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Shortest Path Problems Using fuzzy Weighted Arc Length

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***Abstract*: We have presented a novel method for solving the fuzzy shortest path problem (FSPP) considering Type-I and Type-II weighted trapezoidal fuzzy number (WTpFN) and weighted triangular fuzzy number (WTrFN) and also can predict the crisp shortest path (CSP) length. Additionally, the method is compared with some existing methods. Finally, we have given some numerical experiments to show the effectiveness of the proposed model. Numerical and graphical results show that the new technique is superior to the current methods.**

***Index Terms*: Weighted trapezoidal fuzzy numbers, fuzzy linear programming, ranking method, fuzzy shortest path problem.**

I. INTRODUCTION

There are many situations in real life where we need to find the shortest path (SP) from origin to destination. Traditionally, fundamental graph theory can solve SPP when the parameters (such as time, cost, distance and so on) are a crisp number. For example, Dijkstra's algorithm [1] where the weighted graph is used to find the SP. Bellman-Ford algorithm [2] used the single-source problem to find the SP when edge weight is negative. Floyd–Warshall algorithm [2] solved all SPs in pair.but there are many situations in real life where we have to face with uncertain parameters such as natural calamities (flood, earthquake, heavy rainfall, low visibility during winter) or social disasters such as strike, roadblock or bad human health which may act as hurdle between the origin to destination.To remove these uncertainties from the above mentioned cases, we utilise Zadeh's fuzzy [3] principle. When the SP problem is solved under the fuzzy environment, then such type of problem is known as FSPP. During the last few decades, the topic of FSPP has achieved substantial popularity among researchers because of its widespread applications in different branches of communication, scheduling [4], railway network [5], the broadcast problem [6] and so on.

Dubois and Prade [7] were the first to analyse the FSPP by using the FMB algorithms. However, the primary drawback of their method is that the path derived by the algorithm may not exist in the network. Li et al. [8], used ANN, Zhang et al. [9], applied FPA, Hassanzadeh et al. [10], proposed GA for solving the Type-II weighted FSPP. Kaufmann and Gupta [11], studied the fuzzy arithmetic theory and application which helps us to deal with the arithmetic problems in real life situations. Chuang and Kung [12], solved the FSP length and the corresponding FSPP in a network. Furthermore, numerous methodologies were proposed for FSPP considering order relations between fuzzy numbers Jain [13], was first to introduce the fuzzy ranking index to compare two different fuzzy numbers. Since then, several researchers [14] [15] [16] proposed the different approaches to compare two different fuzzy numbers. FSPP is one of the applications where we used the ranking index to find the SP are available in references [17], [18], [19], [20], [21], [22] [23], [24], [25].

Zimmermann [26] was the first to introduce fuzzy linear programming (FLP) which has been used in our proposed method. Some of the recent works on FLP problem are available in references [27], [28], [29] where different researchers have applied FLP to solve various problems. Fuzzy linear shortest path programming (FLSPP) is one of the applications of FLP. There have been some of significant research works in the area of FSPP with Type-II weighted FAL which are available in references [30], [31]. Fuzzy set theory is well known technique to deal with uncertainty in optimization problem. SPP with fuzzy number etc. are described by few researchers. The main contributions of this paper as follows.

- To the best of our information, there is no algorithmic approach based linear programming model in literature for SPP with Type-I WTpFN and WTrFN.
- Comparison with existing models to prove that the proposed model is performing better on the basis of graphical, logical and numerical evidence.
- The proposed model has the capability to solve the problems which have been already solved by existing models. Moreover, the proposed model can also solve the new set of problems which has not been investigated in any research articles till date.

II. PRELIMINARIES

We present some basic definitions and the arithmetic operations on WTpFN

Definition 2.1: [3]. Let X be a non-empty set of elements x, A fuzzy set F in X is a set of pairs $(x, \mu_F(x))$ for $x \in X$ that $\mu_F(x): X \rightarrow [0,1]$

Definition 2.2: [16]. Let A GnTpFN is

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$\tilde{A}^p = \left(\tilde{a}_1^p, \tilde{a}_2^p, \tilde{a}_3^p, \tilde{a}_4^p : \omega_L^p, \omega_R^p\right)$ where $\tilde{a}_1^p, \tilde{a}_2^p, \tilde{a}_3^p, \tilde{a}_4^p$ real numbers with $\tilde{a}_1^p \le \tilde{a}_2^p \le \tilde{a}_3^p \le \tilde{a}_4^p$, ω_L^p, ω_R^p the left height and the right height of $\tilde{A}^p$ and the membership function (MF):

$$\mu_{\tilde{A}^p}(x) = \begin{cases} 0; & for\ x \prec \tilde{a}_1^p\ or\ x \succ \tilde{a}_4^p \\ \omega_L^p \cdot \dfrac{x-\tilde{a}_1^p}{\tilde{a}_2^p-\tilde{a}_1^p}; & for\ \tilde{a}_1^p \le x \prec \tilde{a}_2^p \\ \omega_L^p + \left(\omega_R^p - \omega_L^p\right)\cdot \dfrac{x-\tilde{a}_2^p}{\tilde{a}_3^p-\tilde{a}_2^p}; & for\ \tilde{a}_2^p \le x \le \tilde{a}_3^p \\ \omega_R^p \cdot \dfrac{x-\tilde{a}_4^p}{\tilde{a}_3^p-\tilde{a}_4^p}; & for\ \tilde{a}_3^p \prec x \le \tilde{a}_4^p \end{cases} \quad (1)$$

