Comparison of Second Order Sliding Mode Control Strategies for Coupled Tank System

B. Amarendra Reddy, P. Vamsi Krishna

Abstract— ***Coordinating a controller to twin tank liquid estimation system under after conditions is a troublesome errand. 2-SMC was proposed for liquid estimation coupled tank structure. The non-direct controller is set in the mood for following the perfect water level for each required point with sporadic state of precision, less rising time. The standard focal motivations behind SMC are less influenced to plant parameter game plans and irritating impacts which avoids the need of reasonable illustrating. Rule of required liquid estimation in tank 2, under different starting conditions is appeared in reenactment with MATLAB programming.***

Keywords: Modelling of twin tank, sliding mode control (SMC), 2-sliding mode control (SMC) twisting, super twisting and drift.

I. INTRODUCTION

Fluid estimation control for course of action related tanks with controlling the fluid passing is preposterous control issue for system encounters. As it required to store some degree of liquid in tanks and after that traded to other one [1],[9]. For controlling fluid estimation in methodology tank structure needs a productive and liberal controller. For this sort sound controller arrangement is required as (SMC) method of reasoning. Along these lines of fluid estimation structure is required for undertakings like iron endeavors, garments and paints affiliations, etc.

In this paper a liberal control methodology is required to keep up the perfect fluid estimation in twin tank system. This can be created by using SMC which is one of the Variable Structure Control Systems (VSCS). Variable structure control systems, as the name prescribes, are a class of structures whereby the 'control law' is deliberately changed in the midst of the control advancement as shown by some predefined rules which depend on the states of the structure [2]. The most observed piece of VSC is its ability towards liberal control structures and using VSC as a rule results invariant control systems. The verbalization "invariant" concludes that the structure is absolutely pitiless toward parametric weakness and outside aggravations. The one of the genuine confinements of first deals sliding mode control is the jabbering wonder. Among a couple proposed answer for this issue, 'Levant ovsky (1986) and Emelyanov et al' proposed the theory for 'Higher mentioning SMC. In paper oversees 2-SMC with bowing and bowing figurings [3] and coast computations to the liquid estimation control system. This paper is overseen as searches for after. Structure Description of a twin tank is analyzed in fragment II. Reliable showing of the twin tank structure is brief in the area III. SMC perspectives and control methodology acknowledgment for summed up structure is amassed in portion IV. Redirection looks at using MATLAB are destitute down in zone V. Consummations are showed up partially VI.

II. SYSTEM DESCRIPTION

The system tank of two tanks coupled using a pipe. T1 and T2 are the two tanks coupled. 'l1' and 'l2' are the cross-zone territories of the tank 1 and tank 2 uninhibitedly. 'b' is the stream rate of the liquid for the tank 1. 'b1' is the stream rate between tank 1 and tank 2. Finally, b2 is the out-stream rate of the liquid from tank2. 'H1' and 'H2' are the statures of the liquid estimation in tank 1and tank 2 independently. The terms B1, B2, B12 that will appear in showing conditions are the proportionality constants. These will depend on the coefficients of discharge, gravitational trustworthy and the cross-sectional zone of each outlet.

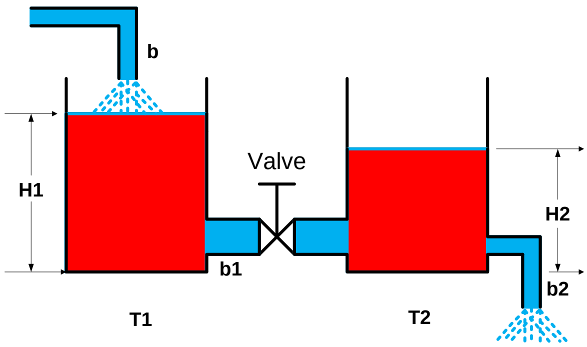

Fig 1. Representing a twin or series tank system

III. MATHEMATICAL MODELING

Fig 1 shows the physical depiction of acoupled tank structure. Siphon supplies water to the tank 1. T1 and T2 were associated using a pipe with the help of a valve. Additionally, there is an outlet related with T2 as having flood of b2. The liquid used in the plant is acknowledged to suffer, non-thick and in compressible. The non-straight state states of the coupled tank are gotten using Bernoulli's condition.

