Wave Analogies for Generalized Description of Geodynamic Zones

Igor Borisovich Movchan, Alexandra Anatol'evna Yakovleva

Abstract*: Recently, a lot of publications about the fractal dimension of fracturing, which forms the branching space-related grids at different scale levels, have appeared in the geological literature. The value of the fractal dimension is close to one, which was derived for the coastal line of England. Considering the automated decoding potential and non-potential fields on the whole territory of the United Kingdom, this dimension varies regarding the value of the fractal dimension of Cantor set, 0.631. Observing the same reconstructions in other regions and with a similar quantitative result, one can assume the universal character of Cantor dimension in the law of fractal fragmentation of the structures of geological half-space. Discovered laws are used for the solution of the problem of the primary forecast.

***Index Terms*: geological fracturing, fractal, self-similarity, standing wave, cellular structure, discordant area.**

I. INTRODUCTION

Geological faults and fractures (geological fracturing) form branching spatial grids that are mapped at different scale levels and have the properties of discreteness and spatial regularity. The coordinated elements are observed in the ratio of different-rank disjunctive systems. These elements are associated with the geological term “inheritance”. Recently, these terms have been replaced by “self-similarity”: there is an analogy between the geometric similarity in the branching structure of the disjunctive system and geometric similarity of the fractal branching structure. The self-similar structure of a disjunctive, detected as the scale increases, is defined by M.V. Gzovsky as the result of several stages of its development. At each stage, a part of the Earth’s crust, regardless of the scale of its study, receives periodically applied loads. The evolution of geological fracturing determines the generations of permeable zones of the Earth’s crust. As a result, factors of structural control of anomalous endogenous objects appear. Quaternary activation of elements of geological fracturing is associated with patterns in the spatial relationship of heterogeneous forms of modern landscape, for example, the forms of continents’ and islands’ coastlines. The manifestation of self-similarity of the elements in the morphostructure of the mentioned objects is one of the unexpected examples of fractals. This phenomenon gave B. Mandelbrot [1] the idea of the fractal geometry of nature. The geometrical commonness of landscape elements may indicate their fractal nature: rhombic form or a form similar to it consisting of particular segments, separated from each other by a set of disjunctives or particular regional faults. While measuring the length of the coast *L*, the indented coastline is replaced by an open polygon consisting of elements with length ε at least. If the parameter ε is minimized, for normal smooth curves, the value $L(\varepsilon)$ reaches the definite limit, which is the length of the curve. One of the first investigations of the applied significance of fractals was developed in the UK. As a result, it is known that for the coast of the UK, *L* depends on ε (in the range from 10 to 1,000 km) according to the power law $L(\varepsilon) \approx \varepsilon^{1-D},\ D \approx 1{,}3$, where D is the scale self-similarity factor or fractal dimension. Recognizing the primacy of British colleagues in the development of the applied aspects of the theory of fractal groups, we focus on the consideration of the UK territory to solve the identified problems.

The statement of the effect of fractal self-similarity and the attempt to parameterize this effect by fractal dimension did not add anything new to geology and geophysics, both from the standpoint of understanding the genesis of investigated objects and from the standpoint of field materials’ interpretation methods. Peculiarity of the method, considered in this article, consists in the development of a criteria set for structural reconstructions based on the data of non-contact measurements, represented by the geofields determined along the surface (in 2D as a function of two spatial coordinates) of an area (field of relief absolute heights, field of optical density of remote sensing, magnetic and gravity fields, etc.). We also consider another key problem of combined interpretation of heterogeneous geofields in terms of localization of responses in their structures from homogeneous (comparable) geological anomalies. As a basic element, including the solution of two problems at once, we consider lineament decoding, adopted in aerospace geology, which in geophysics, has the form of qualitative interpretation methods with tracing of synphasic axes and mapping the elements of geoblock structure. The significance of our development regarding the traditional expert interpretation and automated (LESSA software in the ER DAS shell) processing consists in the application of algorithms, which provide complete perception of the structural-tectonic image of the area of the survey.

