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Radiative MHD Flow past a Vertical Melting Front with Variable Temperature

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Abstract: ***In this paper, a study is made on investigating the melting flow and heat transfer characteristics past a vertical plate embedded in a saturated non-Darcy porous medium. To analyze this flow, continuum equations with the combined impact of magnetic field, thermal dispersion, viscous dissipation and radiation along with variable temperature for Newtonian fluids are developed. The flow analysis is carried out for both assisting and opposing situations. The ensuing system of nonlinear ordinary differential equations is solved numerically using R-K method coupled with shooting technique. The ultimate influence of different governing physical parameters on velocity, temperature profiles and heat transfer rates are numerically discussed and also presented graphically. Further, the results obtained from the current analysis are validated by the results of previously published work.***

Index Terms**:** ***Radiation, Thermal dispersion, Variable temperature, Viscous dissipation.***

I. INTRODUCTION

The study of convective heat transfer accomplished by melting effect in porous media has received some attention within the recent years. This has stemmed from the actual fact that this development has significant direct application in permafrost melting, frozen ground thawing, casting, welding process as well as phase change material (PCM). Radiation effect plays an important role in prevalent heat transfer in producing processes wherever the standard of the ultimate product might rely upon heat management factors. Now a days we can find numerous applications within the cooled nuclear reactors power generation systems etc. Lai et al. [1] studied the problem regarding convective heat transfer of fluid past a vertical plate in a porous medium by considering variable viscosity. Gorlaet al. [2] explored the impact of mixed convection from a vertical surface in a fluid saturated porous medium. They observed that, the melting process decreases the heat transfer at solid liquid interface. Cheng and Lin [3] studied the impact of melting on mixed convective heat transfer over a vertical porous surface. MHD mixed convective flow over a permeable vertical plate by taking radiation effect was studied by Aydin and Kaya [4]. Bakier et al. [5] examined the effect of melting, thermal radiation and temperature difference on MHD mixed convective flow through a vertical plate in a saturated porous medium for opposing flow. Veerraju et al. [6] studied the effects of variable viscosity, variable wall temperature on a mixed convective boundary layer flow along a vertical plate. Olanrewaju et al. [7] presented the effects of heat generation and thermal radiation on the variable viscous fluid with magnetic field through a continuously moving flat plate. They observed that the increase in values of prandtl number reduces the influence of magnetic field intensity and biot number. The effect of slip flow on boundary layer mixed convective flow through a vertical plate was studied by Bhattacharyya et al. [8]. Hemalatha and Prasad [9] analyzed the influence of viscous dissipation on MHD radiative heat flow along a vertical melting surface. Mohammed [10] studied the effects of radiation and melting on MHD mixed convective flow from a vertical plate incorporated in a saturated porous media. He demonstrated that the heat transfer rate decreased with increment in melting parameter and increases with increment in radiation parameter. Umamaheswar et al. [11] considered the influence of chemical reaction and heat absorption/generation on MHD free convective flow over a vertical porous plate.

The combined effect of melting and solute dispersion on non-Darcy mixed convective heat transfer through a vertical plate was studied by Hemalatha et al. [12]. Kameswaran et al. [13] analyzed the effects of radiation and melting with variable permeability over a vertical surface embedded in a non-Darcy porous medium. Chandra Barman et al. [14] presented the effects of melting and magnetic on mixed convective flow through a vertical plate for aiding and opposing external flows along with variable temperature.

In this paper, we made an attempt to study the effects of melting, thermal radiation, dispersion and viscous dissipation with variable temperature in the presence of applied magnetic field. Majority of the above studies in the literature have not considered the effects of thermal dispersion, and melting along with variable temperature in their study.

II. MATHEMATICAL FORMULATION

In present problem, the effect of melting, thermal dispersion, thermal radiation and viscous dissipation on mixed convective heat transfer from a vertical plate is considered. The geometry and flow configuration is shown in the Fig.1. It is assumed that the plate acts as an interface between the Newtonian fluid and the solid phase during melting inside the porous matrix. The plate is at variable temperature T_m at which the material of the porous matrix melts whereas the stagnant free stream temperature$T_\infty > T_m$. The temperature of the solid region is considered less than the melting point i.e. $T_0 < T_m$.

