

# Bianchi Type-VI0 Dark Energy Universe Supporting Accelerated Expansion of the Universe

Priyanka Garg, Rashid Zia, Anirudh pradhan

**Abstract:** In 1998, the surprising discovery of the accelerated expansion of our universe rather than the prior conviction that expansion rate of the universe is constant or decreasing, forced the researchers to construct such cosmological models, which support the accelerated expansion of the universe. In this attempt, there is a trend to modify the earlier models (supporting the constant rate of expansion), by assuming the time-dependent average scale factor  $a(t) = (ABC)^{\frac{1}{3}}$ . In the present paper also, we have revisited the dark energy (DE) model in Bianchi type-V  $I_0$  space-time under the new scenario of accelerated expansion of the universe, assuming the deceleration parameter (DP)  $q$  as a mapping linearly of the Hubble constant  $H$ . From this assumption we find the time-dependent average scale factor  $a(t) = \exp\left[\frac{1}{\beta}\sqrt{2\beta t+k}\right]$

which yields a time dependent  $q(t) = -1 + \frac{\beta}{\sqrt{2\beta t+k}}$ , where  $k$  and  $\beta$  are arbitrary constants of integration. By using new constraints from SN Ia and BAO/CMB data (Giostri et al., JCAP, 2012), we obtain  $k = 0.00001$  and  $\beta = 0.0062$  for a cosmic deceleration to acceleration. Our model and its current range for DE,  $\omega$  (EoS) parameter is observed to be in great concurrence with the ongoing SNeIa observations. For the derived model, we have calculated various physical parameters also and found them supporting current observations.

**Keywords:** Bianchi type-V  $I_0$  universe: dark energy, variable deceleration parameter, accelerated expansion.

## I. INTRODUCTION

Till 1998 it was generally accepted by almost every scientist in cosmology and relativity field that the cosmos is expanding, and the expansion rate is decreasing, because of gravity following up on the matter. The question was, how quickly it is slowing? In 1998 and following years, many groups of astronomers [1–7], in their search for the estimation of the universe expansion rate have observed some surprising results. These groups, in view of SN Ia observations have estimated the separations and predicted the accelerated expansion of the cosmos, which is probably going to continue forever. It is anticipated by these observation that something is responsible for this acceleration. to answer this, again came into picture, the cosmological constant  $\Lambda$ , that can be the best candidate. In this way, the conventional  $(\Omega_\Lambda, \Omega_M) = (0, 1)$  cosmos is discarded and a new pair  $(\Omega_\Lambda \approx 0.7, \Omega_M \approx 0.3)$  is proposed. Such estimation of  $\Omega_\Lambda$  (density parameter) corresponds a very low value of cosmological constant  $\Lambda \approx 10^{-35} s^{-2}$ .

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An extreme research is going ahead, in both observations and theory, to divulge the genuine idea of this cosmic acceleration. It is usually accepted by the cosmology scientist that a type of repulsive force which works as anti-gravity, generated somewhere in the range of 7 billion years back, is the reason of speeding up the expansion of the universe. This obscure physical entity is named as 'dark energy'. Also, on large scales, the universe is having flat geometry as predicted by the cosmic microwave background (CMB) [8]. Since there isn't sufficient matter in the universe (either dark or ordinary matter), so to deliver this flatness, the same 'dark energy' may be a candidate. In addition, the impact of dark energy appears to fluctuate, with the expansion of the universe decreasing and increasing over the period of time. Cosmological constant  $\Lambda$  is the simplest candidate for Dark Energy (DE) yet it should be, to a great degree, tweaked to fulfill the present estimation of the dark energy. On the other hand, to clarify rot of the density, numerous unique models have been recommended, where  $\Lambda$  varies with cosmic time ( $t$ ) [9–11]. Along with this, various researchers considered many alternative candidate as a possible source of dark energy in different scenario [12–24]. Other than these research, a few cosmologists have considered modified gravitational theories as other options to DE causing late-time speeding expansion of the universe [25–32]. The models of DE have been presented in a formal