Revised Manuscript Received on April 07, 2019.

Igor Borisovich Movchan, Saint-Petersburg Mining University, St. Petersburg, Russia.

Alexandra Anatol'evna Yakovleva, Saint-Petersburg Mining University, St. Petersburg, Russia.

II. METHODS

According to standard views, the lineament is a set of linearized (rectified) and heterogeneous elements of landscape, marking geological fracturing under the condition of their Quaternary activation. Consequently, objects of different ranks from linear zones of sharp change of vegetation forms to coastlines and dendritic drainage network of Holocene age are naturally associated with manifestations of disjunctive tectonics. According to practical experience, there are no isolated disjunctives. During the transition from small to large scale, the plane of a single fault or rupture splits into several planes. This split can be traced from planetary and regional fracturing to the microgeological one. The genetic relationship between disjunctive dislocations of various ranks is due to multiple impulses of geological displacements that determine the initiation and development of a fracture. Each of these displacements can be divided into five stages [2]:

1. generation of the net of small primary fractures;
2. merging the part of fractures into a single wave-like surface of destruction and formation of conjugated surfaces associated with it under the conditions of the main change in the stress state of the surrounding rocks;
3. main displacement along relatively large fracture surface, which causes the formation of concomitant small cracks, associated with friction between the wings of a large fracture gap;
4. long-term slip along the gap;
5. generated cavities healing with mineral matter, recovering the continuity of the geological environment and increasing the heterogeneity of the structure of an area of the Earth's crust.

The physical processes generating the dendritic structure of geological fracturing have deterministic character. This is confirmed by the scale factor that functionally joins the chaotic (at first glance) set of geometrically similar (self-similar) faults and ruptures of different ranks [3, 4]. It is not surprising that similar scale coefficients are detected in the ratio between different-rank geomorphological elements of one genesis. Despite the widespread manifestation of self-similarity effect in the geological environment, its quantitative estimations (such as computing of the fractal dimension in the structure of the UK coastline) are still local today. The technique of automated lineament decoding of areal geological and geophysical data, developed in this research, is intended for the extension of the quantitative estimations of the self-similarity effect within an area for any scale of investigation and interpretation of the genesis of this effect.

Parametric decoding of a 2D signal represented by any geological-geophysical scalar field (geofield) is the recalculation of its amplitude values into a set of linear elements, including the following algorithmic steps:

- tracing of elementary lineaments, which includes the localization of extremum points in the geofield $f(x, y)$ structure and in the modulus of its horizontal gradient $\left|\vec{\nabla} f\right| = \sqrt{(\partial f / \partial x)^2 + (\partial f / \partial y)^2}$;

- rotation relative to each extremum point with coordinates (x_0, y_0) of radii-vector (elementary lineament) using the rotation matrix:

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \times \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix}, \tag{1}$$

- selection of the value of the processed component of the geofield for each point $(\tilde{x}, \tilde{y})$ of radius-vector along its length using spline interpolation:

$$i = \mathrm{int}[(\tilde{x} - a)/h],$$

$$f(\tilde{x}) = (x_{i+1} - \tilde{x})^2 (2(\tilde{x} - x_i) + h) f_i h^{-3} + (\tilde{x} - x_i)^2 (2(x_{i+1} - \tilde{x}) + h) f_{i+1} h^{-3} +$$

$$+ (x_{i+1} - \tilde{x})^2 (\tilde{x} - x_i) f_i' h^{-2} - (\tilde{x} - x_i)^2 (\tilde{x} - x_{i+1}) f_{i+1}' h^{-2}, \tag{2}$$

where (x_i, x_{i+1}) are the coordinates of the neighboring nodes of the geofield matrix, f_i and f_{i+1} are the values of the geofield in these nodes, f_i' and f_{i+1}' are the values of the first derivative of the geofield in the mentioned nodes, $x_0 = a$, h is the spacing between the nodes. According to the sample of $f(\tilde{x})$ along elementary lineament, corresponding to its angle of rotation α, the spatial variability of the geofield is calculated as a function of α, which gives dispersion functional during the completion of rotation. Minimum of the functional corresponds to a particular α and reflects the optimal orientation of elementary lineament, which coincides with the axis of the dominant strike of the anomaly in the 2D geofield's structure or with the axis of its gradient zone. Considering the points (x_0, y_0) of codirectional lineaments, the algorithm computes a set of trend lines with the final calculation of intersection points (positions of discordants) of these trends based on Cramer's theorem and on the solution of a system of two linear equations (to determine the spatial relationship between linear approximations of trends).

