International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075, Volume-8 Issue-6, April 2019

Haar Ranking of Linear and Non-Linear Heptagonal Fuzzy Number and Its Application

S. Karthik, N. Punithavelan, Saroj Kumar Dash

Abstract**:** ***Fuzzy numbers play a vital role in all the decision making problems as it represents the value to the linguistic terms which is the input for all the decision making techniques. In the last decade, most of the researchers have used triangular and trapezoidal fuzzy numbers to represent linguistic terms for solving the problem in the uncertain environment. But it is inadequate of dealing a problem when fuzziness arises in seven different points. Therefore, this paper's intent is to propose the linear and non-linear heptagonal fuzzy number under uncertain environments. This paper derives the Haar ranking method for the heptagonal fuzzy number. Furthermore, Algorithm for the fuzzy assignment problem with Haar ranking is proposed. To check the soundness of the proposed ranking method, Fuzzy assignment problem in agriculture has been taken to determine the minimum cost for organic fertilizers applied in the field.***

Index Terms**:** ***Fuzzy sets, α- cut, Fuzzy equation, Heptagonal fuzzy number, Haar ranking, Fuzzy Assignment problem, Agriculture.***

I. INTRODUCTION

In 1965, Zadeh, L.A introduced fuzzy sets [20] to provide a logical way of solving problems when the source of vagueness and imprecision arises. It has a wide range of applications in various fields such as control system, expert system, decision making, artificial intelligence, etc. Fuzzy numbers and its arithmetic operations were helpful for modeling Cognitive computational models, an expert system, measurements, knowledge, and intelligence [17, 18, 19]. Dubois and Prade have defined fuzzy numbers as a fuzzy subset of the real line [8]. A fuzzy number is a multi-valued quantity whose value is precise, rather than a single-valued quantity. Most of the researchers have used triangular and trapezoidal fuzzy numbers to handle imprecision in real life situations [4, 17, 18]. Hexagonal, heptagonal, nonagonal, decagonal fuzzy numbers have also been introduced to tackle the vagueness [10, 11, 14]. Many researchers have concentrated on uncertain linguistic terms in group decision-making systems [4, 17, 20, 21]. In decision-making problems, experts may express their opinions in terms of uncertain linguistic terms when they have difficulties in getting a clear idea or lack of information about the problems. The uncertain linguistic terms are often used as inputs in decision analysis activities [5, 9].

In 2014, Bekheet et.al., introduced polygonal fuzzy numbers for solving fuzzy multi-criterion decision-making problems [2]. Bekheet et.al., proposed a ranking of polygonal fuzzy numbers for the applications in the Multi-Criteria Decision Making Model in 2014 [1]. In 2014, Rathi and Balamohan proposed heptagonal fuzzy number, defined arithmetic operations and derived a ranking method to solve assignment problem for getting optimal solution [14]. Chandrasekaran and his team have used a ranking method on the heptagonal fuzzy number to find an optimal solution for any transportation problem by using zero suffix method in 2015 [3]. In 2017, Menakadevi,et.al., formulated balanced zero assignment problem to crisp assignment problem and solved by ranking index method using heptagonal fuzzy numbers [12]. In 2017, Premalatha and Murugan solved fuzzy assignment problem by a robust ranking method using the fuzzy heptagonal number to calculate optimum objective function [13]. In 2017, Sudha and Karunambigai used the heptagonal fuzzy number to solve the transportation problem, considered linear membership function with symmetry, defined arithmetic operations and proposed a ranking method for solving fuzzy transportation problem [16]. Dhurai and Karpagam proposed a new ranking and a new membership function on heptagonal fuzzy numbers which were used for solving travelling salesman problems in 2017 [7]. In 2017, Sankar and Manimohan adopted pentagonal fuzzy number, derived linear and non- linear pentagonal fuzzy number and solved fuzzy equations using pentagonal fuzzy number [15]. Dhanasekar and Hariharan have proposed Haar Hungarian Algorithm to get the optimal solution, in which the trapezoidal fuzzy numbers were converted into Haar tuples through Haar wavelet concept [6]. Therefore, from this review, it is observed that the research can be done in the following. (i) Deriving Linear and Non-linear heptagonal fuzzy number and (ii) Drawing a new Haar ranking for the heptagonal fuzzy number. To check the validity of the proposed ranking method, Assignment problem in agriculture is taken to determine the minimum cost for organic fertilizers applied in the field.

II. PRELIMINARIES

The following definitions are required in order to understand the fuzzy set, fuzzy number, and membership functions.

Revised Manuscript Received on April 06, 2019.

S. Karthik, Mathematics Division, School of Advanced Sciences, VIT Chennai, Tamilnadu, India.

N. Punithavelan, Physics Division, School of Advanced Sciences, VIT Chennai, Tamilnadu, India.

Saroj Kumar Dash, Mathematics Division, School of Advanced Sciences, VIT Chennai, Tamilnadu, India.

Definition 2.1 A fuzzy set $\tilde{A}$ is a subset of a universe of discourse *X*, which is characterized by a membership function $\mu_{\tilde{A}}(\theta)$ representing a mapping $\mu_{\tilde{A}}: X \to [0,1]$. The function value of $\mu_{\tilde{A}}(\theta)$ is called the membership value, which represents the degree of truth that θ is an element of the fuzzy set $\tilde{A}$.

Definition 2.2 A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number and its membership function $\tilde{A}: R \to [0,1]$ has the following characteristics,

(i) $\tilde{A}$ is convex.

$$\mu_{\tilde{A}}(\lambda\theta_1 + (1-\lambda)\theta_2) \ge \min(\mu_{\tilde{A}}(\theta_1), \mu_{\tilde{A}}(\theta_2)),$$
$$\forall \theta \in [\theta_1, \theta_2], \lambda \in [0,1].$$

(ii) $\tilde{A}$ is normal if $\max \mu_{\tilde{A}}(\theta) = 1$.

(iii) $\tilde{A}$ is piecewise continuous.

Definition 2.3 The α-cut of the fuzzy set $\tilde{A}$ of the universe of discourse X is defined as $\tilde{A}_\alpha = \{\theta \in X / \mu_{\tilde{A}}(\theta) \ge \alpha\}$, where $\alpha \in [0,1]$.