Where $0 \prec \omega_L^p \le 1$ and $0 \prec \omega_R^p \le 1$ If $\omega_L^p = \omega_R^p$ then $\tilde{A}^p$ become $\tilde{A}^p = \left(\tilde{a}_1^p, \tilde{a}_2^p, \tilde{a}_3^p, \tilde{a}_4^p : \omega\right)$ also known as WTpFN.

Case I: when ω lies within zero to less than one,i.e., $0 \prec \omega \prec 1$ then this type of problem known as Type-I WTpFN and the general form of Type-I WTpFN (T1WTpFN) is $\tilde{A}^p = \left(\tilde{a}_1^p, \tilde{a}_2^p, \tilde{a}_3^p, \tilde{a}_4^p : \omega\right)$.

Case II: when $\omega = 1$ then this type of problem known as Type-II WTpFN (T2WTpFN), i.e., $\tilde{A}^p = \left(\tilde{a}_1^p, \tilde{a}_2^p, \tilde{a}_3^p, \tilde{a}_4^p : 1\right)$ or $\tilde{A}^p = \left(\tilde{a}_1^p, \tilde{a}_2^p, \tilde{a}_3^p, \tilde{a}_4^p\right)$

Definition 2.3: The WTpFN $\tilde{J}^r = \left(\tilde{j}_l^r, \tilde{j}_i^r, \tilde{j}_k^r, \tilde{j}_e^r : \omega\right)$ transformed into WTrFN $\tilde{J}^{tr} = \left(\tilde{j}_l^r, \tilde{j}_i^r, \tilde{j}_e^r : \omega\right)$ iff $\tilde{j}_k^r = \tilde{j}_e^r$

Definition 2.4: [16] : Let $\tilde{J}^r = \left(\tilde{j}_l^r, \tilde{j}_i^r, \tilde{j}_k^r, \tilde{j}_e^r : \omega_L^r, \omega_R^r\right)$, and $\tilde{M}_l^s = \left(\tilde{m}_l^s, \tilde{m}_i^s, \tilde{m}_k^s, \tilde{m}_e^s : \omega_L^s, \omega_R^s\right)$ are two GnTpFN where $\tilde{j}_l^r \le \tilde{j}_i^r \le \tilde{j}_k^r \le \tilde{j}_e^r$, and $\tilde{m}_l^s \le \tilde{m}_i^s \le \tilde{m}_k^s \le \tilde{m}_e^s$, then we have:

$$\tilde{J}^r \mp \tilde{M}_l^s = \begin{pmatrix} \tilde{j}_l^r \mp \tilde{m}_l^s, \tilde{j}_i^r \mp \tilde{m}_i^s, \tilde{j}_k^r \mp \tilde{m}_k^s, \tilde{j}_e^r \mp \tilde{m}_e^s : \\ \min\left\langle \omega_L^r, \omega_L^s \right\rangle, \min\left\langle \omega_L^r, \omega_L^s \right\rangle \end{pmatrix}$$

$$\tilde{J}^r \otimes \tilde{M}_l^s = \begin{pmatrix} \tilde{j}_l^r \times \tilde{m}_l^s, \tilde{j}_i^r \times \tilde{m}_i^s, \tilde{j}_k^r \times \tilde{m}_k^s, \tilde{j}_e^r \times \tilde{m}_e^s : \\ \min\left\langle \omega_L^r, \omega_L^s \right\rangle, \min\left\langle \omega_L^r, \omega_L^s \right\rangle \end{pmatrix}$$

$$k \cdot \tilde{J}^r = \left(k\tilde{j}_l^r, k\tilde{j}_i^r, k\tilde{j}_k^r, k\tilde{j}_e^r : \omega_L^r, \omega_R^r\right)$$

Definition 2.5: [16] : Let $\tilde{J}^r = \left(\tilde{j}_l^r, \tilde{j}_i^r, \tilde{j}_k^r, \tilde{j}_e^r : \omega\right)$ and $\tilde{M}_l^r = \left(\tilde{m}_l^r, \tilde{m}_i^r, \tilde{m}_k^r, \tilde{m}_e^r : \omega\right)$ are two WTpFN then :

Case a: If Score ($\tilde{J}^r$) < Score ($\tilde{M}^r$) then $\tilde{J}^r \prec \tilde{M}^r$

Case b: if Score ($\tilde{J}^r$) > Score ($\tilde{M}^r$) then $\tilde{J}^r \succ \tilde{M}^r$

