Synchronization of a Square Fuzzy Transition Probability Relation Matrix using MATLAB

M. Geethalakshmi, G. Kavitha

Abstract: This paper aims to synchronize the square fuzzy transition probability relation matrix which is obtained from the lexical terms along with the help of fuzzy value which in turn calculated from the lexical value. Fuzzy matrices are essential in modeling uncertain situations in various fields. The relation mappings are measured using the lexical terms and the Octagonal fuzzy numbers are specially assigned for lexical values in turn converted to fuzzy value. Along with this, some operations on square fuzzy matrices are discussed by using max-min functions. Here the ambiguity among the relations is easily synchronized and reduced by using the most common tool called the MATLAB by finding the hidden pattern for the square fuzzy transition probability relation matrix. Moreover the major problems faced by IT Professionals are identified by giving suitable numerical example

Index Terms: Lexical Term, Lexical Value, Fuzzy Value, Square Fuzzy transition probability relational Matrix.

I. INTRODUCTION

Fuzzy theory has been introduced by Lotfi A. Zadeh [6] in the year 1965 and it has extensive applications in various fields. Throughout this article the unit interval which is also known as fuzzy value interval is taken as [0, 1], we say x ∈ [0, 1] if 0 ≤ x ≤ 1. Fuzzy matrices are represented as reference function in Dhar [2]. Also, an idea of determinant of the square fuzzy matrix is given in Dhar [1] in 2013. The same study has been extended by Dhar [3] to an idea of finding the determinant of the square fuzzy matrix and adjoint of square fuzzy matrix as well during the same year. Thomson in 1977 [5] first defined Fuzzy Matrices and discussed their convergence criteria of the powers. Kim and Roush [4] developed fuzzy matrix theory and it is an extension of Boolean matrices. Here, a numerical measure of the uncertainty of an outcome about transition probability relation is expressed linguistically. If the model is simulated by a technical expert, linguistic relations would be better than providing approximate probability values. The square fuzzy transition probability relation matrix is developed to handle uncertainty of linguistic decisions. Due to lack of available numerical data, the precise model cannot be keyed out. To grapple with uncertainty in transition probabilities, square fuzzy transition probability relation matrix is used. The paper focuses on how linguistic relations and their inherent fuzzy numbers can be victimized to interpret transition probabilities. Fuzzy number is a peculiar kind of fuzzy set defined over the set of all real numbers and employed for those uncertain entries of a transition matrix.

The Fuzzy Transition Matrix is a matrix with entries as fuzzy numbers. The method is legitimate for any sort of fuzzy number; we have used Octagonal Fuzzy Number, represented as 8-tuples. To contend with uncertainty in the transition probabilities, fuzzy methods are carried out for the desired calculations. The main approach as the path confronted in the literature has been presented. The first approach was traced out by a stochastic matrix employing a fuzzy relation. Numerous works have been brought out and connected to this view is given as both theoretical and application side [7-12]. J. Buckley considered the transition matrix comprising of fuzzy numbers [13-17]. It is quite interesting to note that all the classic probability theory can be fuzzified this way. Due to the natural connection phenomenon that exists between the transition probabilities and fuzzy numbers the proposed work discusses on square fuzzy transition probability relation matrix. The main objective is to manage with unsettled information in stochastic processes, when less or no data are available for the dynamic system conceived. The work focuses on finding the hidden pattern lying behind the dynamic system defined by square fuzzy transition probability relation matrix using MATLAB. Finally, we analyze the problems of IT Professionals in Chennai IT industry in their work place by identifying the hidden pattern and giving the conclusion. Moreover, the data is an unsupervised one and there is uncertainty in the concepts and it is analyzed by using fuzzy tool. The content of the research work is organized as: the methodologies, basic definitions are defined in section 2. The idea of Square Fuzzy Matrix and Synchronization using MATLAB are given in Section 3. Finally, the section 4 covers conclusion based on our study and future directions.

II. METHODOLOGIES

A. Basic Definitions

Definition: 2.1 Fuzzy Matrix (Fuz_M)
The Fuzzy Matrix (Fuz_M) “A” of order m × n is defined as

\[ A = \left[ \begin{array}{ccc} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{array} \right]_{m \times n} \]

where \( a_{ij} \) denotes the membership measure of an element \( a_{ij} \) in A. For simplicity, we write \( A = \left[ a_{ij} \right]_{m \times n} \).