Definition 2.4 A triangular fuzzy number $\tilde{A}$ can be defined as a triplet (*l*, *m*, *r*) and the membership function $\mu_{\tilde{A}}(\theta)$ is defined as:

$$\mu_{\tilde{A}}(\theta) = \begin{cases} 0 & \theta < l \\ \left(\dfrac{\theta - l}{m - l}\right) & l \le \theta \le m \\ \left(\dfrac{r - \theta}{r - m}\right) & m \le \theta \le r \\ 0 & \theta > r \end{cases}$$

Where *l*, *m*, *r* are real numbers and $l \le m \le r$.

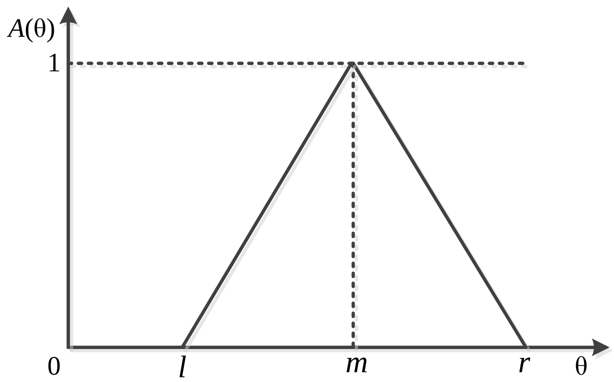

Figure 1: Triangular Fuzzy Number

Definition 2.5 A trapezoidal fuzzy number $\tilde{A}$ can be defined as (a_1, a_2, a_3, a_4) and the membership function is defined as

$$\mu_{\tilde{A}}(\theta) = \begin{cases} \left(\dfrac{\theta - a_1}{a_2 - a_1}\right) & a_1 \le \theta \le a_2 \\ 1 & a_2 \le \theta \le a_3 \\ \left(\dfrac{a_4 - \theta}{a_4 - a_3}\right) & a_3 \le \theta \le a_4 \\ 0 & \theta \le a_1 \ \& \ \theta \ge a_4 \end{cases}$$

Where a_1, a_2, a_3, a_4 are real numbers $a_1 \le a_2 \le a_3 \le a_4$.

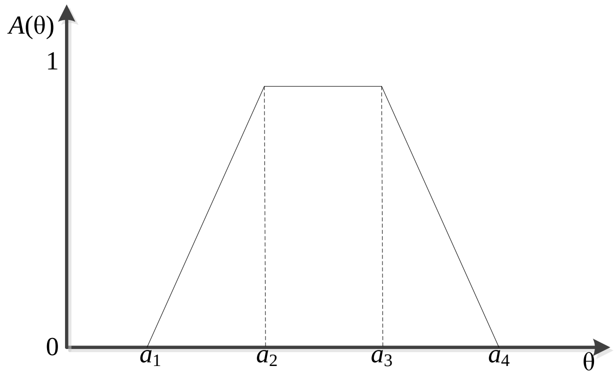

Figure 2: Trapezoidal Fuzzy Number

III. HEPTAGONAL FUZZY NUMBERS AND ITS VARIATION

In the following section, various types of heptagonal fuzzy numbers are proposed in a different perspective.

3.1 Heptagonal Fuzzy Number: A heptagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_7 \in R$ must hold the subsequent conditions

- $\mu_{\tilde{A}}(\theta)$ is a continuous function in the closed interval [0, 1].
- $\mu_{\tilde{A}}(\theta)$ is strictly increasing and continuous function on $[a_1, a_2]$, $[a_2, a_3]$ and $[a_3, a_4]$.
- $\mu_{\tilde{A}}(\theta)$ is strictly decreasing and continuous function on $[a_4, a_5]$, $[a_5, a_6]$ and $[a_6, a_7]$.

3.1.1 Equality of two Heptagonal Fuzzy Numbers: Two heptagonal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7)$ are equal if and only if $a_1 = b_1, a_2 = b_2, a_3 = b_3, a_4 = b_4, a_5 = b_5, a_6 = b_6, a_7 = b_7$.

Next, an attempt is made to classify new types of heptagonal fuzzy numbers in various forms.

3.2 Linear Heptagonal Fuzzy Number with Symmetry

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

3.2.1 Linear Heptagonal Fuzzy Number with Symmetry (LHFNS): A linear heptagonal fuzzy number with symmetry is given as $\tilde{A}_{LS}=\left(a_1,a_2,a_3,a_4,a_5,a_6,a_7;m,n\right)$ where $m,n\in\left(0,1\right)$ an its membership function is defined as

$$\mu_{\tilde{A}_{LS}}\left(\theta\right)=\begin{cases} m\left(\dfrac{\theta-a_1}{a_2-a_1}\right), & a_1\le\theta\le a_2\\ m-\left(m-n\right)\left(\dfrac{\theta-a_2}{a_3-a_2}\right), & a_2\le\theta\le a_3\\ n-\left(n-1\right)\left(\dfrac{\theta-a_3}{a_4-a_3}\right), & a_3\le\theta\le a_4\\ 1, & \theta=a_4\\ n-\left(n-1\right)\left(\dfrac{a_5-\theta}{a_5-a_4}\right), & a_4\le\theta\le a_5\\ m-\left(m-n\right)\left(\dfrac{a_6-\theta}{a_6-a_5}\right), & a_5\le\theta\le a_6\\ m\left(\dfrac{a_7-\theta}{a_7-a_6}\right), & a_6\le\theta\le a_7\\ 0, & \theta\le a_1 \ and \ \theta\ge a_7 \end{cases}$$

3.2.2 α-cut of LHFNS: It is defined as

$$A_\alpha=\begin{cases} A_{1L}\left(\alpha\right)=a_1+\left(\dfrac{\alpha}{m}\right)\left(a_2-a_1\right) \ for \ \alpha\in[0,m]\\ A_{2L}\left(\alpha\right)=a_2+\left(\dfrac{m-\alpha}{m-n}\right)\left(a_3-a_2\right) \ for \ \alpha\in[m,n]\\ A_{3L}\left(\alpha\right)=a_4+\left(\dfrac{n-\alpha}{n-1}\right)\left(a_4-a_3\right) \ for \ \alpha\in[n,1]\\ A_{3R}\left(\alpha\right)=a_5+\left(\dfrac{\alpha-n}{n-1}\right)\left(a_5-a_4\right) \ for \ \alpha\in[n,1]\\ A_{2R}\left(\alpha\right)=a_6+\left(\dfrac{\alpha-m}{m-n}\right)\left(a_6-a_5\right) \ for \ \alpha\in[m,n]\\ A_{1R}\left(\alpha\right)=a_7-\left(\dfrac{\alpha}{m}\right)\left(a_7-a_6\right) \ for \ \alpha\in[0,m] \end{cases}$$

Where $A_{1L}\left(\alpha\right)$, $A_{2L}\left(\alpha\right)$ and $A_{3L}\left(\alpha\right)$ are increasing functions and $A_{3R}\left(\alpha\right)$, $A_{2R}\left(\alpha\right)$ and $A_{1R}\left(\alpha\right)$ are decreasing functions in α.