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A vertical boundary layer flow on the liquid side smoothens the transition from T_m to T_∞. The assisting external flow velocity is taken as u_∞. The system is considered in thermodynamic equilibrium everywhere. Here Newtonian fluid is assumed to be electrically conducting with constant properties except for the density variation in the buoyancy term. Transverse magnetic field is applied to the plate and the effects of flow inertia and radiation are considered. Hall effect and joule effect are also neglected.

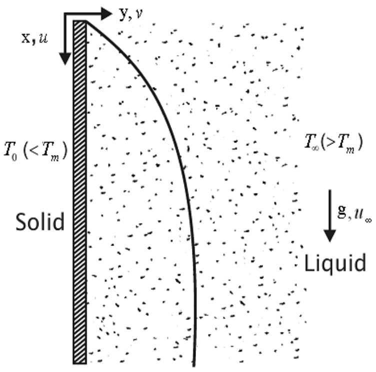

Fig. 1. Schematic model and coordinate system

Under these assumptions and the usual Boussineq approximation, the governing system of steady, laminar boundary layer flow equations may be expressed in usual notation as follows.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\frac{\partial u}{\partial y}+\left(\frac{C\sqrt{K}}{\nu}\,2u\,\frac{\partial u}{\partial y}\right)+\sigma B_0^2\frac{\partial u}{\partial y}\frac{K}{\mu}=-\frac{K\,\rho_\infty\, g\beta}{\mu}\frac{\partial T}{\partial y} \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha\frac{\partial T}{\partial y}\right)+\frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{(\rho_\infty C_p)_f}\frac{\partial q_r}{\partial y} \quad (3)$$

Here, u and v are the velocity components along the x and y directions respectively, T is the temperature in the thermal boundary layer, K is the permeability, C is the inertial coefficient, β is the coefficient of thermal expansion, C_p is the specific heat, ν is the kinematic viscosity, ρ is the density, g is the acceleration due to gravity and B_0 is magnetic flux. Thermal diffusivity $\alpha=\alpha_m+\alpha_d$, where α_m is the molecular diffusivity and α_d is the dispersion thermal diffusivity due to mechanical dispersion. As in the linear model proposed by plumb [15], the dispersion thermal diffusivity α_d is proportional to the velocity component i.e. α_d = γud, where γ is the dispersion coefficient and d is the mean particle diameter. The appropriate boundary conditions for the present problem are defined as follows

At y = 0, $T=T_m=T_\infty+Ax^\lambda$, $k_{eff}\frac{\partial T}{\partial y}=\rho(h_{sf}+C_s[T_m-T_\infty])v=0.$ (4)

$T\to T_\infty$, $u=u_\infty=Bx^\lambda$ as $y\to\infty$. (5)

Where k_{eff} is effective thermal conductivity of the saturated porous medium, h_{sf} is latent heat of melting of solid, C_s is specific heat of solid phase, A and B are constants and λ is referred to as power of index of wall temperature. Utilizing Rosseland diffusion approximation, the radiative heat flux q_r in y-direction is $q_r=\frac{-4\sigma_R}{3k^*}\frac{\partial T^4}{\partial y}$, where σ_R and `k*` are the Stefan – Boltzmann constant and the mean absorption coefficient respectively.

Introducing the stream function ψ such that $u=\frac{\partial\Psi}{\partial y}$ and $v=-\frac{\partial\Psi}{\partial y}$, the equations (2) and (3) transform to

$$\frac{\mu}{K}\frac{\partial^2\Psi}{\partial y^2}+\frac{\mu}{K}\left[\frac{C\sqrt{K}}{\nu}2\frac{\partial\Psi}{\partial y}\frac{\partial^2\Psi}{\partial y^2}\right]+\sigma B_0^2\frac{\partial^2\Psi}{\partial y^2}=-\rho_\infty\, g\beta\frac{\partial T}{\partial y} \quad (6)$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left[\alpha\frac{\partial T}{\partial y}\right]+\frac{\nu}{C_p}\left(\frac{\partial^2\Psi}{\partial y^2}\right)^2+\frac{4\sigma}{\rho C_p\,3k^*}\frac{\partial}{\partial y}\left[\frac{\partial T^4}{\partial y}\right] \quad (7)$$