way by an Equation of State (EoS)  $\omega(t) = \frac{p}{\rho}$ , which can be constant or variable [33] and the current observations appear to marginally support an emerging DE ( $\omega > -1$ ) in early time and ( $\omega < -1$ ) as of now. A few endeavors have been made to develop DE model such that phantom divide ( $\omega = -1$ ) can be crossed by  $\omega$ . The least DE name, which is scientifically proportionate to  $\Lambda$  is the vacuum energy ( $-1 = \omega$ ). Some other options, are phantom energy ( $-1 > \omega$ ) and quintessence ( $-1 < \omega$ ) [34, 35]. Two different cutoff got from observational outcomes originating from SNeIa information [36] and jointly with [37] are  $-0.62 > \omega > -1.67$  and  $-0.79 > \omega > -1.33$ , respectively. The most recent outcomes given in 2009, compel the DE (EoS) to  $-0.92 > \omega > -1.44$ , with confidence limit of 68% [38, 39]. In any case, it isn't at all required to assume a constant estimation of  $\omega$ . Because of the absence of the observational proof in making a refinement amongst variable and constant  $\omega$ , more often than not the EoS parameter is taken as a constant [40] with stagewise estimation 0, -1, +1 and  $-\frac{1}{3}$  for dust fluid, vacuum fluid, stiff and radiation dominated universe, respectively. In any case when all is said in done,  $\omega$  is a function of redshift  $z$ , or scale factor  $a$  or time  $t$  too [41–43].



So, in the present paper, we have discussed Bianchi type-V  $I_0$  DE universe under the new scenario of accelerated expansion of the universe, assuming the time dependent average scale factor  $a(t) = \exp[\frac{1}{\beta}\sqrt{2\beta t + k}]$ . As a consequence, we have found time dependent deceleration parameter  $q(t)$ . Also, for a metric which is homogeneous (spatially), letting that the expansion scalar ( $\theta$ ) is proportional to the shear scalar ( $\sigma$ ). This condition gives  $A = \ell_2 B^m$ , where  $\ell_2$  and  $m$  are constants (Collins [44]). Under these assumption we have calculated various physical parameters and found them in good agreement with three current observations (SNIa supernovae).

**II. THE METRIC AND BASIC FIELD EQUATIONS**

considering **LRS** metric for Bianchi type-V  $I_0$  as:

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x} dy^2 - C^2 e^{-2x} dz^2, \tag{1}$$

where the metric potential  $A$ ,  $B$  and  $C$  are functions of time only.

The Einstein's field equations (EFEs) ( $8\pi G = c = 1$  in gravitational units,) read as :

$$R_j^i - \frac{1}{2} R g_j^i = -T_j^i, \tag{2}$$

here the notations stands for their standard meaning.

The fluid energy momentum tensor is defined as :

$$T_j^i = \text{diag}[T_0^0, T_1^1, T_2^2, T_3^3] \tag{3}$$

Eq. (3) can be parameterized as:

$$\begin{aligned} T_j^i &= \text{diag}[\rho, -p_x, -p_y, -p_z], \\ &= \text{diag}[1, -\omega_x, -\omega_y, -\omega_z]\rho, \\ &= \text{diag}[1, -(\omega + \delta), -\omega, -(\omega + \gamma)]\rho, \end{aligned} \tag{4}$$

where the energy density is denoted by  $\rho$ ,  $\omega_x, \omega_y$  and  $\omega_z$  and  $p_x, p_y$  and  $p_z$  represents the directional EoS parameter and pressures in  $x, y$  and  $z$  directions respectively.  $\omega$  is the deviation-free EoS parameter. Here, we have parametrized the deviation from isotropy by taking  $\omega_x = \omega$ , now introducing two parameters  $\delta$  and  $\gamma$  called skewness parameters as deviations from  $\omega$  in  $y$  and  $z$  directions respectively. Solving the EFE (2), (co-moving coordinate system), taking  $T_j^i$  as per Eq. (4), for the metric (1), we get the system of