Case c:if Score ($\tilde{J}^r$) = Score ($\tilde{M}^r$) then $\tilde{J}^r \simeq \tilde{M}^r$

where, for WTpFN $Score\left(\tilde{J}^r\right) = \dfrac{\omega\cdot\left(\tilde{j}_l^r + \tilde{j}_i^r + \tilde{j}_k^r + \tilde{j}_e^r\right)}{4}$

and WTrFN $Score\left(\tilde{J}^r\right) = \dfrac{\omega\cdot\left(\tilde{j}_l^r + 2.\tilde{j}_i^r + \tilde{j}_e^r\right)}{4}$

A. *The List of Abbreviation And Notation are Listed Below*

FSPP denoted as "fuzzy shortest path problem."
WTpFN denoted as "weighted trapezoidal fuzzy number."
WTrFN denoted as "weighted triangular fuzzy number."
SP denoted as "shortest path."
SPP denoted as "shortest path problem."
ANN denoted as "artificial neural network."
FPA denoted as "Fuzzy Physarum Algorithm"
GA denoted as "genetic algorithm."
ABC denoted as "artificial BEE colony."
FSP denoted as "fuzzy shortest path."
FLP denoted as "fuzzy linear programming."
FLSPP denoted as "fuzzy linear shortest path problem."
CLSPP denoted as "crisp linear shortest path programming."
GnTpFN denoted as "generalize trapezoidal fuzzy number."
T1WTpFN denoted as "Type-I weighted trapezoidal fuzzy number"
T2WTpFN denoted as "Type-II weighted trapezoidal fuzzy number"
CSP denoted as "crisp shortest path."
LPP "linear programming problem"
FAL denoted as "fuzzy arc length."

III. EXISTING LPP IN CSP AND FSP PROBLEMS

In this section, we discussed the current linear model in CSP and FSP.

Notation

α **:** The Starting point of the journey
β **:** Destination point
s_{kl} : The FSPL from k[th] node to l[th] node.

$\sum_{l=1}^{r} x_{kl}$: The total flow out of node *k*.

$\sum_{l=1}^{r} x_{lk}$: The total flow into node *k*.

The CSP problem in LPP model is as follows [32]:

$$Min = \sum_{k=1}^{r}\sum_{l=1}^{r} s_{kl} . x_{kl}$$

Subject to: (2)

$$\sum_{l=1}^{r} x_{kl} - \sum_{l=1}^{r} x_{lk} = \delta_i$$

For all $x_{kl} \in \Re$ where $k, l = 1, 2, ..., r$

Where, $\delta_i = \begin{cases} 1\ if & i = \alpha, \\ 0\ if & i = \alpha+1, \alpha+2, ..., \beta-1 \\ -1\ if & i = \beta. \end{cases}$

Moreover, if we swapped the parameters s_{kl} by the fuzzy parameters,*i.e.* $\tilde{s}_{kl}$, then the CLSPP model in the fuzzy environment is as follows:

$$Min = \sum_{k=1}^{r}\sum_{l=1}^{r} \tilde{s}_{kl} . x_{kl}$$

Subject to: (3)

$$\sum_{l=1}^{r} x_{kl} - \sum_{l=1}^{r} x_{lk} = \delta_i$$

For all $x_{kl} \in \Re$ where $k, l = 1, 2, ..., r$.

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IV. DISCUSSION ON THE SHORTCOMING AND LIMITATION OF EXISTING METHODS

Okada and Soper [33] have considered T2WTpFN to find two non-dominated paths. However, this technique provides no guideline to the decision-maker for choosing the bestroute based on their viewpoint. So, this is more confusing to pick the best SP from source to destination. Mahadavi et al. [17] have considered T2WTpFN and T2WTrFN to find the FSP by dynamic programming. The problem is very complicated, and it contains various calculations concerning a large number of tables. This is the major drawback of the problem as for a new user, it is challenging to understand, and because of numerous calculation it needs more time and space, and it has enormous numbers of reluctance and repeatedly. Also, Deng et al. [19] considered T2WTpFN their proposed methods in which they stated that the number of iteration is equal to the total numbers of node existing in the given network,i.e.,number of repetition is directly proportional to the total number of nodes. As a result, we can see that the above-mentioned methods consume more space and time. Nirmoond et al. [31] proposed method which works with T2WTpFN and provides the range of CSP length but won't be able to predict the actual CSP length.

Moreover, all the authors [9], [17], [19], [20], [31] considered the T2WTpFN or T2WTrFN for solving the FSP and FSP length. After an extensive literature survey, we found that there does not exist any method which works in both the environment i.e.WTpFN and WTrFN.Thus. All the existing methods have shortcoming and limitation. Moreover, we found that there is no such technique exists which can predict the FSP and CSP length under both the Type-I and Type-II WTpFN and WTrFN environment which suggests that there is still a scope of improvement in the weighted fuzzy environment to find the FSP.