The differential states of T1 and T2 are gotten using the stream balance condition and given inequation (1).

$$\frac{dH_1}{dt} = \frac{1}{l_1}\left(b - b_1\right)$$
$$\frac{dH_2}{dt} = \frac{1}{l_2}\left(b_1 - b_2\right) \tag{1}$$

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Where b1 and b2 are defined [2] given as below.

$$b_1 = a_{12}\sqrt{2g(H_1 - H_2)} \quad \text{for } H_1 > H_2$$
$$b_2 = a_2\left(\sqrt{2gH_2}\right) \quad \text{for } H_2 > 0.$$

And a_{12} and a_2 are the cross-section area of pipes connecting the two tanks and outlet respectively. On substitution of the b_1 and b_2, the differential equations of the coupled tank system are obtained and given in equation (2).

$$l_1\frac{dH_1}{dt} = b - B_{12}\sqrt{|H_1 - H_2|}\text{sign}(H_1 - H_2)$$
$$l_2\frac{dH_2}{dt} = -B_2\sqrt{H_2} + B_{12}\sqrt{|H_1 - H_2|}\text{sign}(H_1 - H_2) \quad (2)$$

Since there is no leakage provided in tank 1, assume $B_1 = 0$. Here B_2 is output discharge proportionality constant and mathematically defined using equation (3). And B_{12} is discharge proportionality constant between the tank 1 and tank 2.

$$B_2 = a_2\sqrt{2g}$$
$$B_{12} = a_{12}\sqrt{2g} \quad (3)$$

This system can be modeled in terms of nonlinear functions A and P i.e., A is not a linear function having H1 and H2 containing the root terms as shown in the equation 3

$$\dot{H}(t) = A(H) + P(H)u \quad (4)$$

IV. SLIDING MODE CONTROL

Sliding mode control has seemed valid control law for where the component of chance is more than the parts required or non-direct structures. It has recognized by two important advances (I) delineating a trading surface close to the structure be enough gone to the required way. The system heading are obliged onto a surface in the state space called the sliding surface. (ii) Designing a control law. The cutoff of the control law is to pull in the structure course towards sliding surface in constrained time and notwithstanding keep the headings on the sliding surface.

SMC is fundamentally managed on plan of exchanging surface that is first, second and higher arrangements sliding mode. The higher referencing sliding mode is gave as actuating the state course to reach in obliged time onto the sliding surface sr is tending to condition (5) higher the segment of s higher the piece of SMC

$$s^r = \{x \in \Re^n : s = \dot{s} = \ddot{s} = ...s^{r-1} = 0\} \quad (5)$$

Here r is relationship of sliding mode better the control related smooth law related. The upsides of higher mentioning sliding mode control method [5]-[8] joins together (I) capacity to supported deficiency (ii) Chattering decline (iii) Higher connection precision, etc., in this examination, only second-demand sliding mode control (SOSMC) and particularly turning and super-bowing computations are appeared in reenactment in MATLAB condition for the control of the liquid section of coupled tank structure.

1) First Order Sliding Mode Controller

Consider a non-straight model of the coupled tank system is delineated as in condition (2). Plan of first solicitation incorporates two phases. (a) Defining fittingly the sliding surface. (b)Design of control law so that systemtrajectories pulled in towards the sliding surface.

a) Defining switching surface

The sliding surface can be defined using the equation (6).

$$s = \{x \in \Re^n : s = 0\} \quad (6)$$

Where $x \in \Re^n$ is the state vector and on sliding surface trajectories obey "$s = 0$".

b) Designing Control Law:

The control law of the FOSMC is expressed as follows

$$u = u_{eq} + u_d \quad (7)$$

With u_{eq} is the equivalent control and ud is the unpredictable control. The unpredictable control banner is picked to bring the state bearings towards the trading surface. The equivalent control is a control action imperative to keep up an ideal sliding development toward the day's end it ensures the state course to remain on the sliding surface s=0.

The equivalent control u_{eq} is obtained by equating $\dot{s}(x,u,t) = 0$ where $\dot{s}(x,u,t) = f(x,u,t)$ .The discontinuous signal is expressed as $u_d = k\text{sign}(s)$.

2) Higher order Sliding Mode Controller (HOSMC)

Higher requesting sliding mode control is really headway on a broken course of action of a dynamic framework. Higher sales sliding mode control not just purges a touch of the critical hindrances of the standard strategy yet additionally gives improves following precision under sliding advancement. The higher sales sliding mode is depicted in the equation (5).

