Model of Optimal Funds Allocation with Due Consideration for Risk Minimization

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Abstract: An attempt to study the role of risk management from the standpoint of an entrepreneur is made in the article. The main goal of this article is to find the key patterns describing the features of risk assessment in business as the main element contributing to achieving the economic safety for an organization and to developing a mathematical model of optimal funds allocation for risk management using the dynamic programming techniques. The methods of cognition, retrospective and documentary analysis, as well as synthesis, generalization, and systematization were used in this article. Various risk management methods are used in the modern economic analysis. Diversification, or risk sharing among several business agents, is the most efficient way to reduce risk in the context of instability of the economic and political situation in Russia.

Index Terms: model, optimal allocation, risk, risk management.

I. INTRODUCTION

Risk management is always a relevant task in any sector of the economy. Any entrepreneur wants to run their business with the least possible losses and the biggest possible profit. One should make use of scientific advances in risk management and experience of those who have successfully implemented the risk management system in the enterprise to succeed in this desire.

Each production has its own specific risks, both in the economy and any other field related to its activities [1-4]. If this aspect of enterprise management is neglected, it can result in an increase in the frequency of incidents and accidents, as well as damage caused by unforeseen situations. However, different components of production differently contribute to the total level of risk, depend on each other to a varying degree, and have different structural links. An important economic task is to rationally allocate funds among the elements of the system that differently contribute to the volume of production with different levels of risk, different sizes of expected damage, etc.

II. METHODS OF RISK MANAGEMENT

Risk is an integral part of both people’s lives and the operation of business agents. Everyone understands today that risks in activities cannot be ignored. No entrepreneurship can exist without risks. If the company understands them, it can build an optimal strategy for further development and find a way out of adverse situations; often an entrepreneur also can insure the significant ones [5, p. 25].

Risk management involves several areas: risk transfer, preservation, and reduction. An example of risk transfer is insurance, where all or part of the risk is transferred to an insurer for a certain insurance premium. At the same time, the risk of the insurer itself is not reduced. The insurer may count on the compensation for a part of the damage if an insurance event occurs. The persistence of risk can be demonstrated by the example of creating a reserve fund intended to cover losses from an incident or accident. This measure has no impact on the probability of occurrence of an emergency situation and damage from it. Risk can be reduced through preventive measures, such as installation of alerting and diagnostics systems, maintenance of the current state, staff training and retraining, modernization of fixed assets and replacement of obsolete equipment with more reliable items, etc. Any area of risk management requires certain resources, which raises the problem of optimal funds allocation among different methods of managing risks and objects included in the system, whose risks are taken into account [6-9]. Different level of damage caused by the elements of the system, as well as different efficiency of investing in risk management should be taken into account when allocating funds.

III. MODEL OF FUNDS ALLOCATION OPTIMIZATION

Any enterprise can be treated as a system where its parts interact: workshops, departments, services, etc. At the same time, the enterprise itself can be an element of a system and connected with other enterprises and organizations through a production process [10-13]. The total risks of the system depend on each of its elements and the connection nature. Due to this, the problem of managing risks of each individual element should be considered with due regard to the entire system [14, 15].

Let us consider a system with a center and m agents. Let the funds for risk management in the amount of \( F \) be allocated by the center among the agents. In this case, agents independently take risk management measures using...
funds $f$ allocated to them by the center [16, p. 62].

The system can be represented by an enterprise or an organization with separate elements in its structure.

The developed mathematical model for optimizing the allocation of risk management costs can be applied both for a sole enterprise and for a regional level, as well as for industrial groups. In this case, the degree of interdependence of the agents in the system and their influence on each other have no impact on the course of solving this problem.

As such, the center allocates funds for risk management in the amount of $F$ among $m$ agents in the system. At the same time, let us assume that the more funds are spent on risk management, the less damage there is from unforeseen situations. In other words, if the funds in the amount of $f$ are spent to manage the risks of the $i$-th agent, then it may have damage $X(f)$, where $i = 1, \ldots, m$. (Figure 1).

At this $\forall f_1, f_2, f_1 < f_2$ and the following condition is met: $X(f_1) > X(f_2)$ for $\forall i = 1, m$.

![Fig. 1. Chart of the funds allocation for risk management in the system](image)

This requires to choose the optimal funds allocation for risk management among agents that ensures minimal damage $Y$ to the entire system, where

$$Y = \sum_{i=1}^{m} X_i(f).$$

The damage is measured in monetary units and depends on the likelihood of an unfavorable situation occurrence and the peculiarities of an agent.

The gain in this problem is the total damage $Y$ caused by $m$ agents or the entire system.

The peculiarity of determining the gain in this problem must be noted. Profit or income from some activity are usually taken as gain in such tasks [17-20]. In this case, the goal of the funds allocation for risk management is to reduce damage. This can explain the choice of the objective function. Damage $Y$ from the production risks implementation should be the least with an optimal funds allocation $F$ for risk management. The production (industrial) risk is understood as the likelihood of an event occurrence that would entail a decrease in the enterprise income as a result of the production disruption [21, p. 12680], [22, p. 12658].