In the problem of estimation of the 2D structure of a geofield, there is a qualitative idea of generalization effect: during constant resolution of equipment, continuous change in the height of photographing of the Earth's surface (or continuous zoom of survey) is accompanied by a discrete change in the lateral dimensions (and, accordingly, depth) of the geostructural elements being mapped. Simulations of this effect show that smoothing of the areal signal (2D geofield) does not lead to the result, which is the same as a generalization with increasing photographing height. Let's assume that the generalization phenomenon is associated with the separation of the 2D signal on quasi-independent components $\zeta_n = f_n \otimes (\delta_n - h_n)$, each of which has its own frequency band and, therefore, the depth interval in geological half-space.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

Here, f_n is the initial geofield, h_n is the transfer function of the band-rejection filter, ζ_n is the high-, mid- or low-frequency component of the geofield, obtained by the bandpass filter, δ_n is Kronecker function. The choice of frequency bands for constructing a rejection filter is based on the autocorrelation radius of the geofield associated with the wavelength.

The structure of geodynamic zones at each level of generalization is approximated by a polygonal geoblock image, obtained from the calculation of spatial stationarity parameter r:

$$R(\tau)=\int_{-\infty}^{+\infty} f(x)f(x\pm\tau)dx,\quad r=0.5\left(R(0)\right)^{-1}\int_{-\infty}^{+\infty}R(\tau)d\tau \cdot \tag{3}$$

Considering the principle of physical analogy and the dynamics of the lithosphere, which is similar to the dynamics of viscous fluid in geochronological scale, we associate polygonal geoblock structures with Benard cells formed in nonequilibrium liquid layer with a vertical temperature gradient. As the imbalance in liquid layer increases (the growth of vertical temperature gradient and appropriate change of the rheology of liquid), we observe the transformation of cellular structures along the free surface of layer: linear dimension of the cells is decreased, and the cell geometry is changed (Fig.1a) up to the centers of turbulence and complete destructurization.

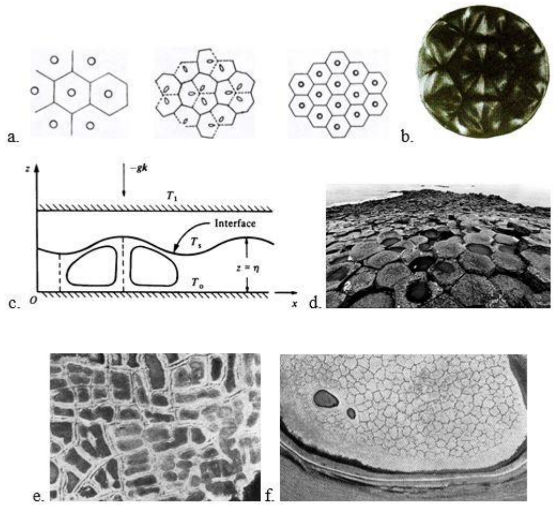

Fig. 1. Physical analogy: cellular structuring of the surface of non-equilibrium flat layer of melt (a-c) and the same structuring along the Earth's surface (d-f.):

a) the dynamics of evolution of time-space fractal during the spontaneous self-organization of layer with increasing imbalance in melt; b) honeycomb-like structure for Benard-Maragoni convection in cylindrical container; c) the wave which corresponds to the structure on Fig. 1b in the vertical section of container. This wave is propagated along interface between the solid and liquid phases of melt [5]; d) cellular structuring of volcanic cover surface in Ireland; e) polygonal roller-lake peatbog in the tundra zone [6]; f) corticated saline-land in desert zone [6].