A. *Proposed Algorithm*

Here, we discuss the proposed method for detecting the absolute FSP along with FSP length. We consider WTpFN and WTrFN for the parameters $\tilde{s}_{ij}$. First of all; we present a way to transform fuzzy linear shortest path programming (FLSPP) into a crisp linear shortest path programming (CLSPP).

Let us consider the Equation 3; then based on the score function we have the following **C**LSPP:

$$Min = \sum_{k=1}^{r}\sum_{l=1}^{r} s_{kl}^{R} \cdot x_{kl}$$

$$Subject\ to\ :$$

$$\sum_{l=1}^{r} x_{kl} - \sum_{l=1}^{r} x_{lk} = \delta_i \qquad (4)$$

$$for\ all\ x_{kl} \in \Re, real\ and\ always\ positive$$

$$where\ \ k,l = 1,2,.....,r$$

where the fuzzy number is $\tilde{s}_{kl}$ corresponding to a real number s_{kl}^{R} concerning a given score function defined in the Definition 2.5 FLSPP and the CLSPP are precisely like each other and the Equation 4, shows that the set of all feasible solution of FLSPP problem and the corresponding CLSPP problem are same.

By the fundamental theorem of LPP Zimmermann [26], for any FLSPP and its corresponding CLSPP arises precisely one of the following cases:

Case 1: CLSPP will have the optimum value iff corresponding FLSPP has an optimum value.

Case 2: CLSPP will have the infinite optimum value iff corresponding FLSPP has infinite optimum value.

Case 3: CLSPP will have no solution when corresponding FLSPP has no solution.

Now, the algorithm is as follows:

Algorithm for finding the shortest path:

Step 1: If the weight of every FAL of the graph is T1WFN option Type I else choose option Type II.

Step 2: Formulate the problem by the model of Equation 3.

Step 3: Using Equation 4, transform the FLSPP into the CLSPP.

Step 4: Solve the CLSPP by using any of the existing optimisation methods/ software such as (MATLAB, GAMS and so on) and obtain the FSP.

Step 5: Substitute all x_{kl} in the objective equation to get the FSP length.

Step 6: To predict the CSP length there are two possibilities

Case a: If problems are of Type-II, then the objective value obtained in step 4 is the CSP length.

Case b: Else the CSP length for the WTpFN is $\frac{1}{4}\sum_{k=1}^{r}\sum_{l=1}^{r}\tilde{s}_{kl}\cdot x_{kl}$ and for the WTrFN is $\frac{1}{3}\sum_{k=1}^{r}\sum_{l=1}^{r}\tilde{s}_{kl}\cdot x_{kl}$

V. APPLICATION OF PROPOSED METHOD IN REAL LIFE PROBLEMS

In this section, the implementation of the proposedmethod is tested using two different real-life issues; one is the distribution network problem, and another is the railway network problem.

Distribution Network Problem:

In Example 5.1 and Example 5.2, we have considered a real-life problem of a distribution network for a soft drink company. Here we consider a soft drink company which is having 23 distribution areas. This configuration is shown in Fig. 1. Now we can assume that the distance between the two distribution centres is a WTpFN where the arc length is given in Tables I and Table II. The company wants to find the SP for distribution between the geographical centres.

Railway Network:

In Example 5.3 and Example 5.4, we have considered a real-life situation of Railway network problem. Here we consider a network of 11

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different railway stations. This configuration is shown in Fig. 2. Now we can assume that the distance between the two railway stations is a WTrFN where the arc length is given in Table III and Table IV. The railway authority wants to find the SP between the considered railway stations.

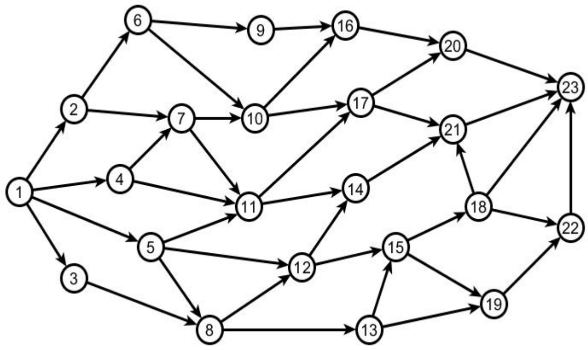

Fig 1: The network contains 1 as a source and 23 as a destination node. [21]

Here, we will test our method for the above-discussed problems where considered in [16,23,25,34]

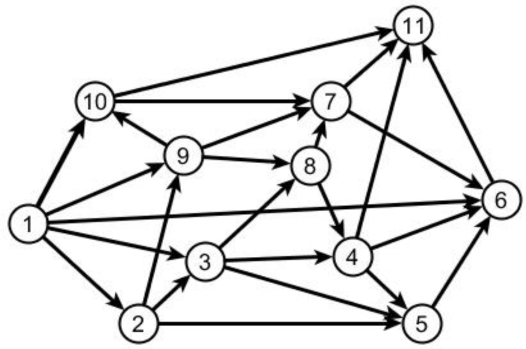

Fig 2: The network contains 1 as a source and 5 as a destination node. [24-25]

Example 5. 1: Consider all the arc length is in T2WTpFN with the conditions of Table 1. Here, we consider a network see Fig 1.