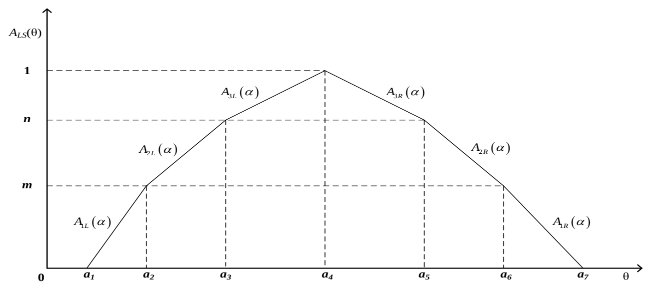

Figure 3: Linear Heptagonal Fuzzy Number with Symmetry

3.3 Linear Heptagonal Fuzzy Number with Asymmetry

3.3.1 Linear Heptagonal Fuzzy Number with Asymmetry (LHFNAS): A linear heptagonal fuzzy number with asymmetry is given as $\tilde{A}_{LAS}=\left(a_1,a_2,a_3,a_4,a_5,a_6,a_7;m,q,n,p\right)$ where $m,n,p,q\in\left(0,1\right)$ and its membership function is as follows

$$\mu_{\tilde{A}_{LAS}}\left(\theta\right)=\begin{cases} m\left(\dfrac{\theta-a_1}{a_2-a_1}\right), & a_1\le\theta\le a_2\\ m-\left(m-n\right)\left(\dfrac{\theta-a_2}{a_3-a_2}\right), & a_2\le\theta\le a_3\\ n-\left(n-1\right)\left(\dfrac{\theta-a_3}{a_4-a_3}\right), & a_3\le\theta\le a_4\\ 1, & \theta=a_4\\ p-\left(p-1\right)\left(\dfrac{a_5-\theta}{a_5-a_4}\right), & a_4\le\theta\le a_5\\ q-\left(q-p\right)\left(\dfrac{a_6-\theta}{a_6-a_5}\right), & a_5\le\theta\le a_6\\ q\left(\dfrac{a_7-\theta}{a_7-a_6}\right), & a_6\le\theta\le a_7\\ 0, & \theta\le a_1 \ and \ \theta\ge a_7 \end{cases}$$

Note:

1. If $q = m$ and $p = n$ the asymmetry heptagonal fuzzy number turns into symmetry heptagonal fuzzy number.
2. To be asymmetry heptagonal fuzzy number, $m\ne q$ and $n\ne p$.

3.3.2 α-cut of LHFNAS: It is defined as

$$A_\alpha=\begin{cases} A_{1L}\left(\alpha\right)=a_1+\left(\dfrac{\alpha}{m}\right)\left(a_2-a_1\right) \ for \ \alpha\in[0,m]\\ A_{2L}\left(\alpha\right)=a_2+\left(\dfrac{m-\alpha}{m-n}\right)\left(a_3-a_2\right) \ for \ \alpha\in[m,n]\\ A_{3L}\left(\alpha\right)=a_4+\left(\dfrac{n-\alpha}{n-1}\right)\left(a_4-a_3\right) \ for \ \alpha\in[n,1]\\ A_{3R}\left(\alpha\right)=a_5+\left(\dfrac{\alpha-p}{p-1}\right)\left(a_5-a_4\right) \ for \ \alpha\in[p,1]\\ A_{2R}\left(\alpha\right)=a_6+\left(\dfrac{\alpha-q}{q-p}\right)\left(a_6-a_5\right) \ for \ \alpha\in[q,p]\\ A_{1R}\left(\alpha\right)=a_7-\left(\dfrac{\alpha}{q}\right)\left(a_7-a_6\right) \ for \ \alpha\in[0,q] \end{cases}$$

Where $A_{1L}\left(\alpha\right)$, $A_{2L}\left(\alpha\right)$ and $A_{3L}\left(\alpha\right)$ are increasing functions and $A_{3R}\left(\alpha\right)$, $A_{2R}\left(\alpha\right)$ and $A_{1R}\left(\alpha\right)$ are decreasing functions in α.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

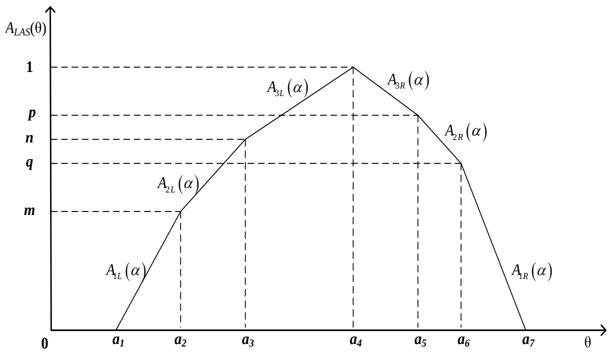

Figure 4: Linear Heptagonal Fuzzy Number with Asymmetry

3.4 Non Linear Heptagonal Fuzzy Number with Symmetry

3.4.1 Non Linear Heptagonal Fuzzy Number with Symmetry (NLHFNS):

A non-linear heptagonal fuzzy number with symmetry is given as

$$\tilde{A}_{NLS} = \left(a_1, a_2, a_3, a_4, a_5, a_6, a_7; m, n,\right)_{(k_1,k_2,k_3;l_1,l_2,l_3)}$$