$$\begin{aligned} \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} &= -\omega\rho, \\ \frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} &= -(\omega + \delta)\rho, \\ \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} &= -(\omega + \gamma)\rho, \\ \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} &= \rho, \\ \frac{\dot{C}}{C} - \frac{\dot{B}}{B} &= 0, \end{aligned}$$

differential equations given below :

$$\tag{5}$$

$$\tag{6}$$

$$\tag{7}$$

$$\tag{8}$$

$$\tag{9}$$

where an over dot denote OD (ordinary differentiation) with  $t$ .

The average scale factor for the metric (1) is defined as

$$a = (ABC)^{\frac{1}{3}}, \tag{10}$$

The expression for  $H$  (Hubble parameter) is defined as:

$$H = \frac{1}{3} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = \frac{1}{3} (H_x + H_y + H_z) = \frac{\dot{a}}{a}, \tag{11}$$

here  $A/A' = H_x, B/B' = H_y$  and  $H_x = H_z$  are the directional parameters (Hubble's) in  $x, y$  and  $z$  directions respectively.

As usual DP (the deceleration parameter)  $q$  is conventionally given as :

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{12}$$

The expansion scalar  $\theta$ , shear scalar  $\sigma^2$  and the anisotropy parameter  $A_m$  are given as :

$$\begin{aligned} \theta &= \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} = 3H, \\ \sigma^2 &= \frac{1}{2} \left[ (H_x^2 + H_y^2 + H_z^2) - \frac{\theta^2}{3} \right] \\ A_m &= \frac{1}{3} \sum_{i=1}^3 \left( \frac{\Delta H_i}{H} \right)^2, \end{aligned} \tag{13}$$

where  $\Delta H_i = H_i - H (i=3,2,1)$ .

**III. Solution of the field equations**

Eq.(9), integration gives :

$$C = \ell_1 B, \tag{16}$$

here constant of integration is  $\ell_1$ .



Putting  $C = \ell_1 B$  in Eq. (6) and subtracting the result from Eq. (7) we obtain  $\delta = \gamma$ . Therefore Eqs. (5)-(9) reduces to system of three independent equations given below :

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{4} \frac{1}{A^2} = -(\gamma + \omega)\rho, \quad (17)$$

$$2\frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 + \frac{1}{A^2} = -\omega\rho, \quad (18)$$

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \rho. \quad (19)$$

Now, we have three independent field equations (17) – (19) in  $A, B, \rho, \omega$  and  $\gamma$  (five unknown parameters). Therefore, to obtain explicit solutions of the system, two additional constraints relating these parameters are required.

First, letting that the expansion scaler ( $\theta$ ) is proportional to shear ( $\sigma$ ). The condition with Eq. (16) (13) and (14) yields

$$A = \ell_2 B^m, \quad (20)$$

where  $\ell_2$  is a constant of integration. The motivation for letting above condition by [45].

Second, motivated by the recent observations, letting DP  $q$  as a linear function of Hubble parameter  $H$  as :

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \beta H + \alpha = \alpha + \beta \frac{\dot{a}}{a}, \quad (21)$$

where  $\alpha$  and  $\beta$  are arbitrary constants. Solving above equation yields

$$a = \exp\left[-\frac{(1+\alpha)t}{\beta} - \frac{1}{(1+\alpha)} + \frac{k}{\beta}\right], \quad (22)$$

*provided  $\alpha \neq -1$*

where  $k$  is a constant of integration. Here we get the value of DP as  $q = -1$ .