At the same time, based on the relationship between the size of the sides and diagonals of polygonal cellular formations of different orders, one can derive the scale factor, which we define as a fractal dimension. To clarify the nature of occurrence of such fractal structures, we pay attention to the experience of S.H. Davis et al. [6] in the study of convective motion in the alloy of lead and thallium. In the process of crystallization of alloy in the transverse section of the sample along the interface between the solid and liquid phases, the waves are formed (Fig .1c). This correlates with the development of polygonal structures in the longitudinal section (Fig. 1b). The rigid walls of the cylindrical container, where the alloy was placed, determine the generation of a standing wave: the nodes are formed on the contact of the melt with the wall in the structure of the disturbed interface. Thus, the development of polygonal dissipative structures in nonequilibrium laterally bounded system is the consequence of the development of standing waves in this system. The evolution of cellular dissipative structure as imbalance, increasing in the system with the formation of self-similar (fractal) polygonal forms of different orders, can be defined as fragmentation of standing wave into ever higher order wave modes. One can assume that this fragmentation phenomenon has inverse dynamics regarding generalization phenomenon and defines the discrete variation of the length λ of standing waves, where λ is proportional to standing waves energy.

We aimed to develop a prototype of the so-called wave zoning method based on the fractal dimension D, that would determine the proportional reduction of the linear size of cells in the process of their fragmentation. In this case, the chosen algorithm of calculation D is similar to the algorithm described for the Cantor set. In a cell, the dominant of either positive or negative structural forms of 2D geofield is assumed. On average, the fragmentation of this cell into higher-order polygonal structures is uniform. It is necessary to count the total number N of marked cells, as well as the number M of cells, corresponding to the dominant shape of the cell of the lower order. Then the fractal dimension $D=\log M/\log N$, and always $M<N$: one can expect the fractal dimension of polygonal fragmentation of lithosphere to be practically the same as the fractal dimension of the Cantor set.

III. RESULTS

Sequential rejection of high- and mid-frequency components from the geofield structure (optical density of remote sensing) determines processing to higher levels of generalization, as well as the reduction of the density of spatial grid of lineaments (geodynamic zones) (Fig. 2a, b). The least generalized structural map is transformed in cellular-block structure, each element of which has its own metallogenic specialization (Fig. 2c).

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

The blocks are approximated by hexagonal polygons due to the hydrodynamic analogy with viscous liquids which are non-equilibrium in terms of density [7, 8]. In these fluids standing internal gravity waves appear and these waves are geometrically reflected by Benard cells (Fig. 2d).

The specific character of lineament analysis algorithm as parametric interpretation technique consists in its applicability to arbitrary genesis geofields. The existence of periodic components in the structure of heterogeneous geofields can be explained by the responses of wave processes in these fields. These periodic components are usually approximated by Fourier series and appear in lineament analysis in the form of regular coaxial structures and cellular geoblock formations. Initially introduced in geomorphology, the definition of lineament for a description of the structure of landscape reflected on remote sensing data is adapted to geophysical fields analysis. In the last case, the term "lineament" is equivalent to the term "inphase axis", introduced, in particular for structural correlation maps deriving. Summarizing the above, one can define the lineament as a rectified element of the structure of scalar geofield that is functionally dependent on two spatial coordinates. This element is traced through heterogeneous forms of geofield (gradient zones; along the axes of extended anomalies; along chains of isometric anomalies; according to a combination of the mentioned features; along the boundaries of spatial stationarity zones of scalar geofield) and marks the lateral structural and tectonic boundaries of lithological and petrographic complexes.