Invoke the similarity variables as

$\psi=f(\eta)(\alpha_m u_\infty x)^{1/2}$, $\eta=\left(\frac{u_\infty x}{\alpha_m}\right)^{\frac{1}{2}}\left(\frac{y}{x}\right)$, $\theta(\eta)=\frac{T-T_m}{T_\infty-T_m}$,

Equations (6) and (7) now reduce to

$$(1+MH+Ff^1)f^{11}+\frac{Ra_x}{Pe_x}\theta^1=0 \quad (8)$$

$$(1+Df^1)\theta^{11}+\left(\frac{1+\lambda}{2}f+Df^{11}\right)\theta^1-Pr.Ec.(f^{11})^2+(1-\theta)(Re.R^*+\lambda f^1)=0 \quad (9)$$

Where the super fix represents the derivative with respect to the similarity variable η, Ra_x is the local Rayleigh number, $\frac{Ra_x}{Pe_x}$ is the mixed convection flow governing parameter, (+ve when the buoyancy is aiding the external flow and is -ve when the buoyancy is opposing the external flow), Pe_x is the Pecklet number, Pr is the Prandtl number, Ec is the Eckert number, MH is the magnetic parameter, F is non-Darcy parameter, D is the dispersion parameter and R^* is the radiation parameter.

$Ra_x=\frac{g\beta K(T_\infty-T_m)x}{\nu\alpha_m}$, $Pe_x=\frac{u_\infty x}{\alpha_m}$, $F=\frac{2C\sqrt{K}u_\infty}{\nu}$, $D=\frac{\gamma\, du_\infty}{\alpha_m}$,

$MH=\frac{\alpha B_0^2 K}{\rho\nu}$, $Ec=\frac{u_\infty^2}{C_p(T_\infty-T_m)}$, $Pr=\frac{\nu}{\alpha_m}$, $R^*=\frac{4}{3}R$, $R=\frac{4\sigma^* T^3}{k_{eff}k^*}$.

The boundary conditions (4) and (5) transform to

$\eta=0, \theta=0, f(0)+\{1+Df^1(0)\}2M\theta^1(0)=0.$ (10)

and $\eta\to\infty$, $\theta=1$, $f^1=1$. (11)

where $M=\frac{C_p(T_\infty-T_m)}{h_{sf}+C_s(T_m-T_0)}$ is the dimensionless melting parameter.

Heat Transfer Coefficient:

The local heat transfer rate is $q_w=-k_{eff}\left(\frac{\partial T}{\partial y}\right)_{y=0}$

The local Nusselt number is $Nu_x=\frac{hx}{k_{eff}}=\frac{q_w x}{k_{eff}(T_m-T_\infty)}$ where h is the local heat transfer coefficient and k_{eff} is the effective thermal conductivity of the porous medium, which is the sum of the molecular thermal conductivity k_m and the dispersion thermal conductivity k_d.

The Nusselt number is obtained as

$$\frac{Nu_x}{(Pe_x)^{\frac{1}{2}}}=\left[1+\frac{4}{3}R+Df^1(0)\right]\theta^1(0) \quad (12)$$

III. SOLUTION PROCEDURE

The equation (8) together with equation (9) is split into a system of first-order ordinary differential equations. Using boundary conditions (10) and (11), they are solved numerically by means of the fourth order Runge-Kutta method combined with a shooting technique. The solution got is coordinated with the values of $f^1(\infty)$ and θ(0). Furthermore, we choose a suitable value to the boundary condition η→∞ i.e η_{max} = 5 which is found adequately substantial for the velocity and temperature to approach the significant free stream properties. Numerical computations are carried out for F=0.5, D=0,0.5,1; R=0,0.5,1; MH=1; λ = 0.1, 0.3, 0.5; Re = 5; Pr = 5; EC=0.1,0.3,1,2; $\frac{Ra_x}{Pe_x}$ = -1, 1. In all figures except Fig. 8.1 and 8.2, λ is taken as fixed value 0.3.