2.1 Second order Sliding Mode Controller

Second-order sliding mode allows eliminating chattering phenomenon and to reach the states to origin by smooth control law. Second- order sliding mode can be done by imposing:

$$s(x,t) = \dot{s}(x,t) = 0 \quad (8)$$

This can be achieved by consider a second order system where switching surface S(x,t) and its derivative $\dot{s}$ (x,t).

i) Defining Sliding Surface:

In second-order sliding mode all the trajectories will converge in finite time to the *origin* $s = \dot{s} = 0$ of the phase plane $(s, \dot{s})$[7]-[10]. Here, the sliding surface can be defined using the equation (9).

$$s = \{x \in \Re^n : s = \dot{s} = 0\} \quad (9)$$

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In matrix representation form, sliding surface "s" is defined as, $s(x)=Sx$ here x is a state vector and $S\in\Re^{n}$ is switching function matrix. The switching function, 's', its first derivative is continuous and second derivative is discontinuous.

$$\dot{s}=\frac{\partial s}{\partial x}\dot{x}=\frac{\partial s}{\partial x}(A(x)+P(x)u) \tag{10}$$

Differentiating the switch surface 's', twice yield to following relation

$$\ddot{s}=\frac{\partial\dot{s}}{\partial x}\left(A(x)+P(x)u\right)+\frac{\partial\dot{s}}{\partial u}\dot{u} \tag{11}$$

$$\ddot{s}=\varphi(x)+\phi(x)\dot{u} \tag{12}$$

It is assumed that if $\left|s(x)\right|<s_0$

$$\begin{aligned}&\left|\varphi(x)\right|<\varepsilon\\&0<\Gamma_m\le\phi(x)\le\Gamma_M\end{aligned} \tag{13}$$

Where ε, Γ_m and Γ_M are positive constants.

ii.a) Control Law: Twisting Algorithm:

The twisting (TW) estimation, known 2-sliding mode tallies, gains a limited time mix of the state bearing to the starting phase of the stage plane (s,) by taking a standard number of turns close begin appeared in Fig. (2), and the streams in light of the switch in charge law. This figuring in a general sense has a broken control and portrayed by condition (14). The critical conditions that guarantee the confined time relationship of headings to the second requesting sliding surface are given in condition (15).

$$\dot{u}_{TW}=\begin{cases}-u \text{ if } |u|>u_{max}\\-k_m\text{sign}(s) \text{ if } s\dot{s}\le 0 \text{ and } |u|\le u_{max}\\-k_M\text{sign}(s) \text{ if } s\dot{s}>0 \text{ and } |u|\le u_{max}\end{cases} \tag{14}$$

$$k_M>k_m>0, k_m>\frac{4\Gamma_M}{s_0}, k_m>\frac{\phi}{\Gamma_m}, \Gamma_m k_M-\phi>\Gamma_M k_m+\phi \tag{15}$$

ii.b) Control Law: Prescribed Twisting Algorithm:

In the prescribed law for bowing puts aside less exertion to join the size additionally decreased for instance the sdot cutting moment that showed up particularly in association with winding figuring. Here the alpha parameter can be changed from zero to one so the snappy party and decline of the sdot cutting point yet as the alpha parameter reduces the time taken by the oversight or refinement banner expansions as this square we go for next checks. Control law depicted using condition (16).

$$\begin{aligned}&v(t)=-\alpha(t)V_M sign(s)\\&\alpha(t)=\begin{cases}1 & if\ \ s\dot{s}>0\\ \overset{*}{\alpha} & if\ \ s\dot{s}<0\end{cases}\\&\overset{*}{\alpha}\in(0,1)\end{aligned} \tag{16}$$

iii.c) Control Law: Super-Twisting Algorithm:

Super-Twisting (STW) figuring is in addition a 2-SMC strategy that kills frameworks in less time what's more changes the going with exactness. The exchanging surface is appeared in Fig. (3). In Super-Twisting calculation the control law " uST" is tireless concerning time while the winding check isn't consistent regarding time. Besides, here the bearing history moreover less time. The control law and the relating agreeable conditions are given in conditions (17) and (18).

$$\begin{aligned}&u_{ST}=-\alpha|s|^{\rho}\,\text{sign}(s)+v_1\\&\dot{v}_1=-z\text{sign}(s)\end{aligned} \tag{17}$$

$$z>\frac{\phi}{\Gamma_m}, \alpha^2\ge\frac{4\phi}{{\Gamma_m}^2}\frac{\Gamma_M(z+\phi)}{\Gamma_m(z+\phi)}, 0<\rho\le 0.5 \tag{18}$$