Solution: After performing the steps 1-6, we get $x_{15} = 1$, $x_{511} = 1$, $x_{1117} = 1$, $x_{1721} = 1$, $x_{2123} = 1$, rest all $x_{ij} = 0$. Hence the SP is $1 \to 5 \to 11 \to 17 \to 21 \to 23$. now, put the value of all x_{ij} in Equation 5 then we get the FSP length as: (38, 49, 58, 65). So, we conclude that the FSP length lies within the range of 38 to 65. Moreover, the maximum possibility of the CSP length lies in between the range of 49 to 58. Additionally, our advanced method predicts the CSP is 52.50.

Table I: Consider T2WTpF Arc length for the given network (Fig. 1)

T	H	T2WTpF	$\tilde{s}_{kl}$	T	H	T2WTpF	$\tilde{s}_{kl}$	T	H	T2WTpF	$\tilde{s}_{kl}$
1	2	(12,13,15,17)	$\tilde{s}_{12}$	7	10	(9,10,12,13)	$\tilde{s}_{710}$	15	18	(8,9,11,13)	$\tilde{s}_{1518}$
1	3	(9,11,13,15)	$\tilde{s}_{13}$	7	11	(6,7,8,9)	$\tilde{s}_{711}$	15	19	(5,7,10,12)	$\tilde{s}_{1519}$
1	4	(8,10,12,13)	$\tilde{s}_{14}$	8	12	(5,8,9,10)	$\tilde{s}_{812}$	16	20	(9,12,14,16)	$\tilde{s}_{1620}$
1	5	(7,8,9,10)	$\tilde{s}_{15}$	8	13	(3,5,8,10)	$\tilde{s}_{813}$	17	20	(7,10,11,12)	$\tilde{s}_{1720}$
2	6	(5,10,15,16)	$\tilde{s}_{26}$	9	16	(6,7,9,10)	$\tilde{s}_{916}$	17	21	(6,7,8,10)	$\tilde{s}_{1721}$
2	7	(6,11,11,13)	$\tilde{s}_{27}$	10	16	(12,13,16,17)	$\tilde{s}_{1016}$	18	21	(15,17,18,19)	$\tilde{s}_{1821}$
3	8	(10,11,16,17)	$\tilde{s}_{38}$	10	17	(15,19,20,21)	$\tilde{s}_{1017}$	18	22	(3,5,7,9)	$\tilde{s}_{1822}$
4	7	(17,20,22,24)	$\tilde{s}_{47}$	11	14	(8,9,11,13)	$\tilde{s}_{1114}$	18	23	(5,7,9,11)	$\tilde{s}_{1823}$
4	11	(6,10,13,14)	$\tilde{s}_{411}$	11	17	(6,9,11,13)	$\tilde{s}_{1117}$	19	22	(15,16,17,19)	$\tilde{s}_{1922}$
5	8	(6,9,11,13)	$\tilde{s}_{58}$	12	14	(13,14,16,18)	$\tilde{s}_{1214}$	20	23	(13,14,16,17)	$\tilde{s}_{2023}$
5	11	(7,10,13,14)	$\tilde{s}_{511}$	12	15	(12,14,16,17)	$\tilde{s}_{1215}$	21	23	(12,15,17,18)	$\tilde{s}_{2123}$
5	12	(10,13,15,17)	$\tilde{s}_{512}$	13	15	(10,12,14,15)	$\tilde{s}_{1315}$	22	23	(4,5,6,8)	$\tilde{s}_{2223}$
6	9	(6,8,10,11)	$\tilde{s}_{69}$	13	19	(17,18,19,20)	$\tilde{s}_{1319}$				
6	10	(10,11,14,15)	$\tilde{s}_{610}$	14	21	(11,12,13,14)	$\tilde{s}_{1421}$				

T: Tail node, H: Head node.

Example 5.2: Consider all the FAL is in T1WTpFN with the same network as mentioned in Example 5.1 and the cond. of Table II. Here, we consider a network see Fig 1.

Solution: After performing the steps 1-6, we get $x_{12} = 1$, $x_{27} = 1$, $x_{710} = 1$, $x_{1017} = 1$, $x_{1720} = 1$, $x_{2023} = 1$ rest all are zero, i.e., $x_{ij} = 0$.

Hence the FSP is $1 \to 2 \to 7 \to 10 \to 17 \to 20 \to 23$; now put the value of all x_{ij} in Equation 7 then we get the FSP length as: (62, 77, 85, 93; 0.2). Moreover, our advanced method predicts the CSP length is 79.25.

