Optimization of spiral plate heat exchanger by gradient based optimizer

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Abstract*: Gradient based search algorithm, 'fmincon' is applied for thermal design optimization of the (SHE) spiral plate heat exchanger. SHE are self-cleaning by virtue of their spiral geometry, which ensures scrubbing of the any adhesives that get attached to the flow surface. This results in very minimal maintenance and greater reliability. The total life cycle cost minimization of the SHE is considered as the objective in the present study, subject to constraints of pressure drop, heat exchanger size and heat load. Results in this optimization study have been compared with those obtained by Genetic Algorithm (GA), Wind driven Algorithm (WDO). It is revealed that the total cost obtained by the FMINCON algorithm, post optimization, is 53.2%, 13% lower than Moretta [4], Bidabadi [1], and 13.157% higher than WDO [2]. However it is observed that the slightly higher cost as obtained by FMINCON is more than offset by the extremely poor value of "Overall heat transfer coefficient", obtained by WDO.

Index Terms*: Spiral plate heat exchanger (SHE), Cost optimization, FMINCON, pressure drop, heat load Wind driven optimization (WDO)

I. INTRODUCTION

Heat exchangers are the devices utilized for the exchange of heat energy starting with one media then onto the next in any physical state [3]. Spiral heat exchangers (SHE) are the perfect sort for cooling slurries and liquids with high viscosity being the trademark that recognizes this heat exchanger from the others [5].

SHE finds applications in sugar industry, paper industry, and chemical industries, for either condensation or evaporation of the fluids [6]. The fluid stream temperatures during the operation are of utmost interest since they effect the economy of the industry.

For example the expulsion of water from food by SHE reduces the destruction of the proteins and induces the microbial stability, thereby decreasing storage and transportation cost. Dairy farms are one of the industrial applications where evaporation and dehydration are used. Novel feature of the SHE is its ability to handle highly viscous fluids and the negligible fouling tendency [5].

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The geometric construction of a spiral plate heat exchanger contains two long plates rolled together in a spiral path. To improve the mechanical strength among the plates they are welded with studs placed between [8]. Since this geometry can provide uniform directional change local turbulence takes place eliminating the stagnant zones and creating self-cleaning flow passage [3]. V. N. KapatkatG and E. Kondhalkar studied the oil extraction system and reported that the spiral tube can be employed for cost reduction [10]. P. Naphon studied the pressure drop and the thermal performance of the helical coil heat exchanger and reported that the mass flow rate of the both the hot and cold fluids have a significant effect on the performance of the heat exchangers [11]. The design approach of the spiral plate heat exchangers was studied by the M. P. Nueza et al and it was noticed that the heat transfer rate for the spiral plate heat exchangers is comparatively more for the spiral plate heat exchanger than another heat exchanger [8].Egner and Burmeister [12] analyzed the flow behavior through the spiral ducts with rectangular cross section, using computational fluid dynamics. With the computational results, Nusselt number correlation was developed as function of mean diameter of the spiral. Results shows that at the entry of the spiral the heat transfer coefficient is maximum and after the entry length the heat transfer coefficient is almost constant. Most important contribution was the estimation of the thermal entry length for the flow through spiral ducts for laminar flows. Burmeister [13] also developed approximate solution to evaluate the thermal effectiveness of SHE for specified thermal units.Heat exchanger optimization is one of the key research field in thermal engineering. A horde of techniques from the classical methods such as Lagrange multiplier, dynamic programming, geometric programming nonlinear programming method and various non-classical techniques such as the discrete maximum, principle random search is employed for such purposes. A detailed survey on different techniques involved for the optimum design of heat exchangers till the 90's with the limitations and merits of the available techniques for the heat exchanger optimization are highlighted. [15]Minton [16] have developed correlations to estimate the heat transfer coefficient and pressure drop for laminar and turbulent flow regimes. Later Martin [17] developed the correlations for pressure drop considering the friction factor.In the present work, *'fmincon'* a gradient based optimization technique is used find the optimal design parameters of the SHE with allowable pressure drop. Case studies reported by Moretta [14], Bidabadi [1], is used to demonstrate the capabilities of the '*fmincon'*.

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A. Mathematical formulations

1) Heat transfer

The thermal equilibrium equation for a heat exchanger stated in [7] is given by the equation below:

$$Q = m_h c_h (T_{hi} - T_{ho}) = m_c c_c (T_{co} - T_{ci}) \quad \text{Eq. (1)}$$

The physical principle being law of conservation of energy i.e. heat lost by the hot fluid stream must be equal to the heat gained by the cold fluid stream with an assumption of negligible heat loss to the surroundings. The thermal equilibrium equation is used in the present study to evaluate the total heat load on the heat exchanger.