For  $\alpha = -1$ , we have a singularity. So, in this case, we find the solution separately. In this case Eq. (21) is reduced to

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \beta H = -1 + \beta \frac{\dot{a}}{a}. \quad (23)$$

The solution of above equation is found to be

$$a = \exp\left[\frac{1}{\beta}\sqrt{2\beta t + k}\right] \quad (24)$$

In this case, we get the value of DP as

$$q = -1 + \frac{\beta}{\sqrt{2\beta t + k}}. \quad (25)$$

Since, from recent observational data, it is established that the universe expansion rate is time dependent instead of a constant. So, we ignore the first case  $\alpha \neq -1$ , because it results into constant DP  $q = -1$ .

Now, we consider the case  $\alpha = -1$ , which gives the time dependent value of DP as in Eq. (25).

Using Eqs. (16), (20) and (24) in Eq. (11) and integrating, we get

$$A = \exp\left[\frac{1}{\beta}\sqrt{2\beta t + k}\right] \quad (26)$$

Using Eqs. (20) and (26), we get

$$B = \frac{1}{\ell_2} \exp\left[\frac{1}{\beta m}\sqrt{2\beta t + k}\right] \quad (27)$$

Eqs. (16) and (27) give the value of  $C$  as

$$C = \frac{\ell_1}{\ell_2} \exp\left[\frac{1}{\beta m}\sqrt{2\beta t + k}\right] \quad (28)$$

Therefore, the geometry of the metric (1) is reduced to

$$ds^2 = -dt^2 + \exp\left[\frac{1}{\beta}\sqrt{2\beta t + k}\right]^2 dx^2 + \frac{1}{\ell_2} \exp\left[\frac{1}{\beta m}\sqrt{2\beta t + k}\right]^2 e^{2x} dy^2 - \frac{\ell_1}{\ell_2} \exp\left[\frac{1}{\beta m}\sqrt{2\beta t + k}\right]^2 e^{-2x} dz^2. \quad (29)$$

#### IV. Results and Discussions

From Eq. (23), we have  $q_0 = -1 + \beta H_0$ , where  $q_0$  and  $H_0$  are the present values of DP and Hubble parameter respectively. Taking  $q_0 = -0.54$  and  $H_0 = 73.8$  (Giostri et al. [46]), we get  $\beta = 0.0062$ . Also, we have  $H = \frac{\dot{a}}{a} = \frac{1}{\sqrt{2\beta t + k}}$ , so that  $H_0 = \frac{1}{\sqrt{2\beta t_0 + k}}$ . Using  $H_0 t_0 = 1$ , we can get  $H_0 = \frac{1}{\sqrt{\frac{2\beta}{H_0} + k}}$

for  $k$ , we have  $k = \frac{1}{H_0} \left[\frac{1}{H_0} - 2\beta\right]$ . Substituting the values of  $H_0$  and  $\beta$ , we can find the value of  $k = 0.00001$ . Thus the values of  $\beta = 0.0062$  and  $k = 0.00001$  are the best fit with the latest observation. These values of  $\beta, k$  have been considered to draw all the figures in the paper.

Now, we know that the average scale factor in terms of redshift  $z$  is given by

$$\frac{a_0}{a} = 1 + z, \quad (30)$$

where  $a_0$  is the present value of the average scalefactor  $a(t)$ . Since factor  $a(t) = \exp\left[\frac{1}{\beta}\sqrt{2\beta t + k}\right]$ . so we obtain

$$\frac{a_0}{1+z} = \exp\left[\frac{1}{\beta}\sqrt{2\beta t}\right]$$



$$= \beta \log \left( \frac{a_0}{1+z} \right) \quad \text{or} \quad z = \frac{a_0}{\exp\left[\frac{1}{\beta}\sqrt{2\beta t + k}\right]} - 1. \quad (31)$$

Figure 1 shows the variation of redshift  $z$  versus cosmic time  $t$ , for  $a_0 = 3.6366$  (calculated from Eq. (24)), which indicates that late time universe is equivalent to redshift  $z = -1$ .