Table II: Consider T1WTpF arc length for the given network (Fig. 1)

T	H	T1WTpF	$\tilde{s}_{kl}$	T	H	T1WTpF	$\tilde{s}_{kl}$	T	H	T1WTpF	$\tilde{s}_{kl}$
1	2	(12,13,15,17;0.2)	$\tilde{s}_{12}$	7	10	(6,7,8,9;0.3)	$\tilde{s}_{710}$	15	18	(8,9,11,13;0.90)	$\tilde{s}_{1518}$
1	3	(9,11,13,15;0.4)	$\tilde{s}_{13}$	7	11	(5,8,9,10;0.4)	$\tilde{s}_{711}$	15	19	(5,7,10,12;1)	$\tilde{s}_{1519}$
1	4	(8,10,12,13;0.6)	$\tilde{s}_{14}$	8	12	(3,5,8,10;0.5)	$\tilde{s}_{812}$	16	20	(9,12,14,16;1)	$\tilde{s}_{1620}$
1	5	(7,8,9,10;0.8)	$\tilde{s}_{15}$	8	13	(6,7,9,10;0.6)	$\tilde{s}_{813}$	17	20	(7,10,11,12;0.25)	$\tilde{s}_{1720}$
2	6	(5,10,15,16;1)	$\tilde{s}_{26}$	9	16	(12,13,16,17;0.8)	$\tilde{s}_{916}$	17	21	(6,7,8,10;0.5)	$\tilde{s}_{1721}$
2	7	(6,11,11,13;0.3)	$\tilde{s}_{27}$	10	16	(6,7,8,9;0.3)	$\tilde{s}_{1016}$	18	21	(15,17,18,19;0.75)	$\tilde{s}_{1821}$
3	8	(10,11,16,17;0.6)	$\tilde{s}_{38}$	10	17	(15,19,20,21;0.2)	$\tilde{s}_{1017}$	18	22	(3,5,7,9;1)	$\tilde{s}_{1822}$
4	7	(17,20,22,24;0.9)	$\tilde{s}_{47}$	11	14	(8,9,11,13;0.5)	$\tilde{s}_{1114}$	18	23	(5,7,9,11;0.1)	$\tilde{s}_{1823}$
4	11	(6,10,13,14;1)	$\tilde{s}_{411}$	11	17	(6,9,11,13;0.6)	$\tilde{s}_{1117}$	19	22	(15,16,17,19;0.2)	$\tilde{s}_{1922}$
5	8	(6,9,11,13;0.25)	$\tilde{s}_{58}$	12	14	(13,14,16,18;0.8)	$\tilde{s}_{1214}$	20	23	(13,14,16,17;0.6)	$\tilde{s}_{2023}$
5	11	(7,10,13,14;0.50)	$\tilde{s}_{511}$	12	15	(12,14,16,17;0.9)	$\tilde{s}_{1215}$	21	23	(12,15,17,18;0.8)	$\tilde{s}_{2123}$
5	12	(10,13,15,17;0.75)	$\tilde{s}_{512}$	13	15	(10,12,14,15;0.11)	$\tilde{s}_{1315}$	22	23	(4,5,6,8;1)	$\tilde{s}_{2223}$
6	9	(6,8,10,11;1)	$\tilde{s}_{69}$	13	19	(17,18,19,20;0.25)	$\tilde{s}_{1319}$				
6	10	(10,11,14,15;0.1)	$\tilde{s}_{610}$	14	21	(11,12,13,14;0.75)	$\tilde{s}_{1421}$				

Example 5.3: *Consider all the FAL is in T2WTpFN with the same network as mentioned in Example 5.3 and the conditions of Table III. Here, we consider a network see Fig. 2 [23].*

Table III: Consider T2WTrF arc length for the given network (Fig. 2)

T	H	T2WTrF	$\tilde{s}_{kl}$	T	H	T2WTrF	$\tilde{s}_{kl}$	T	H	T2WTrF	$\tilde{s}_{kl}$
1	2	(800,820,840)	$\tilde{s}_{12}$	3	5	(730,748,770)	$\tilde{s}_{35}$	8	4	(710,730,735)	$\tilde{s}_{84}$
1	3	(350,361,370)	$\tilde{s}_{13}$	3	8	(425,443,465)	$\tilde{s}_{38}$	8	7	(230,242,255)	$\tilde{s}_{87}$
1	6	(650,677,683)	$\tilde{s}_{16}$	4	5	(190,199,210)	$\tilde{s}_{45}$	9	7	(120,130,150)	$\tilde{s}_{97}$
1	9	(290,300,350)	$\tilde{s}_{19}$	4	6	(310,340,360)	$\tilde{s}_{46}$	9	8	(130,137,145)	$\tilde{s}_{98}$
2	10	(420,450,470)	$\tilde{s}_{210}$	4	11	(710,740,770)	$\tilde{s}_{411}$	9	10	(230,242,260)	$\tilde{s}_{910}$
2	3	(180,186,193)	$\tilde{s}_{23}$	5	6	(610,660,690)	$\tilde{s}_{56}$	10	7	(330,342,350)	$\tilde{s}_{107}$
2	5	(495,510,525)	$\tilde{s}_{25}$	6	11	(230,242,260)	$\tilde{s}_{611}$	10	11	(1250,1310,1440)	$\tilde{s}_{1011}$
2	9	(900,930,960)	$\tilde{s}_{29}$	7	6	(390,410,440)	$\tilde{s}_{76}$				
3	4	(650,667,883)	$\tilde{s}_{34}$	7	11	(450,472,490)	$\tilde{s}_{711}$				