When the temperatures at inlet and outlet are pre-defined, novel method to evaluate the overall heat transfer coefficient is to utilize the concept of logarithmic mean temperature difference (LMTD), the heat duty is calculated as follows:

$$Q = U\,ALMTD \quad \text{Eq. (2)}$$

The overall heat transfer coefficient considering the influence of heat transfer coefficient of cold (h_c) and hot (h_h) streams along with conduction (k_p) in plates and fouling factor (R_f) is expressed as follows:

$$U = \frac{1}{\frac{1}{h_h} + \frac{1}{h_c} + \frac{t}{k_p} + R_f} \quad \text{Eq. (3)}$$

The logarithmic mean temperature difference (LMTD) is calculated by the equation below, the terminology is consistent with the ones in the nomenclature

$$LMTD = \frac{(T_{hi} - T_{co}) - (T_{ho} - T_{ci})}{\ln\left(\frac{(T_{hi} - T_{co})}{(T_{ho} - T_{ci})}\right)} \quad \text{Eq. (4)}$$

The heat transfer area and the Nusselt number are calculated from the expressions below:

$$A = 2 \times L \times H \quad \text{Eq. (5)}$$

$$Nu = 0.239 \times \left(1 + 5.54\frac{D_h}{R_m}\right) Re^{0.806} Pr^{0.268} \quad \text{Eq. (6)}$$

Where D_h is the hydraulic diameter and R_m is the mean diameter of the heat exchanger the Reynolds number, in terms of mass flux (M) can be expressed as follows:

$$Re = \frac{MD_h}{\mu} \quad \text{Eq. (7)}$$

$$M = \frac{m}{A_c} = \frac{m}{HS} \quad \text{Eq. (8)}$$

The mean radius R_m is calculated as follows:

$$R_m = \frac{R_{min} + R_{max}}{2} \quad \text{Eq. (9)}$$

From the physical properties of the stream evaluate Prandtl number from the equation below

$$Pr = \frac{\mu c_p}{k} \quad \text{Eq. (10)}$$

By calculating the Nusselt number from the Reynolds number and Prandtl number, evaluate heat transfer coefficient for the stream by the equation

$$h = \frac{k.Nu}{D_h} \quad \text{Eq. (11)}$$

2) Pressure drop

Pressure drop in both streams and external diameter of the spiral are calculated by the equation below as stated in [2] [11]

$$\Delta p = \frac{1.45(LV^2\rho)}{1.705} \quad \text{Eq. (12)}$$

$$D_s = [1.28L(b_h + b_c + 2t) + C^2]^{0.5} \quad \text{Eq. (13)}$$

3) Cost function

The total cost is obtained by summing up the manufacturing cost (C_i) and operational cost (C_{od}) expressed as

$$C_{total} = C_i + C_{od} \quad \text{Eq. (14)}$$

The manufacturing cost (C_i) is a function of surface of the heat exchanger and is given by Hall equation [21] expressed as follows

$$C_i = e_1 A^j \quad \text{Eq. (15)}$$

Where e_1 = 5973 and j =0.59 as stated in [1]. The operational cost

$$C_{od} = \sum_{k=1}^{ny} \frac{C_0}{(1+i)^k} \quad \text{Eq. (16)}$$

$$C_0 = P \times C_E \times Hr \quad \text{Eq. (17)}$$

$$P = \frac{1}{\eta}\left(\frac{m_h}{\rho_h} \times \Delta P_h + \frac{m_c}{\rho_c} \times \Delta P_c\right) \quad \text{Eq. (18)}$$

II. PROBLEM DESCRIPTION

The goal of the present problem is to minimize the total cost of a spiral heat exchanger subject to certain constraints. This study considers the case study reported by Moretta [4] and the data from it is considered for the optimization. The gradient based search algorithm ‘fmincon’ is applied to this case study and the results are compared to the optimization results obtained by applying GA and WDO.The total cost of a SHE includes the manufacturing and operation cost and this is to minimized

$$Minimize\ J = C_{total} = C_i + C_{od}$$

These are four constraints in this optimization study and they are with regard to the pressure drop that occurs as the fluid stream flow through hot and cold sides, the total heat load

limitation and the size limitation of the SHE
The constraints are

$$\Delta p_h - \frac{1.45\left(LV_h{}^2\rho_h\right)}{1.705} = 0 \qquad \text{Eq. (19)}$$

$$\Delta p_c - \frac{1.45\left(LV_c{}^2\rho_c\right)}{1.705} = 0 \qquad \text{Eq. (20)}$$

$$D_s - [1.28L(b_h + b_c + 2t) + C^2]^{0.5} = 0 \qquad \text{Eq. (21)}$$

$$Q - (U\ (2LH)LMTD) = 0 \qquad \text{Eq. (22)}$$

All these constraints are highly non-linear. The design variables in this study to be optimized are pressure drop in the hot as well as cold fluids, outer diameter length and width of SHE. The values of their upper and lower bounds of all the design variables are given in the table below.