From Eq.(25), we get the value of DP in term of redshift as

$$q(z) = \frac{1}{\log\left(\frac{a_0}{1+z}\right)} - 1. \quad (32)$$

Figure 2(a) corresponding to the Eq. (25), portrays the behaviour of DP ( $q$ ) with cosmic time ( $t$ ), which gives the behavior of  $q$ . The model is emerging from early decelerated stage to an accelerating stage at present (i.e. transit universe) for  $k = 0.00001$  and  $\beta = 0.0062$ . These values of  $\beta$  and  $k$  are obtained as above for consistency with observations Giostri et al. [46]. Figure 2(b) shows the behaviour of DP ( $q$ ) versus redshift  $z$ , and also presents the same nature.

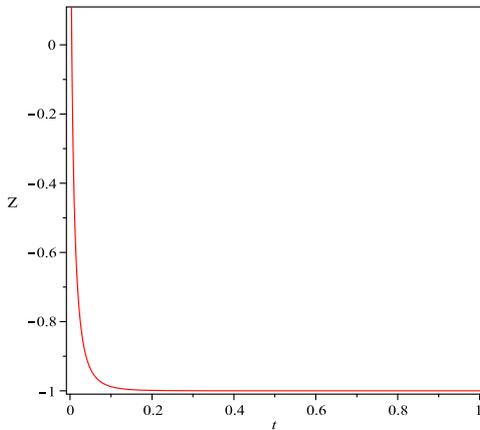


Figure1: the plot of Red Shiftz Vs time t

Here,  $H$  (Hubble parameter),  $\theta$  (scalar of expansion),  $\sigma$  (shear scalar) and  $A_m$  (average anisotropy parameter) are defined as usual and calculated for our model (29) as:

$$H = \frac{\dot{a}}{a} = \frac{1}{\sqrt{2\beta t + k}} = \frac{1}{\beta \log \left( \frac{a_0}{1+z} \right)}, \quad (33)$$

$$\theta = 3H = \frac{3}{\sqrt{2\beta t + k}} = \frac{1}{\beta \log \left( \frac{a_0}{1+z} \right)}, \quad (34)$$

$$\sigma^2 = \frac{1}{2} \left[ (H_x^2 + H_y^2 + H_z^2) - \frac{\theta^2}{3} \right] = \frac{(m-1)^2}{3m^2(2\beta t + k)} = \frac{(m-1)^2}{3m^2 \left( \beta \log \left( \frac{a_0}{1+z} \right) \right)^2}, \quad (35)$$

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left( \frac{\Delta H_i}{H} \right)^2 = \frac{2(m-1)^2}{(m+2)^2}, \quad (36)$$

Equations (34) and (35) lead to

$$\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}} \cdot \frac{(m-1)}{(m+2)}. \quad (37)$$

Here we see that  $\frac{\theta}{\sigma}$  is a constant, which verifies our assumption.

The energy density of the fluid can be found from the Eqs. (19) using Eqs (26),(27) as

$$\rho(t) = \frac{9(2m+1)}{(m+2)^2} (2\beta t + k)^{-1} - \exp \left[ \frac{-6m}{(m+2)\beta} \sqrt{2\beta t + k} \right]. \quad (38)$$

In terms of red shift  $z$ , the energy density  $\rho$  can be given by

$$\rho(z) = \frac{9(2m+1)}{(m+2)^2} \left[ \beta \log \left( \frac{a_0}{1+z} \right) \right]^{-2} - \exp \left[ \frac{-6m}{(m+2)} \log \left( \frac{a_0}{1+z} \right) \right]. \quad (39)$$