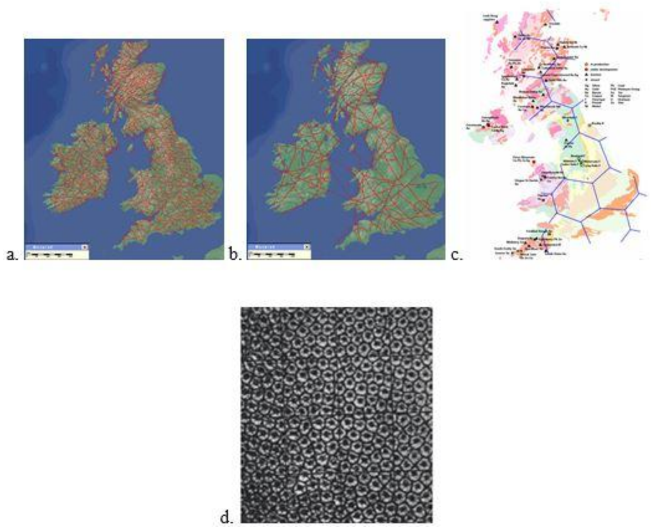

Fig. 2. Maps of automated decoding in the UK and physical analogy:

a), b) lineament fields of different levels of generalization; c) reconstruction of the block fragmentation of the Earth's crust and elements of metallogenic zonality (using [9]); d) polygonal structuring of the surface of the convective layer in Benard's experiment [10].

Fig. 3a, b shows examples of interpretation of the structure of gravity and magnetic fields, which have lower resolution with regard to the map of decoding of remote sensing data (Fig. 2a, b) and, as a result, the geophysical lineaments have relatively distant structure with regard to a set of lineaments derived from the optical density field of the satellite image.

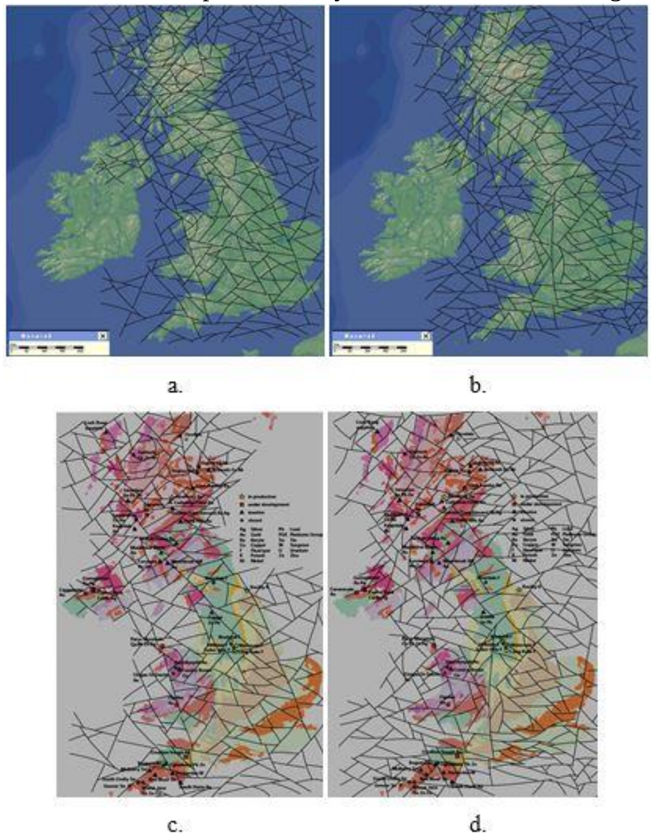

Fig. 3. Lineament decoding of potential fields:

a) lineaments of the magnetic field; b) lineaments of the gravity field in Bouguer reduction. The results of lineament decoding of the magnetic (c) and gravity (d) fields are combined with a map of mineral deposits of the UK with the elements of subquaternary geology (according to [9]). One can see the coincidence of particular lineaments with interfaces of rocks and the correlation of mineral deposits positions with discordant nodes in lineament nets. The maps of mentioned geophysical fields are published in [11].