Definition: 2.2 Square Fuzzy Matrix (SFuz_M)
The Square Fuzzy Matrix (SFuz_M) is defined as a fuzzy matrix having its elements between [0, 1], is called the unit fuzzy interval. A fuzzy matrix of order \( m \times n \), where \( m = n \) with elements belonging to [0, 1] is called a square fuzzy matrix.
Definition: 2.3 Square Fuzzy Transition Probability Relation Matrix

A Square Fuzzy Transition Probability Relation Matrix (SFTPRM) is an \( n \times n \) square fuzzy transition matrix \( A \). The domain of row \( i \) for a given membership degree \( \alpha \in [0,1] \) is the set \( \text{Dom}_i(\alpha) = \{x_{i1}, \ldots, x_{in}\} \cap \Delta_n = \{p_1, \ldots, p_m\} \in R^n: p_{ij} \in [\min_{ij} p_{ij}, \max_{ij} p_{ij}] \land \sum_j p_{ij} = 1 \} \). The domain of the whole matrix for a given \( \alpha \) is \( \text{Dom}(\alpha) = \{x_{i1}, \ldots, x_{in}\} \cap \Delta_n \). Note the elements of \( \text{Dom}(\alpha) \) are matrices of dimensions \( n \times n \) and the number of rows and columns are equal [12].

Definition: 2.4 Membership function for Octagonal Fuzzy Number

The Octagonal Fuzzy Number denoted by \( \text{OctFN} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8) \) where \( a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 \) are the real numbers and their membership function \( \mu_{\tilde{A}}(x) \) is given below:

\[
\begin{align*}
\mu_{\tilde{A}}(x) &= \begin{cases}
0, & x \leq a_1 \\
k \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\
k, & a_2 \leq x \leq a_3 \\
k, & a_3 \leq x \leq a_4 \\
k(1 - k) \frac{x - a_4}{a_5 - a_4}, & a_4 \leq x \leq a_5 \\
k, & a_5 \leq x \leq a_6 \\
k(1 - k) \frac{a_6 - x}{a_7 - a_6}, & a_6 \leq x \leq a_7 \\
k, & a_7 \leq x \leq a_8 \\
0, & x \geq a_8 
\end{cases}
\end{align*}
\]

where \( 0 < k < 1 \).

Here, the Significance of using Octagonal Fuzzy Number (OctFN) is to improve the result by getting the accurate value and the vagueness is also being reduced through this Octagonal Fuzzy Number (OctFN).

Definition: 2.5 Octant Value

The Octant Value of a Square Fuzzy Transition Probability Relation Matrix is given by

\[
\text{Oct Val} = \frac{1}{8} \left[ \sum_{i=1}^{n} \left( R_i, C_j \right) \right]^{1/8}
\]

where \( R_i \) denotes the row of the matrix and \( C_j \) denotes the column.

 Addition of Square Fuzzy Matrices

An addition for two Square Fuzzy Matrices \( \tilde{X} \) and \( \tilde{Y} \) is given by \( \tilde{X} + \tilde{Y} = \max_{SFuz-M} \{ \tilde{X}, \tilde{Y} \} \) or \( \min_{SFuz-M} \{ \tilde{X}, \tilde{Y} \} \).

Let \( \tilde{X} = \begin{bmatrix} 0.4 & 0.8 & 0.9 \\ 0.7 & 0.6 & 1 \\ 1 & 0.5 & 0.7 \end{bmatrix} \) and \( \tilde{Y} = \begin{bmatrix} 0.9 & 0.6 & 0.3 \\ 0.6 & 0.2 & 0.9 \end{bmatrix} \) then

\[
\tilde{X} + \tilde{Y} = \begin{bmatrix} 1 & 0.3 & 0.4 \\ 0.9 & 0.6 & 1 \\ 1 & 0.5 & 0.9 \end{bmatrix}
\]

Similarly, \( \min_{SFuz-M} \{ \tilde{X}, \tilde{Y} \} \) can be calculated. Here the number of rows and columns are equal [18].

 Multiplication of Square Fuzzy Matrices

The Product for the two Square Fuzzy Matrices is given by using max-min operation and min-max operation whose resultant is a fuzzy matrix. The Product is defined to be \( \tilde{X} \ast \tilde{Y} = \max_{SFuz-M} \{ \min_{SFuz-M} ( \tilde{X}, \tilde{Y} ) \} \) or \( \min_{SFuz-M} \{ \max_{SFuz-M} ( \tilde{X}, \tilde{Y} ) \} \).