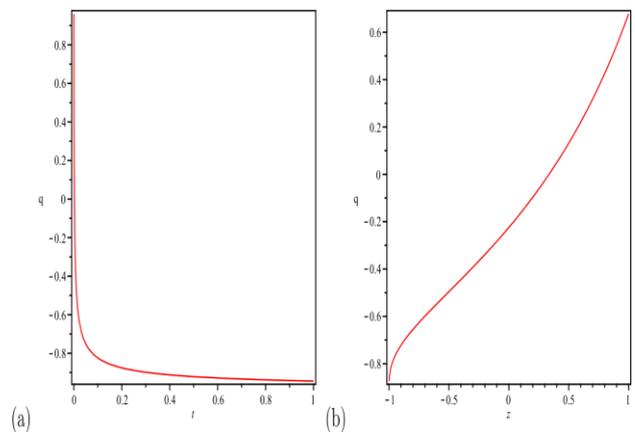


Figure 2: (a)The plot of deceleration parameter  $q$  versus time  $t$ , (b) the plot of deceleration parameter  $q$  versus  $z$

The figures 3(a) and 3(b) depict the variation of energy density  $\rho$  with time  $t$  and redshift  $z$  respectively. From these figures we observe that as  $t \rightarrow 0$ ,  $\rho \rightarrow \infty$  and as  $t \rightarrow \infty$  or  $z \rightarrow -1$ ,  $\rho \rightarrow 0$ .

It is evident that  $\rho$  remains positive and decreasing function of time and converges to zero at  $t \rightarrow \infty$ , as expected.

From Eqs. (18),(26),(27) and (38), we get the deviation-free EoS parameter  $\omega$  as

$$\omega(t) = \frac{1}{\frac{9(2m+1)}{(m+2)^2} (2\beta t + k)^{-1} - e^{\frac{-6m}{(m+2)\beta} \sqrt{(2\beta t+k)}}} \times \left[ \frac{3\beta}{m+2} (2\beta t + k)^{\frac{-3}{2}} - \frac{9(3-m)}{(m+2)^2} (2\beta t + k)^{-1} - e^{\frac{-6m}{(m+2)\beta} \sqrt{(2\beta t+k)}} \right]. \quad (40)$$

In terms of red shift  $z$ , the EoS parameter  $\omega$  can be given by Figure 4(a) portrays the behaviour of the  $(\omega)$  with cosmic time  $t$  as a suitable choices of constants of integration and other physical parameters by reasonably well known situations.

Here, we observe that  $\omega$  is a decreasing function of time  $t$ , and it is always negative. In Figure we get three scenarios

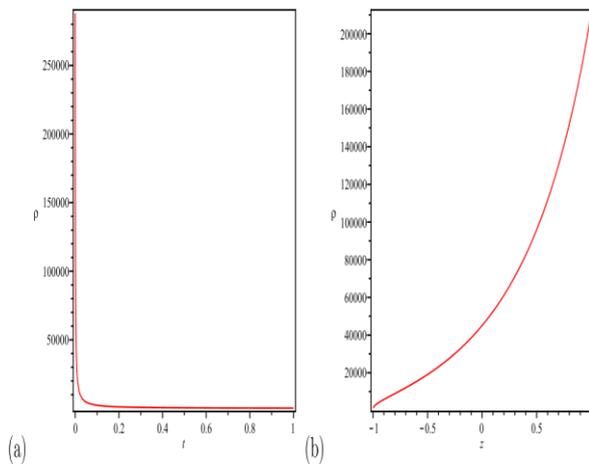


Figure 3: (a)The plot of energy density  $\rho$  versus time  $t$  for  $m = 0.5 \beta = 0.0062$  and  $k = 0.00001$ , (b) the plot of energy density  $\rho$  versus  $z$ , for  $m = 0.5 \beta = 0.0062$