Besides, it is safe to say that there is practically no influence of exogenous factors on the structure of potential fields, which gives a more significant correlation of geophysical lineament map with the elements of the geological and tectonic structure of the region in contrast to optical density lineaments. Considering predictive significance of the results of the lineament and spectrum recalculations, we superimposed Fig. 3a, b on the geological-structural map of the UK with marked positions of known mineral deposits of different genesis (Fig. 3c, d). One can see that almost all deposits are correlated with discordant areas of structural maps of geophysical lineaments. The combination of the latter with the map of coal and multi-metal deposits demonstrates that significant part of these deposits is dependent on a set of periodically located lineament structures attracted to coaxial geodynamic zones (Fig. 4). In application to the mapping of iron-ore and lead-zinc deposits, the magnetic (Fig. 4c) and gravity

(Fig. 4d) fields provide complementary estimations for detection of the controlling structural factor, defined as coaxial periodic structures. Moreover, each kind of deposits is characterized by its own spatial step (length of standing wave), corresponding to a specific depth of appropriate geological anomaly.

The prognostic aspect of such estimations is obvious: if they use a limited set of reference objects (explored deposits, local ore occurrences, geochemical anomalies) within the licensed area then our technique allows determining the position of the prospective target and detailing the contours for surface geological and geophysical survey and subsequent prospecting drilling. The technique has been tested for the forecast of volcanic pipe within Winter Shore (North of Russian platform) and the Anabar shield (North of East Siberian platform), polymetallic deposits of the Kola Peninsula, and gold ore deposits of Egypt.

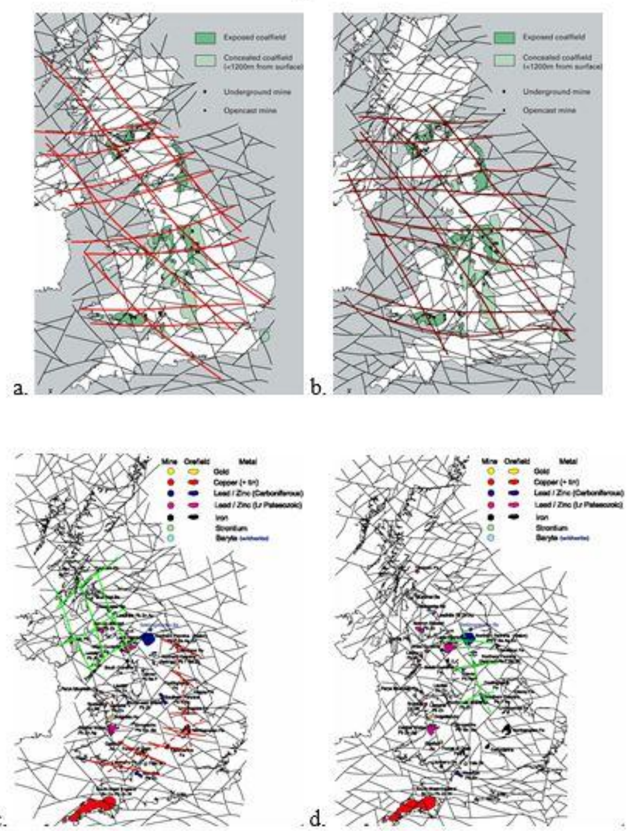

Fig. 4. The structural factor is traced in maps of the magnetic (a) and gravity (b) fields and controls the spatial distribution of coal deposits [12]. Coaxial periodic structures control the distribution of iron ore, lead-zinc deposits of different age [9] and are mapped based on the magnetic (c) and gravity (d) fields.

IV. DISCUSSION

Finally, one can note the physical analogy between the cellular organization of lineament structures and cellular self-organization of non-equilibrium liquid layer, reflected in Fig. 2. If we consider the geological environment a special case of nonequilibrium natural system, it can be said that in the decoding maps in Fig. 2 and Fig. 3, the cellular structure is not explicitly manifested. Nevertheless, mapping of the boundaries of polygonal structures both based on remote sensing data and the maps of gravity and magnetic fields can be considered as a completely correct procedure for three main reasons:

1. existence of the geoblock segmentation in the structure of the Earth's crust and mantle. This phenomenon is an equivalent to polygonal cellular segmentation in nonequilibrium liquid system. Moreover, in relation to the geological environment, we are talking about nonequilibrium distribution of the density of rock complexes of the cross-section in the external gravity field of the planet;

2. observation of the tree-branching effect at faults of different order ratio where these faults are the boundaries of geoblocks of different rank [2];

3. detection of cellular segmentation of the Earth's surface at different scale levels from cellular structuring of soils to polygonal structuring of lava plateaus and geoid surfaces [8, 13].Thus, considering the validity of mapping polygonal forms of different order in the 2D-structure of any geofields, we reconstructed a spatial fractal in the cellular structure of the Earth's crust in the UK based on remote sensing materials (Fig. 5a-c) and equation (3). By superimposing the cellular formations of different order on each other, one can see that each individual cell of the lower order is regularly divided into several cells of the higher order. This process has a discrete nature with a proportional decrease of the size of cells. Discrete fragmentation of cells of order indirectly confirms the concept about the dominating role of standing waves in spontaneous structurization of non-equilibrium geological environment.

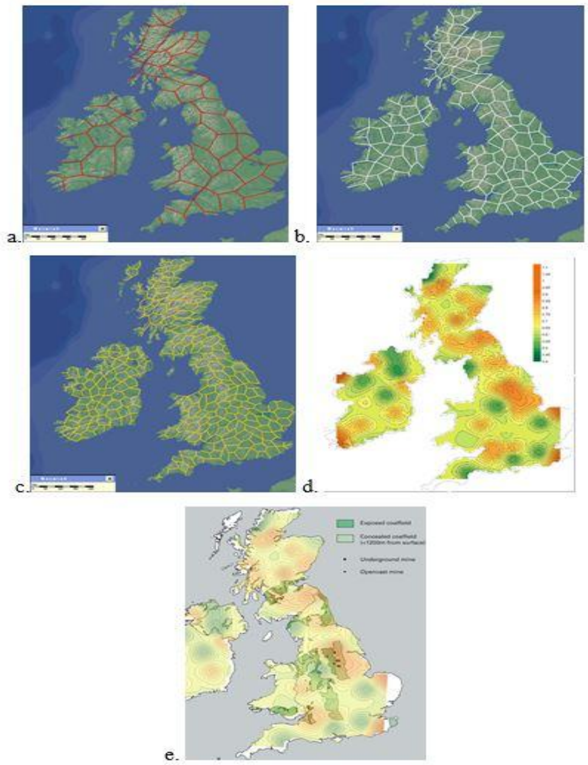

Fig. 5. Spontaneous geoblock structurization in the UK.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

a), b) self-similar cellular structures of a different order; c) map of isolines of scalar field of fractal dimension D; d) superimposing of the map of isolines D on the map of the position of known coal fields.

Each stage of fragmentation is a shift from one mode of standing waves (natural oscillations of interfaces between non-equilibrium geological formations) to other modes. Let's suppose, a single geoblock cell is a positive form of the Earth's surface relief, which has the area occupied by relative elevations with a total area of two-thirds of the area of the mentioned geoblock. During fragmentation, this cell is supposed to be divided into six sub-cells of the higher order that include just four sub-cells in the form of elevation and two sub-cells in the form of depression. This ratio between the cell and sub-cells gives the fractal dimension , whereas the fractal dimension of the famous Cantor's set is equal to 0.631. A similar calculation was realized in our research throughout the UK and the result has the form of a map of isolines of D (Fig. 5). In general, the behavior of parameter D on Fig. 5 can be characterized as oscillation about the equilibrium value of 0.631. In some parts of the coastline of the UK, the computed fractal dimension reaches values of 1.1 1.2, which is almost the value previously calculated by the British colleagues. Primary visual comparison of the map of isolines D with the map of the UK's sub-quaternary complexes (Fig. 5e) demonstrates the correlation between trend azimuth of the axes of scalar field D anomalies and trend azimuths of the structural-tectonic interfaces. Indeed, one can see from Fig. 5d that most of the known coal fields are attracted to gradient zones in the structure of the scalar field . This empirical element can be used as a predictive factor, along with the periodicity criterion.