Let \( \tilde{X} = \begin{bmatrix} 0.2 & 0.6 & 0.7 \\ 0.5 & 0.4 & 0.9 \\ 0.8 & 0.3 & 0.5 \end{bmatrix} \) and \( \tilde{Y} = \begin{bmatrix} 0.1 & 0.1 & 0.2 \\ 0.7 & 0.4 & 0.1 \\ 0.5 & 0.1 & 0.7 \end{bmatrix} \) then \( \tilde{X} \ast \tilde{Y} \) defined using

\[
\max_{SFuz-M, min_{SFuz-M}} \text{function } \tilde{X} \ast \tilde{Y} = \begin{bmatrix} C11 & C12 & C13 \\ C21 & C22 & C23 \\ C31 & C32 & C33 \end{bmatrix}
\]

where,

\[
C11 = \max_{SFuz-M} \{ \min_{SFuz-M} (0.2, 0.1), \min_{SFuz-M} (0.6, 0.7), \min_{SFuz-M} (0.7, 0.5) \} = \max_{SFuz-M} \{ 0.1, 0.6, 0.5 \} = 0.6
\]

\[
C12 = \max_{SFuz-M} \{ \min_{SFuz-M} (0.2, 0.1), \min_{SFuz-M} (0.6, 0.4), \min_{SFuz-M} (0.7, 0.1) \} = \max_{SFuz-M} \{ 0.1, 0.4, 0.1 \} = 0.4
\]

and so on. Thus \( \tilde{X} \ast \tilde{Y} = \begin{bmatrix} 0.6 & 0.4 & 0.7 \\ 0.5 & 0.4 & 0.7 \\ 0.5 & 0.3 & 0.5 \end{bmatrix} \)
Similarly, the \( \text{min}_\text{SFuz}_M, \text{max}_\text{SFuz}_M \) operation can be done and here the number of rows and columns are equal [18].

### III. SQUARE FUZZY MATRIX AND SYNCHRONIZATION USING MATLAB

The following table indicates the lexical terms for which lexical value is assigned with the help of Octagonal Fuzzy Numbers which in turn converted into Fuzzy Value.

<table>
<thead>
<tr>
<th>Table I. Fuzzy Value using Lexical Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Extreme Low (EL)</td>
</tr>
<tr>
<td>Low (Lw)</td>
</tr>
<tr>
<td>Fairly Low (FL)</td>
</tr>
<tr>
<td>Fair (F)</td>
</tr>
<tr>
<td>Fairly High (FH)</td>
</tr>
<tr>
<td>Moderate High (MH)</td>
</tr>
<tr>
<td>High (H)</td>
</tr>
<tr>
<td>Very High (VH)</td>
</tr>
</tbody>
</table>

### IV. ALGORITHM

The algorithm for this study is as follows:

Step1: Assigning Lexical Terms.
Step2: Fixing Lexical Value as Octagonal Fuzzy Numbers.
Step3: Converting Lexical Value to Fuzzy Value.
Step4: Framing Lexical Matrix.
Step5: Replacing Lexical terms by Fuzzy Value by framing Square Fuzzy Transition Matrix.
Step6: Calculate the Octal Value for Square Fuzzy Transition Probability Relation Matrix.
Step7: Identifying Hidden Pattern.
Step8: Analyzing the problems.

### V. NUMERICAL EXAMPLE

The Following are the example for relational maps for the common problems faced by IT Professionals in Chennai taken between the rows and columns as \( RM_1 \) to \( RM_8 \) are

Health Problems followed:\( RM_1 \) - Obesity, \( RM_2 \) - Insomnia, \( RM_3 \) - Frustration, \( RM_4 \) - Depression, \( RM_5 \) - Joint Pain, \( RM_6 \) - Stress, \( RM_7 \) - Vision Problem, \( RM_8 \) - Back/Neck Pain and \( CM_1 \) to \( CM_8 \) are Problems Related to Work Place as follows: \{ \( CM_1 \) - Constant Usage of Monitor, \( CM_2 \) - Project Deadline, \( CM_3 \) - No Proper Communication with Team Leader, \( CM_4 \) - Long Travelling Time, \( CM_5 \) - Sitting in same Posture, \( CM_6 \) - Working in Shift System, \( CM_7 \) - Benching, \( CM_8 \) - Migration from place to place / Transfer\} for the determination of Octant Value in which the Fuzzy Transition Probability Relation Matrix is defuzzified to an Octant Value and the minimum and maximum value of particular problem has been analyzed through this study. The relational mapping for the determination of Octant value is as follows:

![Figure I. Relational Mapping](image)