IV. RESULTS AND DISCUSSION

Fig. 2 illustrates the influence of thermal dispersion parameter D on the velocity distribution for both aiding and opposing external flow cases. It is noticed that for the case of aiding the velocity distribution increase with an increase in the value of D. Opposite behavior is observed just in case of opposing flow.

Fig. 3.1 and 3.2 displays the effect of thermal dispersion D on temperature profile in aiding and opposing cases respectively. It is noticed that the increment in D results in a decrement in temperature profiles in both cases.

Fig. 4 depicts the effect of Eckert number Ec on velocity profile for both aiding and opposing flow cases. The increment in Eckert range causes the increment of the velocity field for aiding flow case where as in case of opposing flow the reverse behavior is observed.

Fig. 5.1 and 5.2 displays the effect of Eckert number Ec on temperature profile for aiding and opposing flow cases. It is noticed that the same behavior in each case exists. An increment in Ec leads to an increment in temperature profile for each the cases.

Fig 6.1 and 6.2 illustrates the behavior of Nusselt number as an increasing function of melting parameter M for different values of thermal radiation parameter R in aiding and opposing cases respectively. It is observed that heat transfer rate increases with an increase in R for both the flow cases.

Fig.7.1 and 7.2 displays the effect of Eckert number Ec on Nusselt number as a function of increasing melting parameter M in aiding and opposing flows respectively. It is seen that the increase in Ec leads to decrease in heat transfer rate for each flow situation.

Fig.8.1 and 8.2 shows the influence of λ on Nusselt number as a function of melting parameter M in both aiding and opposing flows respectively. While the heat transfer rates increases with increase in λ in both aiding and opposing flow situations.

The effect of thermal dispersion D on Nusselt number as an increasing function of melting parameter M is shown in Fig.9.1 and 9.2 in aiding and opposing flows respectively. It is observed that the increase in D leads to an increase in heat transfer rate for both flow cases.

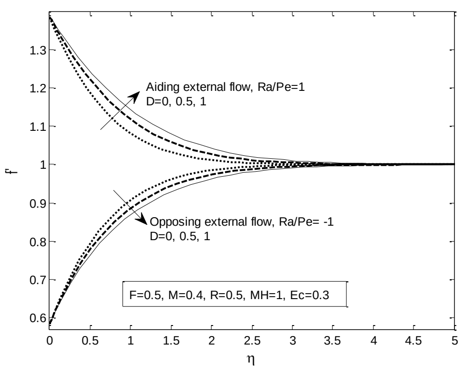

Fig.2. Velocity profiles for different values of D.

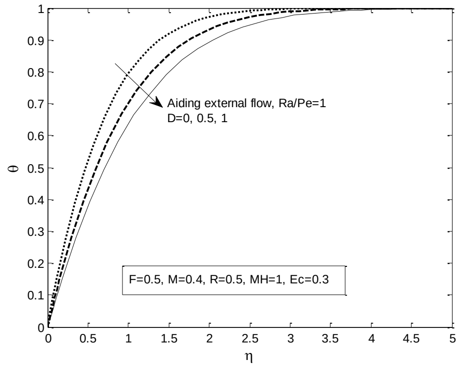

Fig.3.1. Temperature profiles for different values of D.

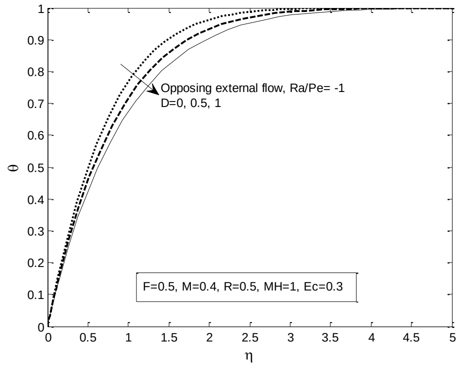

Fig.3.2. Temperature profiles for different values of D.