V. CONCLUSION

An algorithm and appropriate software for the lineament decoding of potential and non-potential geofields at different levels of generalization have been developed. It has been shown that the lineament presentation is invariant regarding the genesis of geofields. Lineament maps can be used in the formal comparison of spatial structures of geofields in the problems of metallogeny and regional forecast. The comparison is performed in terms of the block (cellular) fragmentation of lithosphere and tracing of coaxial periodic structures in the Earth's crust. The regularity of the elements of structural control of deposits and ore occurrences, along with the cellular form of the block structure of lithosphere, determine the hydrodynamic analogies with the classical Benard experiment. According to these analogies, the evolution of cellular structures during increasing of density instability determines the generation of space- or time-related fractal that is quantitatively parametrized in terms of fractal dimension.

A similar evolution of cellular structures is detected in the geological environment based on the processing of multiband remote sensing materials. The value of fractal dimension D is near the fractal dimension of the Cantor's set 0.631, while our calculation of D gives the value that is practically the same as the a priori known value 1.3. Considering similar reconstructions in other regions and a similar quantitative result, one can admit the universal nature of Cantor's dimension for the description of the pattern of fractal fragmentation of geological structures.

REFERENCES

1. B. B. Mandelbrot, "How Long Is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension", *Science*, 156, 1967, pp. 636-638.
2. M. W. Gzovsky, "*Mathematics in geotectonics*", Publ.house "Mir", Moscow, 1971, pp. 97-99
3. I. B. Movchan, A. A. Yakovleva, "Experience of qualitative and quantitative interpretation of nonpotential geofields with surface and deep morphostructural reconstructions on the example of Unica ore province (Kareljya, Russia). *International Journal of Mechanical Engineering and Technology*, 8(12), 2017, pp. 926-932.
4. I. B. Movchan, A. A. Kirsanov, A. A. Yakovleva, "Qualitative interpretation of remote sensing materials in environmental and geological problems", *World applied sciences journal*, 30(1), 2014, pp. 39-45
5. S.H. Davis, U. Müller, C. Dietsche, "Pattern selection in single-component systems coupling Bénard convection and solidification", *Journal of Fluid Mechanics*, 144, 1984, pp. 133-151.
6. I. Gudilin, I. Komarov, "*Application of aerometodes in engineering-geological and hydrogeological studies*", Moscow, Nedra, 1978, pp. 318.
7. O. V. Petrov, I. B. Movchan, "Dissipative structures of the Earth and their wave nature". *Journal "Regional geology and metallogeny"*, 30-31, 2007, pp.12-37.
8. O. V. Petrov, I. B. Movchan, "Dissipative structurization of the Earth crust and mantle as a reflection of wave processes", *Journal "Regional geology and metallogeny"*, 12, 2002, pp. 205-216
9. T. B. Colman, D. C. Cooper, "*Exploration for metalliferous and related minerals in Britain*": a guide. Keyworth, Nottingham: British Geological Survey, 2000, pp. 4-67.
10. H. Benard, "Les tourbillons cellulaires dans une nappe liquide transportant de la chaleur par convection en régime permanent", *Ann. Chem. Phys*, 23(62), 1901, pp. 62-144.
11. British Geological Survey, "View Maps", n.d. Retrieved from: http://www.bgs.ac.uk/data/mapViewers/home.html?src=topNav
12. L.E. Taylor, T. J. Brown, P. A. J. Lusty, K. Hitchen, T B. Colman, D. E. Highley, "United Kingdom Minerals Yearbook 2005". Keyworth, Nottingham: British Geological Survey, 2006, pp. 37.
13. O.V. Petrov, "Internal gravity waves of the Earth and nonlinear paleogeodynamic dissipative structures", Proceedings of the Russian Academy of Science, 326(2), 1992, pp. 323-326.

Published By: Blue Eyes Intelligence Engineering & Sciences Publication