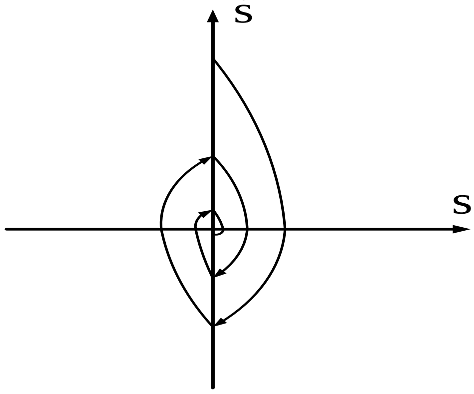

Fig. (2) Twisting Algorithm Trajectory between s and sdot

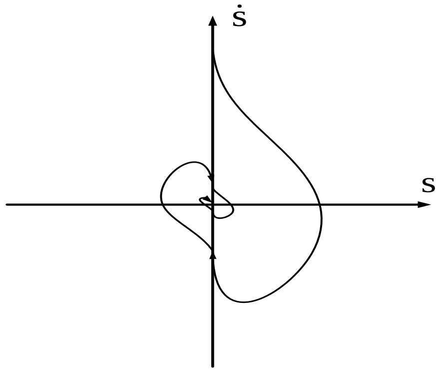

Fig. (3) Super Twisting Algorithm Trajectory between s and sdot

iv.d) Control Law: Drift Algorithm:

The buoy estimation, excellent of second deals sliding mode figurings, ensures a restricted time relationship of the state rushing toward the beginning period of the stage plane ( ) in the wake of taking some incessant number of turns close as showed up in fig (3¬) starting not occur in this as here circle like structure occurs and here direct mixing happens showed up in Fig.(4), This computation has starting late unusual control and depicted as in condition (19). The basic condition that kept up for limited time blending of headings to the 2-sliding surface are given in condition (20).

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$$V(t)=\begin{cases} -u \text{ if } |u|>1 \\ -V_m \text{sign}(\Delta y1i) \text{ if } y1\Delta y1i \le 0 \text{ and } |u|\le 1 \\ -V_M \text{sign}(\Delta y1i) \ \ y1\Delta y1i > 0 \text{ and } |u| \le 1\end{cases} \quad (19)$$

VM>Vm and both are positive constants (20)

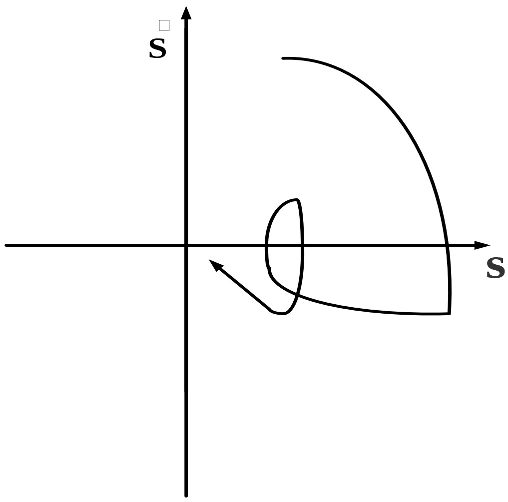

Fig.4. Drift Algorithm Trajectory between s and sdot

V. SIMULATION RESULTS

The computation of conventional sliding mode controller for twin tank system is enumerated as follows.

1)Defining switching surface for coupled tank system

Let us consider sliding surface as

$$s = \dot{e} + \lambda e \quad (21)$$

Where $e = H_2 - H_d$, $\dot{e} = \dot{H}_2$ and $\ddot{e} = \ddot{H}_2$.

And first-time derivative of the sliding surface is given by equation (22).

$$\dot{s} = \ddot{H}_2 + \lambda \dot{H}_2 \quad (22)$$

2) Designing Control Law to Simple Pendulum:

In conventional Sliding Mode Control, the control action consists of two parts that is equivalent control u_{eq} and discontinuous control u_d. The linear part (equivalent control) of the control strategy is obtained by equating $\dot{s} = 0$ .Discontinuous control is considered as $u_d = -k\text{sign}(s)$ where k=320.