Where $m, n \in (0,1)$ and its membership function is as follows

$$\mu_{\tilde{A}_{NLS}}(\theta) = \begin{cases} m\left(\dfrac{\theta - a_1}{a_2 - a_1}\right)^{k_1}, & a_1 \le \theta \le a_2 \\ m-(m-n)\left(\dfrac{\theta - a_2}{a_3 - a_2}\right)^{k_2}, & a_2 \le \theta \le a_3 \\ n-(n-1)\left(\dfrac{\theta - a_3}{a_4 - a_3}\right)^{k_3}, & a_3 \le \theta \le a_4 \\ 1, & \theta = a_4 \\ n-(n-1)\left(\dfrac{a_5 - \theta}{a_5 - a_4}\right)^{l_1}, & a_4 \le \theta \le a_5 \\ m-(m-n)\left(\dfrac{a_6 - \theta}{a_6 - a_5}\right)^{l_2}, & a_5 \le \theta \le a_6 \\ m\left(\dfrac{a_7 - \theta}{a_7 - a_6}\right)^{l_3}, & a_6 \le \theta \le a_7 \\ 0, & \theta \le a_1 \text{ and } \theta \ge a_7 \end{cases}$$

3.4.2 α-cut of NLHFNS: It is defined as

$$\tilde{A}_\alpha = \begin{cases} A_{1L}(\alpha) = a_1 + \left(\dfrac{\alpha}{m}\right)^{k_1}(a_2 - a_1) \ \ for \ \alpha \in [0, m] \\ A_{2L}(\alpha) = a_2 + \left(\dfrac{m-\alpha}{m-n}\right)^{k_2}(a_3 - a_2) \ \ for \ \alpha \in [m, n] \\ A_{3L}(\alpha) = a_4 + \left(\dfrac{n-\alpha}{n-1}\right)^{k_3}(a_4 - a_3) \ \ for \ \alpha \in [n, 1] \\ A_{3R}(\alpha) = a_5 + \left(\dfrac{\alpha-n}{n-1}\right)^{l_1}(a_5 - a_4) \ \ for \ \alpha \in [n, 1] \\ A_{2R}(\alpha) = a_6 + \left(\dfrac{\alpha-m}{m-n}\right)^{l_2}(a_6 - a_5) \ \ for \ \alpha \in [m, n] \\ A_{1R}(\alpha) = a_7 - \left(\dfrac{\alpha}{m}\right)^{l_3}(a_7 - a_6) \ \ for \ \alpha \in [0, m] \end{cases}$$

Where $A_{1L}(\alpha)$, $A_{2L}(\alpha)$ and $A_{3L}(\alpha)$ are increasing functions and $A_{3R}(\alpha)$, $A_{2R}(\alpha)$ and $A_{1R}(\alpha)$ are decreasing functions in α.

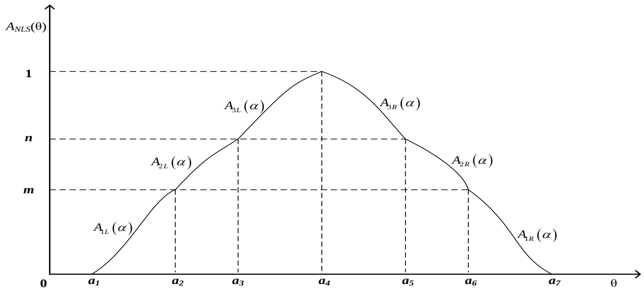

Figure 5: Non Linear Heptagonal Fuzzy Number with Symmetry

3.5 Non Linear Heptagonal Fuzzy Number with Asymmetry

3.5.1 Non Linear Heptagonal Fuzzy Number with Asymmetry (NLHFNAS):

A non-linear heptagonal fuzzy number with asymmetry is given as

$$\tilde{A}_{NLAS} = \left(a_1, a_2, a_3, a_4, a_5, a_6, a_7; m, q, n, p\right)_{(k_1,k_2,k_3;l_1,l_2,l_3)}$$

Where $m, n, p, q \in (0,1)$ and its membership function is as follows

$$\mu_{\tilde{A}_{NLAS}}(\theta) = \begin{cases} m\left(\dfrac{\theta - a_1}{a_2 - a_1}\right)^{k_1}, & a_1 \le \theta \le a_2 \\ m-(m-n)\left(\dfrac{\theta - a_2}{a_3 - a_2}\right)^{k_2}, & a_2 \le \theta \le a_3 \\ n-(n-1)\left(\dfrac{\theta - a_3}{a_4 - a_3}\right)^{k_3}, & a_3 \le \theta \le a_4 \\ 1, & \theta = a_4 \\ p-(p-1)\left(\dfrac{a_5 - \theta}{a_5 - a_4}\right)^{l_1}, & a_4 \le \theta \le a_5 \\ q-(q-p)\left(\dfrac{a_6 - \theta}{a_6 - a_5}\right)^{l_2}, & a_5 \le \theta \le a_6 \\ q\left(\dfrac{a_7 - \theta}{a_7 - a_6}\right)^{l_3}, & a_6 \le \theta \le a_7 \\ 0, & \theta \le a_1 \text{ and } \theta \ge a_7 \end{cases}$$

Note: If $k_1 = k_2 = k_3 = l_1 = l_2 = l_3 = 1$, then non-linear heptagonal fuzzy number turns into a linear heptagonal fuzzy number.

3.5.2 α-cut of NLHFNAS: It is defined as

$$A_\alpha=\begin{cases} A_{1L}(\alpha)=a_1+\left(\dfrac{\alpha}{m}\right)^{k_1}(a_2-a_1)\ \text{for}\ \alpha\in[0,m]\\ A_{2L}(\alpha)=a_2+\left(\dfrac{m-\alpha}{m-n}\right)^{k_2}(a_3-a_2)\ \text{for}\ \alpha\in[m,n]\\ A_{3L}(\alpha)=a_4+\left(\dfrac{n-\alpha}{n-1}\right)^{k_3}(a_4-a_3)\ \text{for}\ \alpha\in[n,1]\\ A_{3R}(\alpha)=a_5+\left(\dfrac{\alpha-p}{p-1}\right)^{l_1}(a_5-a_4)\ \text{for}\ \alpha\in[p,1]\\ A_{2R}(\alpha)=a_6+\left(\dfrac{\alpha-q}{q-p}\right)^{l_2}(a_6-a_5)\ \text{for}\ \alpha\in[q,p]\\ A_{1R}(\alpha)=a_7-\left(\dfrac{\alpha}{q}\right)^{l_3}(a_7-a_6)\ \text{for}\ \alpha\in[0,q]\end{cases}$$

Where $A_{1L}(\alpha)$, $A_{2L}(\alpha)$ and $A_{3L}(\alpha)$ are increasing functions and $A_{3R}(\alpha)$, $A_{2R}(\alpha)$ and $A_{1R}(\alpha)$ are decreasing functions in *α*.