Table 1 Table specifies the lower and upper bounds as specified by Moretta [4]

Design variable	Lower bound	Upper bound
Pressure drop in hot fluid	0	172
Pressure drop in cold fluid	0	172
Outer diameter	0.5	1.5
Length	5	22
Width	0.05	2.3

Mathematically problem can be described as follows

$$Minimize\ J = C_{total} = C_i + C_{od}$$

$$Subjected\ to:$$

$$\Delta p_h - \frac{1.45\left(LV_h{}^2\rho_h\right)}{1.705} = 0$$

$$\Delta p_c - \frac{1.45\left(LV_c{}^2\rho_c\right)}{1.705} = 0$$

$$D_s - [1.28L(b_h + b_c + 2t) + C^2]^{0.5} = 0$$

$$Q - (U\ (2LH)LMTD) = 0$$

The optimization results are compared with the ones obtained by wind driven optimization [2]. The physical properties of hot and cold stream are adapted from Moretta *et al.* [4] and cross verified in Bidabadi [1], are tabulated in the table below

Table 2 physical properties of hot and cold stream

Property	Hot stream	Cold stream	Units
Mass flow rate	127.75	18.92	[kg.s^{-1}]
Inlet temperature	298.60	273.15	[K]
Outlet temperature	298.15	285.93	[K]
Specific heat capacity	4186.80	3768.12	[J.kg^{-1}.K^{-1}]
Thermal conductivity	0.6228	0.5813	[W.m^{-1}.K^{-1}]
Density	1350	1000	[kg.m^{-3}]
Viscosity	0.10353	0.00122	[N.s.m^{-2}]
Velocity	4.4006	3.2597	[m.s^{-1}]

Additional information needed to calculate the operating costs and the heat transfer coefficients are tabulated below:

Table 3 Additional parametric information essential for optimization

Parameter	Symbol	Value	Units
Number of years	ny	15	[Years]
Annual discount rate	i	10%=0.1	-
Energy cost	C_E	0.00012	[Euros/W.hr]
Annual work hours	Hr	8000	[hr. Year^{-1}]
Thermal conductivity of material	$K_{material}$	14.532	[W.m^{-1}.K^{-1}]
Fouling factor	R_f	0.00034682	[m^2.K.W^{-1}]
Core diameter	C	0.3048	[m]

III. RESULTS AND DISCUSSION

Fmincon, which is a gradient based optimizer utilizing the Lagrangian multiplier technique is used in the present study with total cost Eq. (14) as the objective function satisfying the nonlinear constraints specified in the section (II). The total cost is a summation of operating and capital costs. The optimized value of the length (L) and width (H) are essential as the capital cost is dependent on these parameters.
The total heat duty (Q) was constant as the inlet and outlet temperatures were fixed. The average properties of the hot and cold streams as stated in the table (dataset) was used. The stopping criteria for the optimization was set to 1e-12 in the optimization tool box in MATLAB.

Table 4 Design variable values from literature and Fmincon

Parameter	Moretta(Case study)	Bidabadi (GA)	V.C. Mariani (WDO)	fmincon
$(\Delta p)_{hot}$	110.190	12.00	111.198	107.254
$(\Delta p)_{cold}$	85.430	0.889	45.195	43.592
D_s	0.849	0.726	2.280	1.5
L	7.8170	5.9300	5	5
H	0.9144	2.2880	0.6720	0.855
U	1113	586	75	1551.314
C_{total}	44813	22491	18186	20941.279 1

It can be observed from Table 4 that the ‘fmincon’ converges at a cost of 20941.2791 which is about 15% higher than the one in the literature as shown in the table above. Overall heat transfer coefficient (U) which signifies the physics of heat transfer has been increased by 20 times. Fig. 1 represents in a bar chart, the optimized values of the design variables stated in Table 4The pressure drop in hot and cold streams, length remains almost similar with difference being less than 10%. The variation in the outer spiral diameter and width contributes more to the total cost of the heat exchanger which in turn considers the effect of the pressure drop as a constraint in both fluid streams