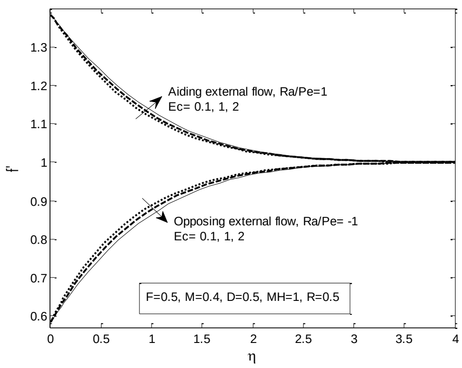

Fig.4. Velocity profiles for various values of Ec

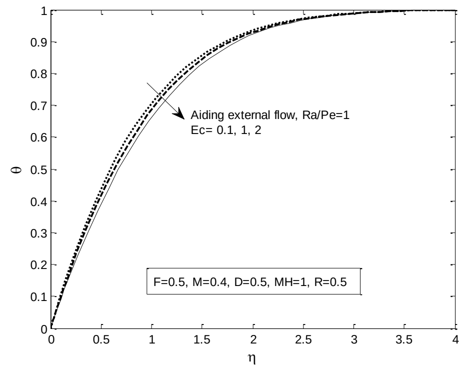

Fig.5.1. Temperature profiles for various values of Ec.

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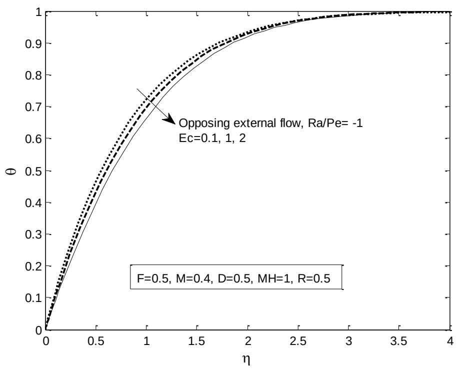

Fig.5.2. Temperature profiles for various values of Ec.

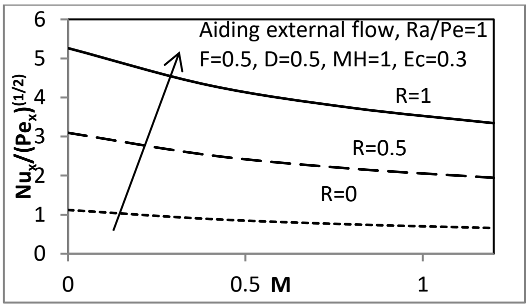

Fig. 6.1 Variation of Nusselt number with melting parameter M, for different R values.

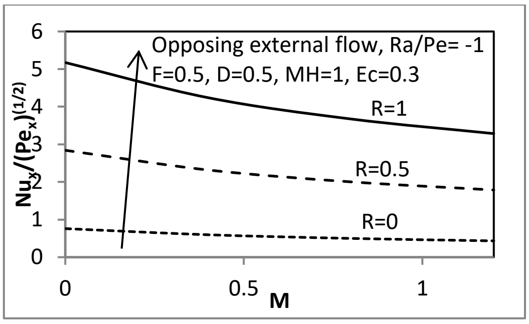

Fig. 6.2 Variation of Nusselt number with melting parameter M, for different R values.

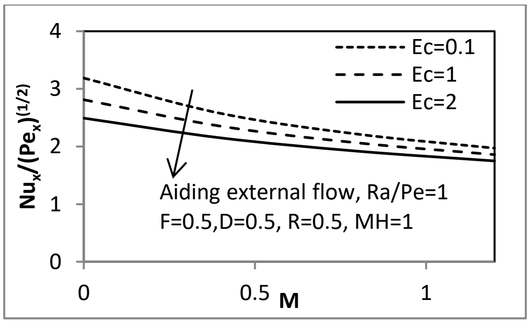

Fig. 7.1 Variation of Nusselt number with melting parameter M, for different values of Ec.

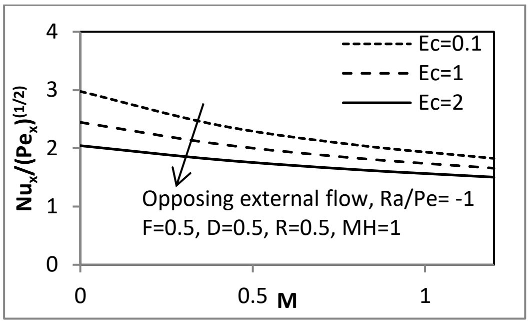

Fig. 7.2 Variation of Nusselt number with melting parameter M, for different values of Ec.