Solution: After performing the steps 1-6, we get $x_{13} = 1$, $x_{35} = 1$ rest all $x_{ij} = 0$ .Hence, the FSP is $1 \to 3 \to 5$; Now put the value of all x_{ij} in Equation 8 then we get and the FSP length as: (1080, 1109, 1140). So, we conclude that the FSP length lies within the range of 1080 to 1140 and the maximum possibility of the CSP length is 1109. Moreover, ouradvanced method predicts the CSP is 1109.5.

Example 5.4: *Consider Fig. 2 with the conditions of Table IV*

Table IV: T1WTrFN with different membership function for Example 5.4 (fig. 2)

$\tilde{s}_{kl}$	T1WTrFN	$\tilde{s}_{kl}$	T1WTrFN	$\tilde{s}_{kl}$	T1WTrFN
$\tilde{s}_{12}$	(800,820,840;0.1)	$\tilde{s}_{35}$	(730,748,770;0.75)	$\tilde{s}_{84}$	(710,730,735;1)
$\tilde{s}_{13}$	(350,361,370;0.5)	$\tilde{s}_{38}$	(425,443,465;1)	$\tilde{s}_{87}$	(230,242,255;0.3)

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$\tilde{s}_{16}$	(650,677,683;0.2)	$\tilde{s}_{45}$	(190,199,210;0.20)	$\tilde{s}_{97}$	(120,130,150;0.6)
$\tilde{s}_{19}$	(290,300,350;0.9)	$\tilde{s}_{46}$	(310,340,360;0.40)	$\tilde{s}_{98}$	(130,137,145;0.9)
$\tilde{s}_{210}$	(420,450,470;0.1)	$\tilde{s}_{411}$	(710,740,770;1)	$\tilde{s}_{910}$	(230,242,260;1)
$\tilde{s}_{23}$	(180,186,193;0.2)	$\tilde{s}_{56}$	(610,660,690;1)	$\tilde{s}_{107}$	(330,342,350;0.25)
$\tilde{s}_{25}$	(495,510,525;0.3)	$\tilde{s}_{611}$	(230,242,260;0.2)	$\tilde{s}_{1011}$	(1250,1310,1440;0.5)
$\tilde{s}_{29}$	(900,930,960;0.25)	$\tilde{s}_{76}$	(390,410,440;0.6)		
$\tilde{s}_{34}$	(650,667,883;0.50)	$\tilde{s}_{711}$	(450,472,490;0.8)		

Solution: After performing the steps 1-6, we get $x_{12} = 1$, $x_{25} = 1$ rest all $x_{ij} = 0$. Hence the FSP is $1 \to 2 \to 5$. Now put the value of all x_{ij} in Equation 10 we get the FSP length as: (1295, 1330, 1365; 0.1). Moreover, our advanced method predicts the CSP length is 1330.

VI. RESULT AND DISCUSSION

The result obtained from Example 5.1 for FSP length and FSP are (38, 49, 58, 65) and $1 \to 5 \to 11 \to 17 \to 21 \to 23$, respectively which is precisely the same as shown in [17,19,31,34]. We have already discussed the flaws of the existing method of [17, 19], however, in our proposed method we overcome those flaws. Further, in Example 5.3 the FSP length and the FSP are obtained as (1080, 1109, 1140) and $1 \to 3 \to 5$ which is precisely the same as shown in [17]. However, our proposed methods consume less space and computations and last but not the least, no complexity which helps a newcomer to understand. Moving further, the proposed method has been tested for both the above discussed Example 5.1 and Example 5.3 which have been already examined by many researchers. When our method has been compared (in Table V and Fig. 3-5) with the other existing methods, we can clearly observe that the CSPL is smaller than or equal to other discussed methods. In Table VI, we compare the objective value of the related techniques.

Table V: Logical comparative analysis with current methods.

Example 5.1	*Our advanced method* ≈ *Ji et al.* [34] < *Deng et al.* [19] < *Mahdavi et al.* [17]
Example 5.3	*Our advanced method* < *Mahdavi et al.* [17]