Using the above table, the lexical matrix is framed for the relation mappings which are given in-order to give the accuracy for an ambiguous fuzzy value. The Lexical Matrix is given as follows:

\[
\begin{bmatrix}
CM_1 & CM_2 & CM_3 & CM_4 & CM_5 & CM_6 & CM_7 & CM_8 \\
RM_1 & EL & Lw & FL & F & Dw & FL & H & VH \\
RM_2 & F & EL & FL & FL & F & FH & MH & H \\
RM_3 & FL & FH & EL & Lw & F & Lw & FL & MH \\
RM_4 & FL & FH & EL & Lw & F & VH & EL & FL \\
RM_5 & F & FH & FL & F & EL & H & VH & FL \\
RM_6 & MH & H & F & FH & VH & EL & FH & FL \\
RM_7 & FH & VH & FH & MH & H & Lw & EL & FL \\
RM_8 & F & FL & FL & Lw & VH & VH & H & EL & .
\end{bmatrix}
\]

The Lexical Matrix is converted as Square Fuzzy Transition Probability Relation Matrix with the help of fuzzy values are as follows: The Square Fuzzy Transition Probability Relation Matrix is given by
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\[ \begin{bmatrix}
CM_1 & CM_2 & CM_3 & CM_4 & CM_5 & CM_6 & CM_7 & CM_8 \\
RM_1 & 0 & 0.35 & 0.45 & 0.55 & 0.35 & 0.45 & 0.81 & 1 \\
RM_2 & 0.55 & 0 & 0.45 & 0.45 & 0.55 & 0.65 & 0.74 & 0.81 \\
RM_3 & 0.45 & 0.65 & 0 & 0.35 & 0.55 & 0.35 & 0.45 & 0.74 \\
SFTPRM = RM_4 & 0.35 & 0.45 & 1 & 0 & 0.55 & 1 & 0.81 & 0.35 \\
RM_5 & 0.55 & 0.65 & 0.45 & 0.55 & 0.81 & 1 & 0.45 & \\
RM_6 & 0.74 & 0.81 & 0.55 & 0.65 & 1 & 0 & 0.65 & 0.45 \\
RM_7 & 0.65 & 1 & 0.65 & 0.74 & 0.81 & 0.35 & 0 & 0.45 \\
RM_8 & 0.55 & 0.45 & 0.45 & 0.35 & 1 & 1 & 0.81 & 0 \\
\end{bmatrix} \]

The Octant Value for the Square Fuzzy Transition Probability Relation Matrix is obtained as follows:

\[ \text{Octant Value} = \begin{bmatrix}
0.808 \\
0.818 \\
0.800 \\
0.820 \\
0.823 \\
0.833 \\
0.826 \\
0.823
\end{bmatrix} \]

The hidden sequence of length 100, say for instance \{'EL'; 'Lw'; 'FL'; 'F'; 'FH'; 'MH'; 'H'; 'VH'\} of the Square Fuzzy Transition Probability Relation Matrix found through MATLAB is given by:

\{'F'; 'MH'; 'EL'; 'VH'; 'MH'; 'FH'; 'H'; 'Lw'\}
\{'VH'; 'MH'; 'FH'; 'H'\}

These hidden sequences of lexical terms are generated and it is useful in solving and giving solutions to any real-life problem existing with ambiguity. The toughest task in handling any dynamic problem is finding the hidden pattern or sequence that exists within them. Also, it was observed that the major problems face by IT Professionals are obtained as $RM_6$ - Stress and the minor problem occurs as $RM_3$ - Frustration.

VI. CONCLUSION

In this research paper, we have synchronized the square fuzzy transition probability relation matrix obtained from the lexical terms with the help of fuzzy value which was calculated from the lexical value. The relation mappings were measured using the lexical terms and the Octagonal fuzzy numbers were specially assigned for lexical values which in turn converted to fuzzy value. Along with this, some operations on square fuzzy matrices were discussed by using max-min functions. The ambiguities among the relations were synchronized and reduced using MATLAB by finding the hidden pattern for the square fuzzy transition probability relation matrix. Here the common problems of IT professionals in Chennai are analyzed in which, the health problems are taken in rows as $RM_1$ to $RM_8$ and the problems related to work place are taken in columns as $CM_1$ to $CM_8$ in order to determine Octant Value in which the Fuzzy Transition Probability Relation Matrix is defuzzified to an Octant Value and the minimum and maximum value of particular problem has been analyzed throughout this study. Moreover, the maximum octal value is observed to be 0.833 such that the most major problems among IT Professionals are identified as $RM_6$. Stress and the minimum value is observed to be 0.800 such that the most minor problem among IT Professionals are identified as $RM_3$ - Frustration.

REFERENCES


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