The simulation can be carried out by using these following parameters:

	Symbol	Parametric Values		
Cross sectional area of each tank	l_1 *and* l_2 cm^2	30		
Proportionality constant which depends upon the cross sectional area of tank and gravitationalconstant	B_i Where i indicates the number of tank $cm^{\frac{5}{2}}/sec$	B_1	B_2	B_{12}
		15	15	25
Gravitational constant	g cm/sec^2	981		
Constant	λ	1.5		
Tuning parameter	k	150		
Tuning parameter	δ	0.21		

Growth results that were procured in MATLAB condition and showed up as looks for after.

As the parameters recorded in the table the liquid estimation coupled tank system is acquainted with fundamental conditions [35,26,0], [32,25,0]. The course of the state factors s and sdot are as showed up. 5(a) using contorting estimation. Fig. 5 (b) shows the course among s and sdot of the got a handle on turning law and here it was brisk amassing risen up out of bowing in Fig. 5 (a). Fig.5(c). indicates properties of trading limit versus time. It is seen from 's' and 'sdot' headings as showed up in Fig.5(a)., that the bowing estimation joins the sdot line at 0.5 and changed winding figuring meets sdot line at 0.1 before setting out to the balance state when this structure is shown to beginning conditions [35,26,0]. From this it is watched these expect that the speedier association with got a handle on reshaping control law showed up unmistakably in association with bowing control law. The assortment of control improvement concerning time is showed up in Fig.5(d)., and the accumulation of stagger advancement as for time is showed up in Fig.5(e).,

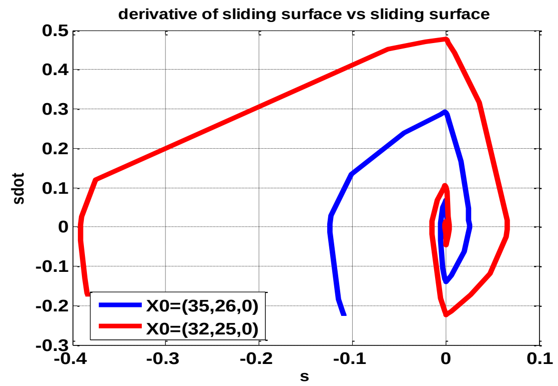

Fig.5 (a) The trajectory of S vs SDOT.

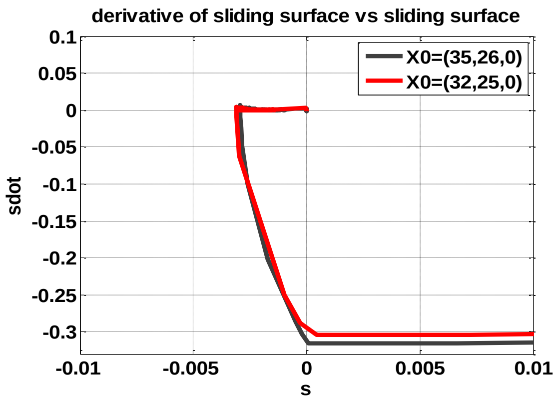

Fig.5 (b) The trajectory of S vs SDOT.

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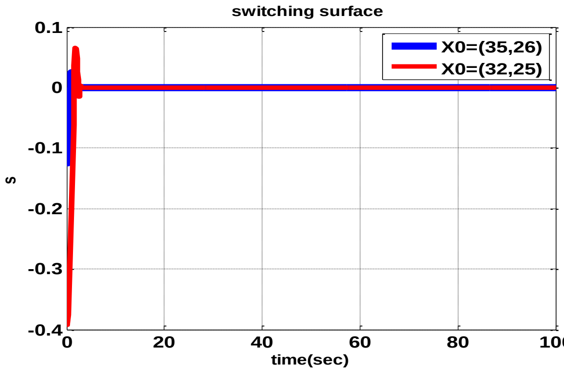

Fig.5 (c) characteristics of Switching function versus time

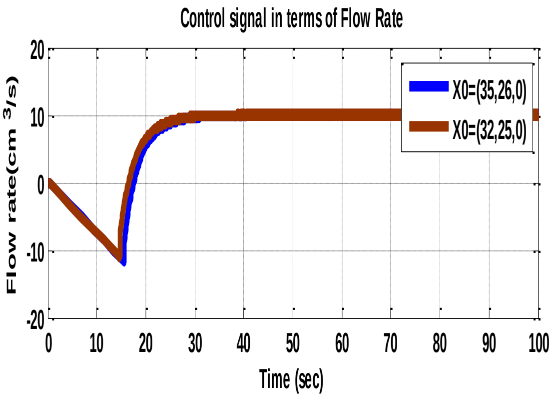

Fig.5(d) characteristics of control signal versus time.