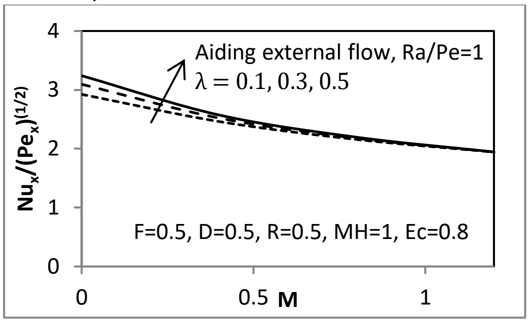

Fig. 8.1 Variation of Nusselt number with melting parameter M, for different values of λ.

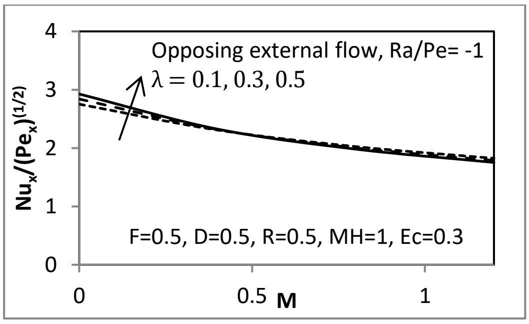

Fig. 8.2 Variation of Nusselt number with melting parameter M, for different values of λ.

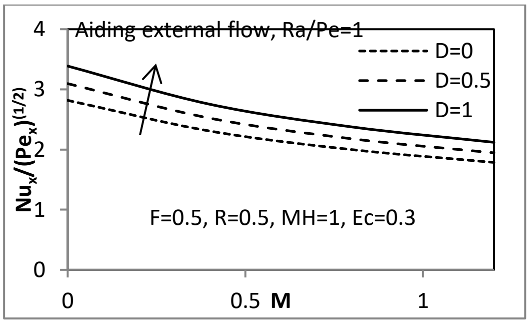

Fig. 9.1 Variation of Nusselt number with melting parameter M, for different values of D.

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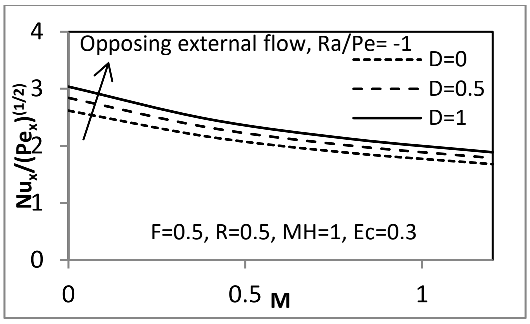

Fig. 9.2 Variation of Nusselt number with melting parameter M, for different values of D.

Our results are collated with those of previously produced results of Sobhaet al. [16] and Cheng and Lin [3] for exceptional cases of the problem under consideration. Table.1 and Table.2 exhibit that the present numerical values are in good concurrent with the earlier published results.

Table.1 $f^1(0)$ values obtained by Sobha et al. [16], Cheng and Lin [3] and present in an aiding flow with M = 2.0

Ra/Pe	Sobha et al. [16]	Cheng and Lin [3]	Present
0.0	1.00	1.00	1.00
1.4	2.40	2.40	2.40
3.0	4.00	4.00	4.00
8.0	9.00	9.00	9.00
10	11.00	11.00	11.00
20	21.00	21.00	21.00

Table.2 $\theta^1(0)$ values obtained by Sobha et al. [16], Cheng and Lin [3] and present in an aiding flow with M = 2.0

Ra/Pe	Sobha et al. [16]	Cheng and Lin [3]	Present
0.0	0.27062	0.2706	0.27108
1.4	0.3801	0.3801	0.38009
3.0	0.4745	0.4745	0.4745
8.0	0.69019	0.6902	0.69012
10	0.75939	0.7594	0.75929
20	1.0384	1.0383	1.03812

V. CONCLUSION

Melting with thermal dispersion and power of index of plate temperature (λ = 0.3) increases heat transfer more significantly than when melting occurs with thermal dispersion and λ = 0. It is hoped that the paper will be of interest to experimentalists in the same field in the presence of various parameters.

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