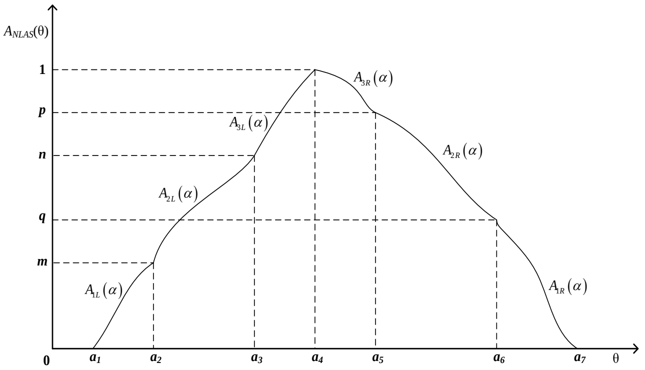

Figure 6: Non Linear Heptagonal Fuzzy Number with Asymmetry

IV. ARITHMETIC OPERATIONS ON LINEAR HEPTAGONAL FUZZY NUMBER WITH SYMMETRY

Let $\tilde{A}_{LS}=(a_1,a_2,a_3,a_4,a_5,a_6,a_7;m_1,n_1)$ and $\tilde{B}_{LS}=(b_1,b_2,b_3,b_4,b_5,b_6,b_7;m_2,n_2)$ be two linear heptagonal fuzzy numbers with symmetry, then

(i)The addition of two heptagonal fuzzy numbers is defined as

$$\tilde{C}_{LS}=\tilde{A}_{LS}+\tilde{B}_{LS}=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4,a_5+b_5,a_6+b_6,a_7+b_7;m,n)$$

Where $m=\min\{m_1,m_2\}$ and $n=\min\{n_1,n_2\}$.

(ii) The subtraction of two heptagonal fuzzy numbers is defined as

$$\tilde{D}_{LS}=\tilde{A}_{LS}-\tilde{B}_{LS}=(a_1-b_7,a_2-b_6,a_3-b_5,a_4-b_4,a_5-b_3,a_6-b_2,a_7-b_1;m,n)$$

Where $m=\min\{m_1,m_2\}$ and $n=\min\{n_1,n_2\}$.

(iii) If *c* is a positive scalar, then the scalar multiplication is defined as

$$c\tilde{A}_{LS}=(ca_1,ca_2,ca_3,ca_4,ca_5,ca_6,ca_7;m,n)$$

If *c* is a negative scalar, then

$$c\tilde{A}_{LS}=(ca_7,ca_6,ca_5,ca_4,ca_3,ca_2,ca_1;m,n).$$

V. HAAR RANKING METHOD FOR HEPTAGONAL FUZZY NUMBER

Let $\tilde{A}=(a_1,a_2,a_3,a_4,a_5,a_6,a_7)$ be a Heptagonal fuzzy number. For the convenient of using Haar wavelet ranking, the Heptagonal fuzzy number is rewritten as $\tilde{A}=(a_1,a_2,a_3,a_4,a_4,a_5,a_6,a_7)$. The average and detailed coefficients namely the scaling and wavelet coefficients of the Heptagonal fuzzy number can be calculated as follows.

Step-1: Group the Heptagonal fuzzy numbers in pairs.

$$[a_1,a_2],[a_3,a_4],[a_4,a_5],[a_6,a_7]$$

Step-2: The first four elements of $\tilde{A}$ with the average of these pairs (approximation coefficients) and replace the last 4 four elements of $\tilde{A}$ with half of the difference of these pairs (detailed coefficients).

$$\alpha_1=\left(\frac{a_1+a_2}{2}\right),\alpha_2=\left(\frac{a_3+a_4}{2}\right),\alpha_3=\left(\frac{a_4+a_5}{2}\right),\alpha_4=\left(\frac{a_6+a_7}{2}\right)$$

$$\beta_1=\left(\frac{a_1-a_2}{2}\right),\beta_2=\left(\frac{a_3-a_4}{2}\right),\beta_3=\left(\frac{a_4-a_5}{2}\right),\beta_4=\left(\frac{a_6-a_7}{2}\right)$$

Then $\tilde{A}$ changed into $\tilde{A}_1=(\alpha_1,\alpha_2,\alpha_3,\alpha_4,\beta_1,\beta_2,\beta_3,\beta_4)$

Step-3: Group the pair of approximation coefficients of $\tilde{A}_1$. Then, find the new approximation coefficients and the detailed coefficients for the pair of approximation coefficient of $\tilde{A}_1$ $[\alpha_1,\alpha_2],[\alpha_3,\alpha_4]$

$$\gamma_1=\left(\frac{\alpha_1+\alpha_2}{2}\right),\gamma_2=\left(\frac{\alpha_3+\alpha_4}{2}\right)\quad \eta_1=\left(\frac{\alpha_1-\alpha_2}{2}\right),\eta_2=\left(\frac{\alpha_3-\alpha_4}{2}\right)$$

Then $\tilde{A}_1$ changed into $\tilde{A}_2=(\gamma_1,\gamma_2,\eta_1,\eta_2,\beta_1,\beta_2,\beta_3,\beta_4)$

Step-4: Find the pair of approximation coefficient in $\tilde{A}_2$. Then, find the new approximation and detailed coefficients for the pair of approximation coefficient of $\tilde{A}_2$.$[\gamma_1,\gamma_2]$

$$\delta_1=\left(\frac{\gamma_1+\gamma_2}{2}\right),\delta_2=\left(\frac{\gamma_1-\gamma_2}{2}\right)$$

Then $\tilde{A}_2$ changed into $H(\tilde{A})=(\delta_1,\delta_2,\eta_1,\eta_2,\beta_1,\beta_2,\beta_3,\beta_4)$

Step-5: Determine the Ranking.

- $\tilde{A}\prec\tilde{B}$, if the first element of the ordered tuple of $H(\tilde{A})$ is less than the first element of the ordered tuple of $H(\tilde{B})$.
- If the first element is the same in both $H(\tilde{A})$ and $H(\tilde{B})$ then compare the 2nd element of both $H(\tilde{A})$ and $H(\tilde{B})$ it will continue till the last

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

element. And in such cases: $\tilde{A} \approx \tilde{B}$.