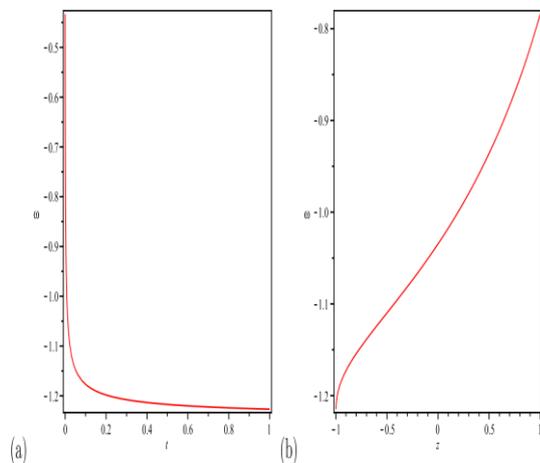


Figure 4: (a)The plot of equation of state parameter  $\omega$  versus time  $t$ , for  $m = 0.5 \beta = 0.0062$  and  $k = 0.00001$  (b) the plot of equation of state parameter  $\omega$  versus  $z$ , for  $m = 0.5 \beta = 0.0062$

like  $(\omega = -1)$ , which are mathematical equivalent to cosmological constant, quintessence  $(\omega \geq -1)$  and phantom energy  $(\omega \leq -1)$ . Figure 4 (b) depicts the variation of the EOS parameters  $(\omega)$  versus redshift  $z$ , which also shows that, at late time when  $z \rightarrow -1$ ,  $\omega$  tends to a small negative constant due to negative pressure, indicating an expanding universe.

Since we have already obtained  $\delta = \gamma$ , we can now obtain  $\gamma$  by using Eqs. (18),(26),(27) in Eq. (17) as

$$\gamma(t) = \frac{1}{(2\beta t + k)^{-1} \left( \frac{2}{m} + \frac{1}{m^2} \right) - e^{\frac{-2}{\beta} \sqrt{(2\beta t+k)}}} \times \left[ (2\beta t + k)^{-1} \left( 1 + \frac{1}{m} + \frac{2}{m^2} \right) + (2\beta t + k)^{\frac{-3}{2}} \left( -\beta + \frac{\beta}{m} \right) - 2e^{\frac{-2}{\beta} \sqrt{(2\beta t+k)}} \right]. \quad (42)$$

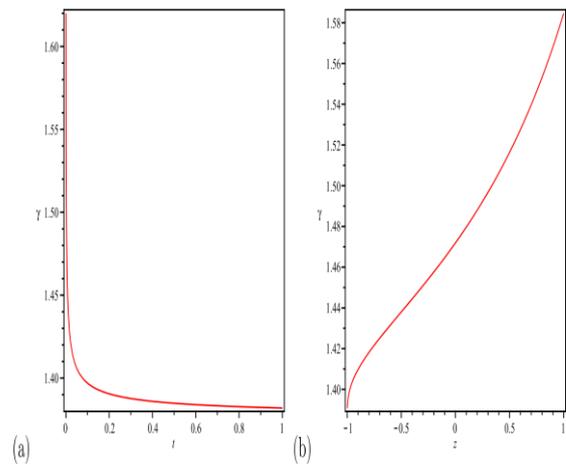


Figure 5: (a)The plot of skewness parameter  $\gamma$  versus time  $t$ , for  $m = 0.5 \beta = 0.0062$  and  $k = 0.00001$  (b) the plot of skewness parameter  $\gamma$  versus  $z$ , for  $m = 0.5 \beta = 0.0062$

In terms of red shift  $z$ , the skewness parameter  $\gamma$  can be given by

$$\gamma(z) = \frac{1}{\left( \beta \log \left( \frac{a_0}{1+z} \right) \right)^{-2} \left( \frac{2}{m} + \frac{1}{m^2} \right) - e^{\log \left( \frac{a_0}{1+z} \right)}} \times \left[ \left[ \beta \log \left( \frac{a_0}{1+z} \right) \right]^{-2} \left( 1 + \frac{1}{m} + \frac{2}{m^2} \right) + \left[ \beta \log \left( \frac{a_0}{1+z} \right) \right]^{-3} \left( -\beta + \frac{\beta}{m} \right) - 2e^{-2 \log \left( \frac{a_0}{1+z} \right)} \right]. \quad (43)$$

Figure 5(a) and 5(b) show the variation of skewness parameter  $\delta = \gamma$  ( which are deviations from  $\omega$  along  $y$  and  $z$  axes respectively),

with respect to cosmic time  $t$ . The figure shows that  $\gamma$  is a decreasing function of

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time and always positive as expected. Figure 5 (b) shows variation of  $\gamma$  and redshift  $z$ , which also shows that at late time, when  $z \rightarrow -1$ ,  $\gamma$  tends to a small positive constant, indicating anisotropy of the universe.