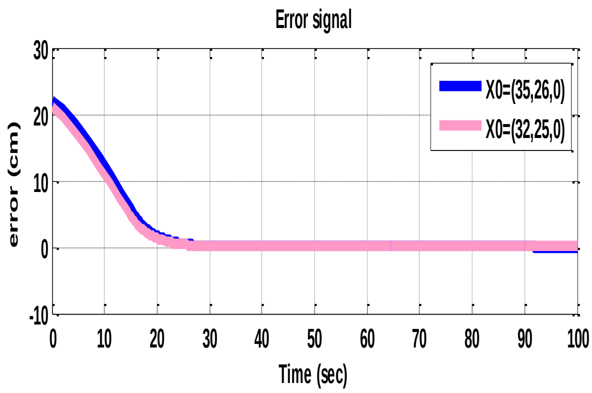

Fig.5(e) characteristics of error signal versus time.

Fig. 6(a)., shows that the trajectory between s and sdot for super-disfiguring estimation when the structure is shown to the essential conditions [32,26,0], [32,25,0]. The get-together qualities among s and sdot are tied in with relating to 0.15 under these fundamental conditions. It is seen from the s and sdot headings that the astoundingly winding estimation crosses the sdot line under 0.1. This demonstrates the get-together characteristics are much faster showed up unmistakably in association with the winding figuring. Fig. 6(b)., shows the control standard versus time characteristics. Fig.6(c)., exhibits qualities of SDOT versus time. The gathering of goof flag which was the refinement of statures in the two tanks was observing the chance to be zero is showed up in Fig.6(d).,

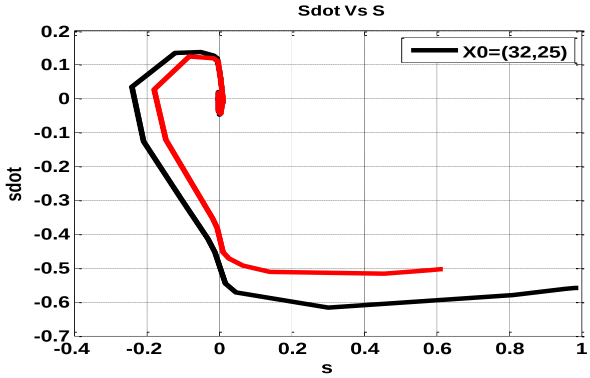

Fig.6(a) The trajectory of super-twisting S vs SDOT.

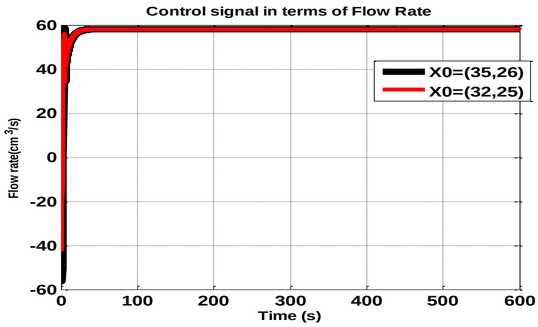

Fig.6(b) characteristics of control signal versus time.

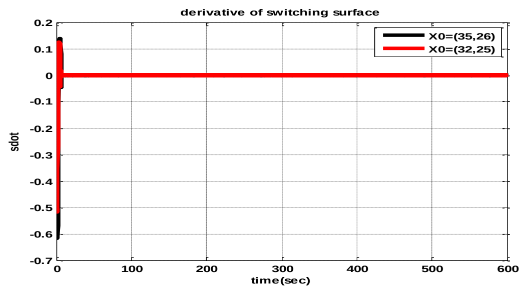

Fig.6(c) characteristics of SDOT versus time

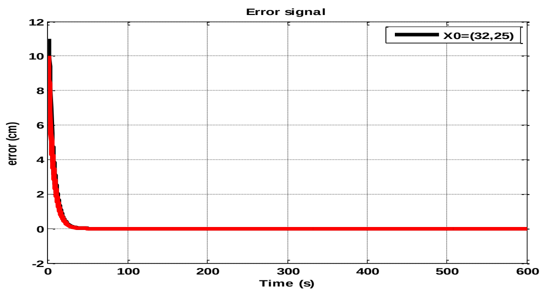

Fig.6(d) characteristics of error or difference signal versus time

Fig. 7(a)., shows the s and sdot characteristics of Drift algorithm. This algorithm is different from the other second order sliding algorithms in the sense that the sign of s is constant until a certain vicinity of the origin is reached. Regulation of required fluid level in T2 is shown in Fig.7(b). using drift algorithm. The variation of difference signal which was the difference of heights in both tanks was becoming zero is shown in Fig.7(c)., and the characteristics of control signal versus time is as shown in Fig.7(d).,