5.1 Numerical illustration of Haar Ranking

Let $\tilde{A} = (1,4,7,11,15,18,21)$ and $\tilde{B} = (11,14,17,21,25,28,31)$ be two symmetric Heptagonal fuzzy numbers. For the convenient of using Haar wavelet ranking, the Heptagonal fuzzy numbers are rewritten as $\tilde{A} = (1,4,7,11,11,15,18,21)$ and $\tilde{B} = (11,14,17,21,21,25,28,31)$. The average and detailed coefficients namely the scaling and wavelet coefficients of Heptagonal fuzzy numbers can be calculated as follows,

Step 1: Group the Heptagonal fuzzy numbers in pairs. For $\tilde{A}$, the pairs will be $[1,4],[7,11],[11,15]$ and $[18,21]$. For $\tilde{B}$, the pairs will be $[11,4],[17,21],[21,25]$ and $[28,31]$.

Step 2: The first four elements of $\tilde{A}$ with the average of these pairs (approximation coefficients) and replace the last 4 four elements of $\tilde{A}$ with half of the difference of these pairs (detailed coefficients).

$$\alpha_1 = \left(\frac{1+4}{2}\right), \alpha_2 = \left(\frac{7+11}{2}\right), \alpha_3 = \left(\frac{11+15}{2}\right), \alpha_4 = \left(\frac{18+21}{2}\right)$$

$$\beta_1 = \left(\frac{1-4}{2}\right), \beta_2 = \left(\frac{7-11}{2}\right), \beta_3 = \left(\frac{11-15}{2}\right), \beta_4 = \left(\frac{18-21}{2}\right)$$

Then $\tilde{A}$ changed into $\tilde{A}_1 = (2.5, 9, 13, 19.5, -1.5, -2, -2, -1.5)$

Similarly $\tilde{B}$ changed into $\tilde{B}_1 = (12.5, 19, 23, 29.5, -1.5, -2, -2, -1.5)$

Step 3: Group the pair of approximation coefficients of $\tilde{A}_1$. Then, find the new approximation coefficients and the detailed coefficients for the pair of approximation coefficient of $\tilde{A}_1$ $[2.5,9]$ and $[13,19.5]$.

$$\gamma_1 = \left(\frac{2.5+9}{2}\right), \gamma_2 = \left(\frac{13+19.5}{2}\right),$$

$$\eta_1 = \left(\frac{2.5-9}{2}\right), \eta_2 = \left(\frac{13-19.5}{2}\right)$$

Then, $\tilde{A}_1$ changed into $\tilde{A}_2 = (5.75, 16.25, -3.25, -3.25, -1.5, -2, -2, -1.5)$

Similarly, $\tilde{B}_1$ changed into $\tilde{B}_2 = (15.75, 26.25, -3.25, -3.25, -1.5, -2, -2, -1.5)$

Step 4: Find the pair of approximation coefficient in $\tilde{A}_2$. Then, find the new approximation and detailed coefficients for the pair of approximation coefficient of $\tilde{A}_2$. $[5.75, 16.25]$

$$\delta_1 = \left(\frac{5.75+16.25}{2}\right), \delta_2 = \left(\frac{5.75-16.25}{2}\right)$$

Then, $\tilde{A}_2$ changed into $H(\tilde{A}) = (11, -5.25, -3.25, -3.25, -1.5, -2, -2, -1.5)$

Similarly, $\tilde{B}_2$ changed into $H(\tilde{B}) = (21, -5.25, -3.25, -3.25, -1.5, -2, -2, -1.5)$

Step 5: Determine the Ranking

$\tilde{A} \prec \tilde{B}$, since the first element of the ordered tuple of $H(\tilde{A})$ is less than the first element of the ordered tuple of $H(\tilde{B})$.

5.2 Comparison between existing and proposed ranking method

Let $\tilde{A} = (1,4,7,11,15,18,21)$ and $\tilde{B} = (11,14,17,21,25,28,31)$ be two symmetric Heptagonal fuzzy numbers. The following table depicts the comparison between existing and proposed ranking method.

S. No	Ranking methods	Ranking values	Rank
1	New ranking function [8]	$R(\tilde{A}) = 10.54$ $R(\tilde{B}) = 20.54$	$A \prec B$
2	Robust ranking technique [1, 2, 3]	$R(\tilde{A}) = 34.75$ $R(\tilde{B}) = 64.75$	$A \prec B$
3	Ranking of Heptagon Number using Zero Suffix Method [7]	$R(\tilde{A}) = 7$ $R(\tilde{B}) = 17$	$A \prec B$
4	Haar ranking (proposed)	$R(\tilde{A}) = 11$ $R(\tilde{B}) = 21$	$A \prec B$

Table 1: Comparison between existing and the proposed ranking method

VI. FUZZY ASSIGNMENT PROBLEM

Suppose there are m jobs which are to be executed and m persons are available for doing these jobs. Assume that each person can do each job at a time.

Let $\tilde{P}_{ij}$ be a fuzzy cost of allocating the i^{th} person to the j^{th} job.

Let the decision parameter y_{ij} denote the allotment of the i^{th} person to the j^{th} job.

The problem is to determine an assignment (which job should be given to which person on one- one basis) so that the total cost of executing all jobs is optimum. These kinds of problems are said to be assignment problem. Mathematically, the problem is defined as

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$$\min \text{ (or) } \max X = \sum_{j=1}^{m}\sum_{i=1}^{m} \tilde{P}_{ij} y_{ij}$$

Subject to

$$\sum_{i=1}^{m} y_{ij} = 1 \quad for\ i = 1, 2, \ldots, m.$$

$$\sum_{j=1}^{m} y_{ij} = 1 \quad for\ j = 1, 2, \ldots, m.$$

Where

$$y_{ij} = \begin{cases} 1, \text{ if the } i^{th} \text{ crop is planted in the } j^{th} \text{ field} \\ 0, \text{ if the } i^{th} \text{ crop is not planted in the } j^{th} \text{ field} \end{cases}$$

$$\tilde{P}_{ij} = (\tilde{P}^1_{ij}, \tilde{P}^2_{ij}, \tilde{P}^3_{ij}, \tilde{P}^4_{ij}, \tilde{P}^5_{ij}, \tilde{P}^6_{ij}, \tilde{P}^7_{ij})$$

6.1 Algorithm for Solving Fuzzy Assignment Problem through Haar Wavelet:

Step 1: Construct the fuzzy cost matrix using the heptagonal fuzzy number.

