Investigating Newman’s Error
In Integral Calculus

Nurul Shida, Norulhuda, Nurul Aini, Sharifah, Abdul Halim

Abstract: The main objective is to investigate the types of
difficulties and errors faced by polytechnic students when
solving integral calculus problem in Engineering Mathematics.
The types of error selected in this study were motivation,
understanding, transformation, process skills, coding, and
negligence. The study respondents consisted of 128 Semester 2
students from Diploma in Electrical Engineering and Diploma
in Mechanical Engineering at Polytechnic Ibrahim Sultan. Data
analysis revealed that mostly students made errors in
understanding and processing skills. The results verified that
there were serious difficulties among polytechnic students in
learning integral calculus through problem solving. The input
from this study is expected to be a guideline for additional
mathematics lectures, especially in planning and implementing
better teaching so that mistakes made by students can be
minimised.

Index Terms: Newman, Integral Calculus, error, polytechnic.

I. INTRODUCTION

Most misconceptions in mathematics are like
challenges to students [9], make error in understanding the
importance of a problem [10]. Weakness in understanding
the basic concepts may lead them to use the wrong strategy
when solving mathematical questions [5].

Transformation error means that students do not
understand well the formulas that are used to solve problem;
students are unable to determine which formula to use for
solving a problem; students cannot appropriately determine
a mathematical operation or set of operations to solve problems in a question; or students can't recognize the
operation or a set of operations [7]. It also means that
students cannot plan a solution to solve the problem; students
cannot determine which formula; students do not have many
exercises; or students cannot determine the mathematical
operations to be used. Transformation errors happen at every
number. This shows that students do not understand formulas well. It is because students still memorise the
formulas without understanding them. Error in transforming
a word problem into a proper mathematical problem [10] will
indicate that students have problems in computations and
applying the formula [2], or students forget a formula that
utilized or technique on what to do [3]. Therefore, the
students cannot select the appropriate mathematical
operations or procedures [6]. The transformation error
happens because the student can’t choose the formula or plan
an answer to solve the problem [8]. Process skills occur when
students are not aware of making mistakes in a summation
operation and cannot operate to simplify fractions correctly
[7], and thus causing errors in performing mathematical
procedures [10]. In the meantime, students also encounter
challenges in replacing the positive and negative signs,
resulting in errors when the formula is used [2]. Process skill
errors mean that students do not understand the
mathematical operation concept well [7]. They are not aware
of making mistakes in the summation operation. Students
cannot simplify fractions correctly [7] or accurately perform
the mathematical calculation or the procedure (Execution of
mathematical processing) [6]. Unfinished answer is when
students use a correct formula or procedure, but they do not
finish it. Coding error is to represent the mathematical
solution into an acceptable written form [4] but the students
are unable to effectively interpret and validate the
mathematical solution in terms of the real world problem.
This error is reflected by an impossible or not reasonable
answer [10]. Encoding error means that students cannot give
an appropriate conclusion because there is an error in the
calculation result [7], since the students cannot answer
according to the question [3]. Students’ negligence must get
consideration because isn't tended to instantly, the error will
affect the next math problem. Negligence happens in
transcribing information from the question [9]. Carelessness
error means that students are not careful when they calculate
but can correct it immediately without guidance from the
researcher [7]. Students are not careful in doing the
calculations, and do not check before the exam answers are
collected. Students do not read carefully to what is given [7].
Errors are caused by students’ carelessness in [3].

II. LITERATURE REVIEW

For negligence error, students were needed to
discover the cause of their errors. Therefore, it needed to
understand calculation operation procedures again and do
more exercises. Next, students should not disregard basic
mathematical skills that include addition, subtraction,
multiplication, and division [2]. Mastery of these skills is
deficient if the students were not able to show the
mathematical steps in a neat and efficient way [9].
As for coding error, it involved the interpretation of mathematical solution in connection to the original problem situation, validating interpreted mathematical solution by checking whether this is appropriate and reasonable for its purpose [10]. This implies that teachers are proposed to use scaffolding to overcome errors so that then errors don’t occur once more [8]. Therefore, transformation errors can be avoided if respondents understand the concepts and mathematical sentences [1]. Transformation error occurs in a solution involving a mathematical problem, that is, the respondents face problems in translating the mathematical problems from the word form and description into the mathematical form. If the students cannot interpret the question to the appropriate form, the students may choose the wrong strategy and operation to resolve the problem. Most respondents found difficulty in concluding the mathematical sentences because they might not be exposed to the connection between their existing experiences with the concept to be taught and were not given the opportunity to think and learn the common way or strategy in problem solving.