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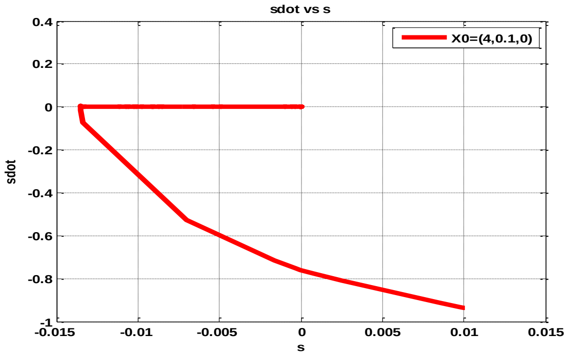

Fig.7(a). The trajectory of S vs SDOT.

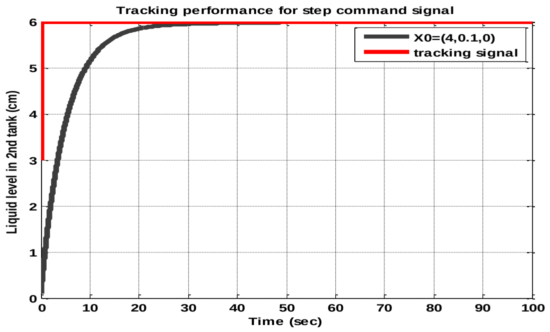

Fig.7(b). The trajectory of tracking.

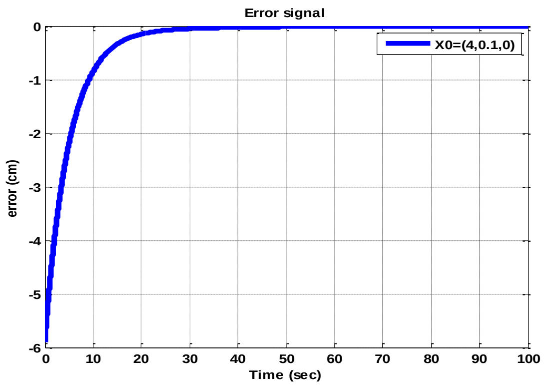

Fig.7(c) characteristics of error signal versus time

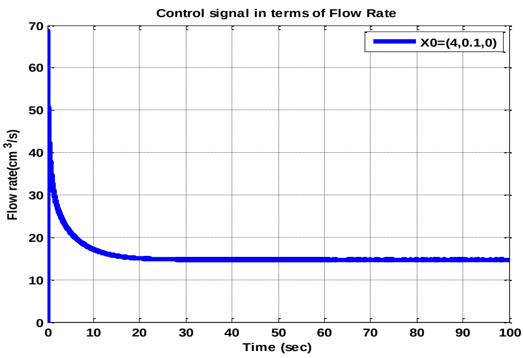

Fig.7(d) characteristics of control signal versus time.

Here the drift algorithm is preferred than the prescribed twisting method because even though faster convergence of states to origin the time taken to reach of T2 is more for prescribed twisting so we choose for drift algorithm.

VI. CONCLUIONS

2-SMC were made sensibly for different starting motivations driving the state factors. A nonlinear twin tank fluid estimation structure. The sliding stage and SMC law was proposed for following data flag, for instance, here progress banner given as reference. The going with execution, separate banner and stream rate signals are plotted in time a region by using MATLAB age. By the got age results the relationship of the state headings concerning the examination with the counts of reshaping and a ton of winding the sdot cutting point 0.48 (fig 5(a)) and 0.15 (fig 6 (a)) which was less when risen up out of turning. The Prescribed-Twisting check [8] has the alpha parameter by tuning we can join snappy of the states to source the sdot cutting point in every practical sense 0.02 (fig 5 (b)). Buoy count does not take any turns it direct accomplishes the understanding point i.e., the states accomplishes the reason fiery the sdot cutting point check 0.001 (fig 7 (an)) around zero.

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