Step 2: Apply Haar technique to each entries of the heptagonal fuzzy cost matrix.

Step 3: Normalize the Haar fuzzy number by the following accuracy function

$$N\left(H(\tilde{A})\right) = \left(\frac{a_1 + ma_2 + na_3 + a_4 + a_5 + na_6 + ma_7 + a_8}{8}\right)$$

Where $m, n \in (0,1)$

Step 4: Check whether the problem is balanced or unbalanced.

(i) If the number of rows and columns are equal, the problem is said to be balanced.

(ii) Otherwise, unbalanced.

Step 5: If the problem is balanced, then move to Step 6. If the problem is unbalanced, add dummy variable whose entries are zeros and make it balanced, then move to Step 6.

Step 6: Use the following Hungarian method to solve the assignment problem

Hungarian Algorithm:

Step (i): Choose the smallest element in each row and subtract with each element in the corresponding row.

Step (ii): Choose the smallest element in each column and subtract with each element in the corresponding column.

Step (iii): Cover all the zero entries of the cost matrix by drawing minimum number of lines through appropriate rows and columns.

Step (iv): To check optimality,

- The problem is solved if the minimum number of lines covering zero elements in the cost matrix (square matrix) equal to n (the number of rows or columns)
- If the minimum number of lines is less than n, then proceed to Step (v).

Step (v): Determine the smallest element which is not covered by any line. Add that element with the intersected elements by lines and subtract with uncovered elements by lines. Go to Step (iii) until the optimality is reached.

Step 7: Optimum allocation is determined for the problem.

6.2 Numerical Illustration for Fuzzy Assignment Problem:

Let us consider a fuzzy assignment problem. Suppose a farmer aims to plant four different crops in each of four equal sized fields. Rows represent different crops and columns represent different fields. The crop yield depends on enriched nutrients present in the soil. In order to enrich the soil quality, organic fertilizers, manures, and composts must be applied in the fields. Let $[\tilde{P}_{ij}]$ be the cost matrix whose elements are given by heptagonal fuzzy numbers. The farmer's objective is to find the optimal assignment of crops to fields in such a way that the total cost utilized for organic fertilizers becomes minimized. The fertilizer cost between crops (C_i, i= 1, 2, 3, 4) and fields (F_j, j= 1, 2, 3, 4) is given below as Fuzzy Heptagonal Cost Matrix in thousands (in rupees).

	F_1	F_2
C_1	(2, 5, 8, 12, 16, 19, 22)	(10, 12, 14, 15, 16, 18, 20)
C_2	(2, 4, 5, 7, 9, 10, 12)	(7.5, 9.5, 10.5, 12.5, 14.5, 15.5, 17.5)
C_3	(7, 9, 11, 13, 15, 17, 19)	(6, 8, 9, 11, 13, 14, 16)
C_4	(4, 6, 7, 9, 11, 12, 14)	(1, 3, 5, 7, 9, 11, 13)
	F_3	F_4
C_1	(4, 5, 6, 8, 10, 11, 12)	(7, 10, 12, 14, 16, 18, 21)
C_2	(4, 6, 9, 10, 11, 14, 16)	(2, 3, 4, 6, 8, 9, 10)
C_3	(10, 12, 14, 15, 16, 18, 20)	(10, 12, 15, 17, 19, 22, 24)
C_4	(7.5, 9, 10.5, 11.5, 12.5, 14, 15.5)	(8.5, 10.5, 12, 14.5, 17, 18.5, 20.5)

Table 2: Fuzzy Heptagonal Cost Matrix.

Then, Fuzzy Heptagonal Cost Matrix is transformed into Haar Fuzzy Heptagonal Matrix.

	F_1	F_2
C_1	(12, -5.25, -3.25, -3.25, -1.5, -2, -2, -1.5)	(15, -2.25, -1.75, -1.75, -1, -0.5, -0.5, -1)
C_2	(7, -2.5, -1.5, -1.5, -1, -1, -1, -1)	(12.5, -2.5, -1.5, -1.5, -1, -1, -1, -1)
C_3	(13, -3, -2, -2, -1, -1, -1, -1)	(11, -2.5, -1.5, -1.5, -1, -1, -1, -1)
C_4	(9, -2.5, -1.5, -1.5, -1, -1, -1, -1)	(7, -3, -2, -2, -1, -1, -1, -1)
	F_3	F_4
C	(8, -2.25, -1.25, -1.25, -0.5,	(14, -3.25, -2.25, -2.25, -1.5, -1, -1,

$_1$	-1, -1, -0.5)	-1.5)
C_2	(10, -2.75, -2.25, -2.25, -1, -0.5, -0.5, -1)	(6, -2.25, -1.25, -1.25, -0.5, -1, -1, -0.5)
C_3	(15, -2.25, -1.75, -1.75, -1, -0.5, -0.5, -1)	(17, -3.5, -2.5, -2.5, -1, -1, -1, -1)
C_4	(11.5, -1.875, -1.375, -1.375, -0.75, -0.5, -0.5, -0.75)	(14.5, -3.125, -1.875, -1.875, -1, -1.25, -1.25, -1)

Table 3: Haar Fuzzy Heptagonal Matrix.

Next, the optimal solution is obtained through the following Normalized accuracy function of Haar Heptagonal fuzzy number.

$$N\left(H(\tilde{A})\right)=\left(\frac{a_1+ma_2+na_3+a_4+a_5+na_6+ma_7+a_8}{8}\right)$$

Where $m,n\in\left(0,1\right)$

	F_1	F_2	F_3	F_4
C_1	0.053125	1.225	0.621875	0.73125
C_2	0.25625	0.94375	0.465625	0.371875
C_3	0.875	0.75625	1.225	1.23125
C_4	0.50625	0.125	0.963281	1.082031

Table 4: Normalized Haar Fuzzy Heptagonal Matrix.

To get the optimal allocation, the above assignment problem is solved using the Hungarian method. After solving, the minimum cost utilized for organic fertilizers is 1.775 thousand and the corresponding optimal allocation of crops in the fields is (1, 1), (2, 4), (3, 3) and (4, 2). It means that the crop C_1 is assigned to the field F_1, the crop C_2 is assigned to the field F_4, the crop C_3 is assigned to the field F_3 and the crop C_4 is assigned to the field F_2 to minimize the total cost utilized for organic fertilizers.