III. METHODOLOGY

Selected students were given final exam questions at the end of the 2017/2018 Semester, which was conducted in April 2018. The final exam questions in integral calculus had five questions. Question 1 had 3 sub-questions and Question 2 had 2 sub-questions. Various forms of question were given, and they involved the simplest, medium and hardest questions. Question 1 and Question 2 were direct questions. These questions should be easy for respondents to answer because they were given through simple sentences. While Question 3, Question 4 and Question 5 were questions in the form of problem solving. Respondents should think first before resolving the problems or questions. The problems can indicate students’ difficulties by using Newman’s error categories. The research procedures included three steps: (1) Sample selection from students who were enrolled in the Engineering Mathematics 2 course, (2) Instrument used, which is the integral calculus problem solving question in the Final Examination Session for December 2017/2018; and the (3) Findings from the examination result and the level of student miscalculation according to Newman’s fault analysis procedure were identified. Figure 1 illustrates the research procedure in the students’ written solutions.

IV. RESULTS

The findings were based on the study purpose, which was to analyse the difficulties and types of error in integral calculus. It involved quantitative and qualitative findings. All answer papers were examined by the lecturer for each class and analysed by the researcher. Quantitative results were descriptively given to indicate the mean of students’ marks and rate of students who could answer the problems. This could indicate whether they had difficulties or not. The first incorrect work done by the students in their solution will be assessed for the type of error. The mistakes made will be classified based on the Type Classification Guide from the Newman Error Hierarchy Model. Therefore, the types of student error in this study were classified either from the type of motivation, understanding, transformation, process skills, coding, or negligence. The types of errors for each item will be analysed by frequency and percentage. All data was obtained in the analysis by using the SPSS 22. The students’ marks were calculated and analysed by questions. The mean of their marks was calculated to find out how much they could solve the problem. The quantitative results of this study were the mean of students’ marks and frequencies. The mean marks were calculated to identify their performance and rate students who could answer the problems. Then students’ responses to the problems were qualitatively analysed to identify their difficulties. The aim was to identify the difficulties and determine the type of student problem solving errors in integral calculus. In analysing the students’ mistakes in solving mathematical problems, models made by Newman (1977) were used. The Newman Error Hierarchy Model will divide the types of error according to the problem solving stage performed by someone who solved the mathematical problems. The types of student error in this study were classified either from the types of motivation, reading, understanding, transformation, skills process, coding, or negligence. Guidance for each type of error is presented in Table 1.

<table>
<thead>
<tr>
<th>Type of Error</th>
<th>Guidance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Motivation</td>
</tr>
<tr>
<td>II</td>
<td>Understanding</td>
</tr>
<tr>
<td>III</td>
<td>Transformation</td>
</tr>
<tr>
<td>IV</td>
<td>Process skills</td>
</tr>
<tr>
<td>V</td>
<td>Coding</td>
</tr>
<tr>
<td>VI</td>
<td>Negligence</td>
</tr>
</tbody>
</table>

Table2 shows the number of students who gave correct and complete answers to the problems. The result indicated that students had difficulties in learning integral calculus. The data below indicated that students could not answer Question 4 and Questions 5, which were in the problem solving form. Moreover, the percentage of students who gave correct and complete answers showed that most students were unable to answer the problems.
Therefore, the students had weakness in responding to the problems.