**Energy condition:** The weak energy conditions(WEC) and dominant energy conditions(DEC) are given by (i)  $\rho \geq 0$  (ii)  $\rho - p \geq 0$  and (iii)  $\rho + p \geq 0$ ,  $\rho + 3p \geq 0$  is the strong energy condition (SEC).

From Figure 6 and Figure 3(a), we observe that WEC and DEC are satisfied most of the period of evolution, (except  $\rho + p < 0$  for a very short period at the early phase of the universe). But, the SEC violates throughout the evolution history ( $\rho + 3p \leq 0$ ), although it approaches to zero at late time. This behavior is acceptable in case of dark energy models. So, we may say that our model is consistent on energy condition front also.

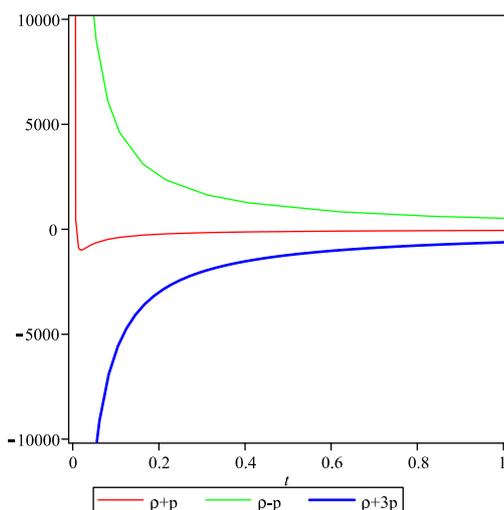


Figure 6: the plot of energy conditions versus time  $t$  for  $m = 0.5$ ,  $\beta = 0.0062$  and  $k = 0.00001$ .

## V. Concluding remarks

A new Bianchi Type-V  $I_0$  DE anisotropic universe with time dependent DP  $q$  and time dependent EoS parameter  $\omega$  has been explored which is different from other author's solutions.

The main features of the model are as follows:

- The EFEs solutions have been found by letting average scale factor  $a(t) = \exp\left[\frac{1}{\beta}\sqrt{2\beta t + k}\right]$  which gives the time dependent deceleration parameter that is recently observed. Thus our DE model represents a transit universe, where the cosmic expansion takes place from early decelerated stage to the present accelerating stage.
- This model of DE gives the dynamics of  $\omega$  given by Eq. (40). In this derived model,  $\omega$  is obtained as time dependent which is a negative decreasing with time and remains negative throughout the evolution of the universe. This DE model presents the early and late time universe and it reveals that the expansion of universe is accelerating at present. The range of  $\omega$

obtained in the model is in the limits of acceptable range.

- The anisotropic parameter  $A_m$  is a constant and is non-zero for  $m \neq 1$ . ie. our model is anisotropic and may attain isotropy if  $m = 1$ .
- The skewness parameter  $\gamma$  is positively decrease with time, which also shows the deviation from isotropy at current epoch. It tends to zero as time  $t \rightarrow \infty$ , i.e., isotropy is attained at late time.
- The model represents non-rotating, shearing and expanding universe.
- At  $t = -\frac{k}{2\beta}$ , the parameters  $\sigma$ ,  $\theta$  and  $H$  diverge at the initial Singularity.
- The model has constant anisotropy when  $m \neq 1$  and it reaches to the phase of isotropy when  $m = 1$ .

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