I. CONCLUSION

In this paper, the linear and non-linear heptagonal fuzzy numbers are formulated under uncertain environment and also the Haar ranking for the heptagonal fuzzy number is proposed. To verify the validity of the proposed ranking method, Assignment problem in agriculture is taken to determine the minimum cost for organic fertilizers applied in the field. The important outcomes of this research are:

- The derived various types of heptagonal fuzzy numbers are helpful to handle imprecise information and proposed the Haar ranking method will assist all kinds of decision making models for ranking the criteria.
- Under different environment, different types of heptagonal fuzzy number can be used by the decision makers with respect to the situations.

All the Multi Criteria Decision Making (MCDM) models such as DEMATEL, TOPSIS, VIKOR, etc., can be extended using this fuzzy number. Also, the linear and non- linear heptagonal fuzzy numbers will be useful to handle imprecision in real life problems such as control system, automatic system, engineering, science and technology.

REFERENCES

1. Bekheet, S., Mohammed S., & Hefny, H. A. A Generalized Polygon Fuzzy Number for Fuzzy Multi Criteria Decision Making, Springer International Publishing Switzerland 2014, pp. 415–423, 2014.
2. Bekheet, S., Mohammed, A., & Hefny, H. A. An Enhanced Fuzzy Multi Criteria Decision Making Model with a proposed Polygon Fuzzy number, International Journal of Advanced Computer Science and Applications, Vol. 5, No. 5, 2014.
3. Chandrasekaran, S., Kokila, G., & Junu Saju. Ranking of Heptagon Number using Zero Suffix Method, International Journal of Science and Research (IJSR), Volume 4 Issue 5, May 2015.
4. Devadoss, A.V. & Felix, A. A new Fuzzy DEMATEL method in an Uncertain Linguistic Environment, Advances in Fuzzy Sets and Systems, 16(2), (2013), 93-123.
5. Devadoss, A.V., & Felix, A. A Fuzzy DEMATEL approach to Study cause and effect relationship of Youth Violence, International Journal of Computing Algorithm, 2, (2013), 363-372.
6. Dhanasekar, S., Hariharan, S, Haar Hungarian Algorithm to Solve Fuzzy Assignment Problem, 113(7), (2017), 58- 66.
7. Dhurai, K., & Karpagam, A. New Membership Function and New Ranking Function on Heptagon Fuzzy Number, International Journal of Mathematics And its Applications, Volume 5, Issue 4-F (2017), 875-878.
8. Dubois, D., & Prade, H. Operations on Fuzzy Numbers, International Journal of Systems Science, 9(6), (1978), 613-626.
9. Fan, Z. P., & Liu, Y. A method for Group Decision-Making based on Multi-Granularity uncertain Linguistic Information, Expert Syst Appl, 37(5), (2010), 4000-4008.
10. Felix, A., & Devadoss, A.V. (2015). A new Decagonal Fuzzy Number under Uncertain Linguistic Environment, International Journal of Mathematics and Its Applications, 3(1), 89-97.
11. Felix, A., Christopher S., & Devadoss, A.V. A Nonagonal Fuzzy Number and Its Arithmetic Operation, International Journal of Mathematics and Its Applications, 3(2), (2015), 185-195.
12. Menakadevi, S., Abarna, B., & Tharani, R. Ranking of Heptagonal Fuzzy Number Method to Solve Balanced Zero Assignment Problem, IJSRSET, September-October-2017 [(3)6: 148-151].
13. Premalatha, D., & Murugan, P. Solving Fuzzy Assignment Problem for Heptagonal Fuzzy Number using Ones Assignment Method, IJSRSET, September-October-2017 [(3)6: 539-542].
14. Rathi, K., & Balamohan, S. Representation and Ranking of Fuzzy Numbers with Heptagonal Membership Function Using value and Ambiguity Index, Applied Mathematical Sciences, Vol. 8, 2014, no. 87, 4309 – 4321.
15. Sankar Prasad Mondal, Manimohan Mandal, Pentagonal fuzzy number, its properties and application in fuzzy equation, Future Computing and Informatics Journal 2 (2017) 110- 117.
16. Sudha, S., & Karunambigai, S. Solving a Transportation Problem using a Heptagonal Fuzzy Number, International Journal of Advanced Research in Science, Engineering and Technology, 4(1) ,2017.
17. Wang,Y. The Cognitive Processes of Formal Inferences}, International Journal of Cognitive Informatics and Natural Intelligence, 1(4), (2007), 75-86
18. Wang,Y., & Berwick, R.C. Towards a Formal Framework of Cognitive Linguistics, Journal of Advanced Mathematics and Applications, 1(2), (2012a), 250-263.

19. Zadeh, L. A. Fuzzy Logic and Approximate Reasoning, Syntheses, 30, (1975b), 407-428.
20. Zadeh, L. A. Fuzzy Sets, Information and Control, 8(3) (1965), 338-353.
21. Zadeh, L. A. The concept of a Linguistic variable and its Application to Approximate Reasoning (Part II), Information Science, 8, (1975), 301-357.

AUTHORS PROFILE

Mr. S. Karthik has completed his B.Sc Mathematics in Bharathidasan University and M.Sc Mathematics in University of Madras, Tamilnadu, India. He is currently doing Ph.D. in mathematics division, School of Advanced Sciences, VIT Chennai. His research interests are Fuzzy Decision-Making System, Fuzzy Inference System, and Machine Learning.

Dr. N. Punithavelan joined in VIT Chennai in 2011, as an Assistant Professor. He received his B.Sc in Physics from Madurai Kamarajar University, M.Sc in Materials Science from Bharathiar University, India and Ph.D. in MEMS from GIST, South Korea. His current research interests are theoretical and experimental investigations in Thin Film Technology, Solid State Physics, Fluid Dynamics and Fuzzy Logic.

Dr. Saroj Kumar Dash is working in VIT Chennai since 2011. Before joining this Institution he was doing his Post-Doctoral work in IIT Madras during 2009 to 2011 by NBHM Post-Doctoral fellowship. He finished his Ph.D. work from the Utkal University, India. His main areas of interests are: Random Fourier series, Control and Game Theory, Fuzzy logic, Web-page ranking algorithm.