Table 2 Quantitative results of students’ responses to problem solving

<table>
<thead>
<tr>
<th>Item</th>
<th>Number of students who give the right answer</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (i)</td>
<td>63</td>
<td>49.2</td>
</tr>
<tr>
<td>1 (ii)</td>
<td>2</td>
<td>1.6</td>
</tr>
<tr>
<td>1 (iii)</td>
<td>6</td>
<td>4.7</td>
</tr>
<tr>
<td>2 (i)</td>
<td>10</td>
<td>7.8</td>
</tr>
<tr>
<td>2 (ii)</td>
<td>3</td>
<td>2.3</td>
</tr>
<tr>
<td>3.</td>
<td>11</td>
<td>8.6</td>
</tr>
<tr>
<td>4.</td>
<td>0</td>
<td>0.0</td>
</tr>
<tr>
<td>5.</td>
<td>0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 3 presents the most common type of mistakes made by respondents in regard to understanding and process skills. The findings showed that the highest percentage of respondents who made a process skill error was 57.8%, which involved Question 1 (ii). Process skill error occurred because respondents failed to finish the phrase \( \int dx \) that was formed. Some students could understand the given questions but failed to incorporate them into the first step of solution to form expressions \( \int dx \) or \( \int dy \), failed to select the appropriate mathematical operation, and complete the expression perfectly.

Table 3 Descriptive analysis of students’ tendency levels to make errors according to types.

<table>
<thead>
<tr>
<th>Item No.</th>
<th>Type of Error</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (i)</td>
<td>6 (4.7%)</td>
<td>30 (23.4%)</td>
<td>29 (22.7%)</td>
<td>41 (32.0%)</td>
<td>6 (4.7%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (ii)</td>
<td>5 (3.9%)</td>
<td>61 (47.2%)</td>
<td>74 (57.8%)</td>
<td>75 (58.6%)</td>
<td>71 (55.5%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (iii)</td>
<td>8 (6.3%)</td>
<td>65 (50.8%)</td>
<td>67 (52.3%)</td>
<td>53 (41.4%)</td>
<td>47 (36.7%)</td>
<td>72 (56.3%)</td>
<td></td>
</tr>
<tr>
<td>2 (i)</td>
<td>10 (7.8%)</td>
<td>50 (39.1%)</td>
<td>49 (38.3%)</td>
<td>49 (38.3%)</td>
<td>25 (19.5%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 (ii)</td>
<td>7 (5.5%)</td>
<td>69 (53.9%)</td>
<td>69 (53.9%)</td>
<td>64 (50.0%)</td>
<td>14 (10.9%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>2 (1.6%)</td>
<td>10 (7.8%)</td>
<td>10 (7.8%)</td>
<td>6 (4.7%)</td>
<td>3 (2.3%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>5 (3.9%)</td>
<td>25 (19.5%)</td>
<td>21 (16.4%)</td>
<td>19 (14.8%)</td>
<td>15 (11.7%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>3 (2.3%)</td>
<td>19 (14.8%)</td>
<td>21 (16.4%)</td>
<td>23 (18.0%)</td>
<td>19 (14.8%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Understanding error may occur in all questions given in the final examination. This matter happened because students still did not understand how to solve the questions. This understanding error occurred because the students have read or seen the given questions, but they did not understand the terminology, specific terms, or the whole meaning of the verse. Although the given questions were very clear and easy to understand, they required understanding of the formulas and memorization to solve them. The third highest error analysis was the coding error, which involved all questions. The reason was that students could not express things in writing by using the correct words or proper symbols in solving problems. Moreover, students did not use the formula correctly and were still confused with the topic formula. This was because the respondents could not remember the formula or ways to solve the problem. Coding error can occur when a mathematical solution is represented in an acceptable written form. The students could not give an appropriate conclusion because there was an error in the calculation result. The students could not answer according to the question asked. Also, the students could not represent the answer appropriately. Moreover, some students could not represent their results from the mathematical processing, while negligence error will occupy the fourth place. It occurred because there were some students who made mistakes in algebraic expansion and put positive and negative signs when calculating the replacement value for the final answer. There were also careless students who wrote numbers after reading the questions. Question 1 (iii) showed that 56.3% of students committed this offense.