Let us build a mathematical model of this problem by taking the following steps:

1. Defining the number of steps. Number of steps $m$ is equal to the number of agents among which the funds for risk management $F$ are allocated.

2. Defining the system states. The system state at each step is determined by the amount of funds $\varphi$ available before this step $\varphi \leq F$.

3. Choosing stepsize controls. The control at the $i$-th step $f_i$, $i = 1, m$, is the amount of funds allocated for managing the risks of the $i$-th agent.

$$X(f)$$ (1)

4. The gain function at the $i$-th step

This is the damage caused by the $i$-th agent when investing its $f$ of funds in safety.

Let $Y = \sum_{i=1}^{m} X_i(f)$.

5. Definition of the function of transition to a new state

$$w_i(\varphi, f) = \varphi - f$$

If the system was in the $\varphi$ state at the $i$-th step and control $f$ was selected, then the system will be in the $\varphi - f$ state at the next $(i+1)$-th step. In other words, if $\varphi$ of funds is available and $f$ of funds is invested in improving the safety of the $i$-th agent, then $\varphi - f$ of funds remains for the further risk management for the remaining agents.

6. Combination of a functional equation for $i = m$:

$$Y_m(f) = X_m(f), f_m(\varphi) = \varphi$$ (3)

At the final step, before investing funds in reducing the risk of the last $m$-th agent, the conditional optimal control corresponds to the amount of available funds; this means that all the remaining funds should be invested in the last agent. The conditional optimal gain is equal to the damage caused by the activity of the last $m$-th agent.

7. Combination of the basic functional equation.

The following functional equation is obtained using the above relations:

$$Y_i(\varphi) = \min_{f \leq \varphi} \{X_i(f) + Y_{i+1}(\varphi - f)\}$$ (4)

Let us explain this equation. Suppose $\varphi$ of funds remained for further risk management before the $i$-th step. Then the center can spend $f$ of funds on managing the risk of the $i$-th agent, while the damage from the activities of the $i$-th agent will amount to $X(f)$, and the remaining $(\varphi - f)$ of funds should be invested in the safety of the other agents from the $(i+1)$-th to the $m$-th. The conditional optimal gain from such funds allocation is damage $Y_{i+1}(\varphi - f)$ that agents from the $(i+1)$-th to the $m$-th will cause. The conditional control $f$ is optimal if the sum of $X(f)$ and $Y_{i+1}(\varphi - f)$ is minimal. In other words, the funds should be distributed at the $i$-th step so as to minimize the damage from the activities of the $i$-th agent and all subsequent agents up to the $m$-th.

The created model allows to solve the problem of funds allocation for risk management within the system. Building a mathematical model using dynamic programming techniques allows to explore various options of the funds allocation among agents and identify the best way of the funds allocation, which will minimize the overall risk [23-25].

**Example 1.** Suppose 5,000 conventional units were allocated for the risk management at a company with three workshops. That is, $F = 5,000$, $m = 3$. For the sake of simplicity, it can be assumed that only sums that are multiples of 1,000 conventional units are allocated. Values $X_i(f)$, $i = 1, 2, 3$, are listed in Table 1.

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<tr>
<th>$f$, thous. conventional units</th>
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<th>$X_i(f)$, thous. conventional units</th>
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As such, it can be stated that the developed model of optimal funds allocation for risk management in the "center – agents" system allows not just to reduce damages from unforeseen situations, but also to gain such benefits as risk reduction, insurance discounts, increasing environmental sustainability, and improving the reliability of the system in general and agents in particular.

**REFERENCES**


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<th>Table 2. Result of solving Example 1</th>
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<td>$\phi$</td>
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<td>$f_1(\phi)$</td>
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<th>Table 3. Funds allocation options</th>
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For $f_1 > f_2 X(f_1) < X(f_2)$, $i = 1, 3$.

Let us perform conditional optimization and gradually fill out Table 2 following its results.

Below is the description for one of the options. The cells of Table 2 corresponding to the explored results of unconditional optimization are highlighted in gray. $\phi = F = 5$ is at the beginning of the funds allocation. The optimal option for the given initial conditions is to spend $f_1(5) = 3$ thous. conventional units of funds on risk management of the first subdivision. In this case, the company management has 2 thous. conventional units of funds before the second step corresponding to the determination of funds for risk management of the second subdivision. Based on the above conditional optimization, it is clear that the costs of managing the risk of the second object are inexpedient, and the remaining 2 thous. conventional units of funds are allocated for the risk management of the third subdivision (Table 3).

**IV. CONCLUSION**

Application of the developed mathematical model of the optimal funds allocation for risk management will allow to reduce damage from unforeseen situations by 10 – 13%. The profit of the system will grow accordingly. Aside from the direct damage reduction through the optimal funds allocation for risk management, agents and the entire system gain some advantages. Reducing damage to the system indicates the reduction of risk for the system and agents. This fact allows to reduce insurance costs – both compulsory (compulsory insurance of civil liability of the owner of a hazardous object) and voluntary – by reducing the insurance premium, its size depending on the risk level of the insurer. Reduction in the risk level of the system and agents indicates their sustainability and reliability, which positively influences business relationships with partners.