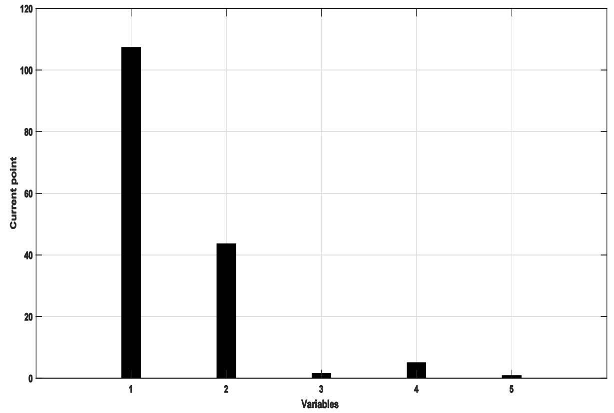

Fig. 1 Optimized parameter plot

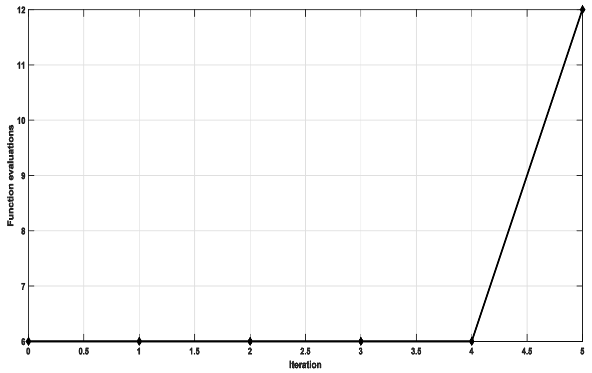

Fig. 3 Function evaluation count

Figure (2) depicts the variation of the objective function with iterations. It can be observed that from iteration 0 to 1 the variation is abrupt but after the iteration 2 the variation is asymptotic.

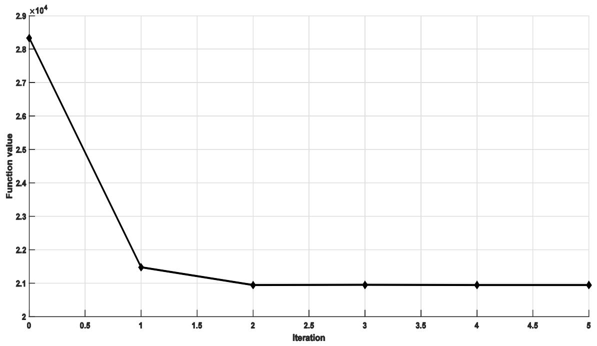

Fig. 2 Variation of objective function with iterations

Even after achieving the goal of minimization at iteration number 2, there are further iterations to satisfy the constraints and to achieve the first order optimality criterion. The variation of number of functional evaluations per iterations are shown in the figure [3]. It can be seen that, at the convergence, the functional evaluations are maximum, which indicates the exhaustive search along the gradient for minima. The total cost obtained after optimization using 'fmincon' algorithm is about 13.157% higher than the cost obtained by WDO algorithm, implying the inferiority of 'fmincon'. However, this conclusion is misleading and unfound for, as a more meticulous examination of the optimization results reveal that the value of overall heat transfer coefficient, as obtained in the present study is disproportionately superior than that obtained by WDO. The value of the 'U' is 20.6841 times larger than the value of 'U", optimized by WDO. This clearly highlights the superiority of the 'fmincon' algorithm over WDO.

A comparison with the other two studies Moretta [4] and Bidabadi [1], reveals that the cost obtained by FMINCON is lower. Also the value of 'U' is thrice that of study [1] and 28.2541 % more than that of Moretta [4].

IV. CONCLUSION

Thermal design optimization of SHE is carried out using a gradient based search algorithm, "fmincon", with the objective of minimizing the total cost. The optimized design parameter values are 107.254, 43.592, 1.5, 5 and 0.855 for pressure drop on hot side, pressure drop on cold side, overall diameter, length and width of SHE. Results reveal that the cost of SHE obtained is lesser than the ones repeated by Moretta [4], Bidabadi [1] earlier and only slightly more than the value reported by WDO algorithm [2]. But the value of overall heat transfer coefficient obtained in the present study is superior to all the above three studies.

Therefore, in future, a study with dual objective of minimum the cost and maximization of overall heat transfer coefficient can be carried out for getting better insights into design optimization.

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