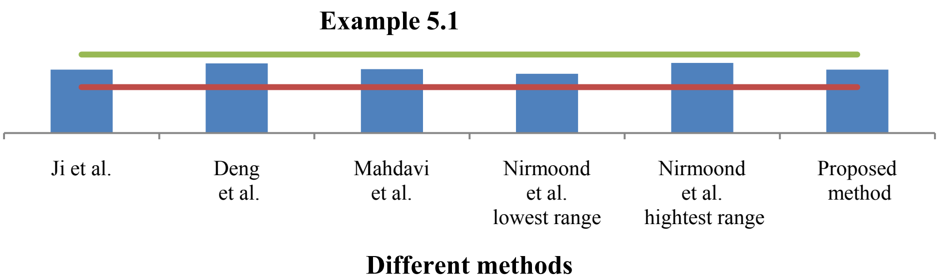

Fig. 3: CSPL data with fuzzy region under different methods

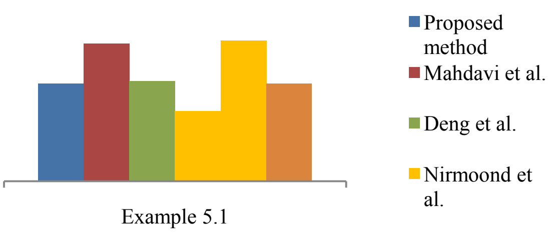

Fig 4: Objective value vs. existing method

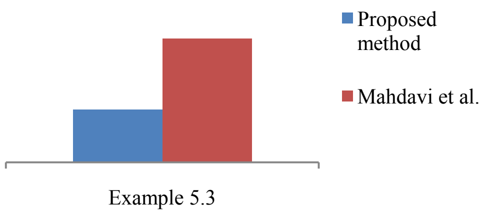

Fig. 5: Objective value vs. existing method

From Fig 3, Fig 4 and Fig 5, we can conclude that the objective value for all methods always lies within the fuzzy range.

To the best of our information, Example 5.2 and Example 5.4 are not taken into consideration by any researcher till date. We have discussed these problems as they are also the applications of fuzzy weighted SPP. In Example 5.2, the FSPL and the FSP are obtained as (62, 77, 85, 93; 0.2) and $1 \to 2 \to 7 \to 10 \to 17 \to 20 \to 23$. In Example 5.4, FSPL and the FSP are obtained as (1295, 1330, 1365; 0.1) and $1 \to 2 \to 5$. The proposed method not only works under weighted trapezoidal fuzzy environment but also applicable under weighted triangular fuzzy environment. However, most of the current methods are either solved as Type-II weighted triangular fuzzy set problems or Type-II weighted trapezoidal fuzzy set problems; very few methods exist which works in both the environment. Furthermore, the proposed method works in both environments without any limitation. So, due to this advantage, we overcome the limitation mentioned in Section 4.1. Also, we have concluded that our method is more reliable than any of the current methods. Table VI, clearly shows that the reliability of our advance method when compared to any of the current methods.

Table IV: Comparison with our proposed model with some existing models

Example	The methods	Shortest path
Example 5.1 (Type-II WTpFN)	The methods of [17]	$1 \rightarrow 5 \rightarrow 11 \rightarrow 17 \rightarrow 21 \rightarrow 23$
	The methods of [19]	$1 \rightarrow 5 \rightarrow 11 \rightarrow 17 \rightarrow 21 \rightarrow 23$
	The methods of [31]	$1 \rightarrow 5 \rightarrow 11 \rightarrow 17 \rightarrow 21 \rightarrow 23$
	Proposed method	$1 \rightarrow 5 \rightarrow 11 \rightarrow 17 \rightarrow 21 \rightarrow 23$
	Predicted FSP length	(38, 49, 58, 65)
Example 5.2 (Type-I WTpFN)	The methods of [17]	NA
	The methods of [19]	NA
	The methods of [31]	NA
	Proposed method	$1 \rightarrow 2 \rightarrow 7 \rightarrow 10 \rightarrow 17 \rightarrow 20 \rightarrow 23$
	Predicted FSP length	(62, 77, 85, 93; 0.2)
Example 5.3 (Type-II WTrFN)	The methods of [17]	$1 \rightarrow 3 \rightarrow 5$
	The methods of [19]	NA
	Proposed method	$1 \rightarrow 3 \rightarrow 5$
	Predicted FSP length	(1080, 1109, 1140)
Example 5. 4. (Type-I WTrFN)	The methods of [17]	NA
	The methods of [19]	NA
	The methods of [31]	NA
	Proposed method	$1 \rightarrow 2 \rightarrow 5$
	Predicted FSP length	(1295, 1330 , 1365;0.1)

NA: Not Applicable

VII. ADVANTAGES OF THE PROPOSED MODEL

i) We can easily find out the FSP and the FSPL.
ii) This algorithm uses some elementary idea of a scoring index and FLP theory which is easy for implementations.
iii) The vast knowledge of FLP is not required.
iv) The proposed method has an added advantage of solving the problems where arc lengths have varied weight.
v) The ultimate advantage of the advance algorithm is to solve a new set of a problem apart from the existing problems.
vi) This algorithm can predict the CSP length.

VIII. CONCLUSION

The proposed paper present a novel method to solve the Type-I and Type-II WTpFN and WTrFN. This method is capable of finding FSP, FSP length and also able to predict CSP length. Moreover, it reflects that our advanced method reduce the time, effort and space complexity over the current methods. From the numerical and graphical results, we can explain that the new novel method can solve different types of problems which can't be addressed by any of the existing methods. In future, we extend this single objective WTpFN and WTrFN method in bi-objective and multi objective environment.

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