probability of lot acceptance, $L(p)$. For the number of minimum groups, g, it decreases as the consumer's risk, β, the specified constant, a, the number of products, r and the number of allowable preceding lots, i increase. For the probability of lot acceptance, $L(p)$, it increases as the mean ratio, $\frac{\mu}{\mu_0}$ increases.

The performances of the GChSP are then compared with the GSP. It has been shown that the GChSP has better performances compared to the GSP. By saying that, the GChSP has lower number of minimum groups, g and probability of lot acceptance, $L(p)$, compared to the GSP.

However, this article only develops the GChSP for Gamma distribution. The GChSP can be further developed by using other distributions since there are many distributions in the literature. Besides that, it can be further studied by using different quality parameter as this article focuses only at the mean of a product.

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