Transformation error occurred because the students were unable to convert a problem which is in a sentence form into a symbol or a mathematical language. This made the students unable to identify the operations or strategies needed to solve a problem. Transformation error existed for Question 1 (ii), 1 (iii), 2 (ii), and Question 2 (iii). These questions required the students’ skills to create questions from the square root, by using power, part operation, and the basic concept of integration to solve the question correctly. This indicated that the difficulty level of the question had also influenced the students’ ability to correctly interpret the question to the solution form. Motivational errors occurred because respondents did not have the confidence to answer the given questions. The immediate respondents did not try to solve the given questions. Respondents could not answer because they did not understand the questions. Respondents read the given questions but could not understand the symbols. Therefore, respondents could not answer most questions that involved problem solving. Process skill error means that students do not understand well the mathematical operation concept. Process skill errors occur when students were not aware of making mistakes in summation operation because students could not simplify fractions correctly; hence, error in performing mathematical procedures [10]. The student could not perform the mathematical calculation or the procedure accurately, and students had problems in formula computation and application. Furthermore, the students couldn’t choose the appropriate mathematical operations or procedures.

V. DISCUSSION

Students did not understand well the formulas that were used to solve problems. Some students were not able to determine which formula that should be used to solve the problems and determine a mathematical operation or set of operations to solve problems in the question appropriately.
Moreover, they were not able to identify the operation or a set of operations. Students could not plan the solutions to solve problems. Students could not determine the mathematical operations to be used. Transformation errors happened at every number. This showed that students did not understand formulas well. It was because the students were still memorising formulas without understanding them. The results verified that there are serious difficulties among polytechnic students in learning integral calculus through problem solving. From the analysis, several reasons at can be used to identify which student has each type of error for each problem and sometimes has two or three simultaneous errors in one problem. In addition, students had the most difficulties in answering problem solving questions. The reasons for difficulties in integral calculus problem solving is because students did not pay attention in problem solving framework steps by using prior knowledge in order to solve unfamiliar problems. Moreover, students were still confused with the previous topic, which is differentiation. Analysing the types of mistake made by students in solving these mathematical problems could provide information on the disadvantages and weaknesses of the respondents’ understanding in a topic, especially the difficult topics and trying to find solutions. Lecturers can devise a special or enhancement programme to help students to their level. If this weakness is not overcome, it is feared that the students will cultivate phobia, be lonely, bored, not happy, easy to give up, and then generally fail in the mathematics examination, especially in integral calculus. According to the results of the analysis, students had not used the problem solving framework and lack of critical thinking elements while finding a solution. Students were not motivated to continue solving a problem. Students mostly lack in transformation and process skills. Students were also unable to code the problems properly. A majority of students could not consider critical thinking elements, which were identification and interpretation, information analysis, evaluation evidence, and argument. Furthermore, students lacked in the problem solving framework, which involved holistic understanding, identifying the method, applying the method, and checking accuracy.

VI. CONCLUSION

Lecturers need to emphasise on the understanding of terms and symbols in teaching so as not to violate mathematical terminology. Some lecturers often take the effortless way by considering this trivial and wasted past ignorance and continue to emphasise formula memorisation and only memorise the solving method. The input from this study is expected to be a guideline for additional mathematics lectures, especially in planning and implementing better teaching so that mistakes made by students can be minimised.

REFERENCES


AUTHORS PROFILE

Nurul Shida
Lecturer Polytechnic Ibrahim Sultan
Master of Education Mathematics UTM

Dr. Norullhuda Ismail
PENSYARAH KANAN (DSS1)
Fakulti Sains Sosial Dan Kemanusiaan
Sekolah Pendidikan
Fakulti Pendidikan Jabatan Pendidikan Sains, Matematik Dan Multimedia Kreatif
p-norullhuda@utm.my

Nurul Aini Mohd Ahyan
PENSYARAH KANAN (DSS1)
Fakulti Sains Sosial Dan Kemanusiaan
Sekolah Pendidikan
Sekolah Pendidikan Fakulti Sains Sosial Dan Kemanusiaan Universiti Teknologi Malaysia

Dr. Sharifah binti Osman
B.Sc. (Hons) in Chemical (UKM)
Dip.Ed. in Chemistry (MPBP)
Ph.D in Engineering Education (UTM)

Dr. Abdul Halim bin Abdullah
B.Sc. Comp.Ed. (Mathematics) (UTM)
M.Ed. (Mathematics Education) (UTM)
Ph.D (Mathematics Education)(UKM)
E-mail : p-halim@utm.my

Published By:
Blue Eyes Intelligence Engineering & Sciences Publication

Retrieval Number: F3417048619/190BEJESP

1117