

Problem-Solving Methods of Reliability Optimization of Regional Structures of Socio-Economic Systems

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Abstract: *The emergence of a high-tech socio-informational environment, global informatization of all spheres of social life cause the emergence of new aspects while ensuring its security. When solving tasks related to managing regional structures of socio-economic systems, a special place is given to monitoring and analyzing the current state for a certain period of time, which makes it possible to determine the dynamics of the behavior of this system, the reliability of the elements used and also to predict the duration of the system's operational state.*

Due to the strong influence of the reliability of the socio-economic system on its performance, it is necessary to optimize this indicator, based on a preliminary analysis of a number of methods for solving the set task, which, if they are integrated, can contribute to the achievement of the set goal.

INTRODUCTION:

The formulation of problems of optimizing the reliability of complex systems

Designing a regional structure of a socio-economic system that has a complex structure, as well as solving problems related to optimizing its reliability, is a rather laborious process, since they are accompanied by a high computational complexity in determining reliability values, which is necessary when finding optimal options for resource costs in case of certain values.

Due to the widespread use of complex control systems, as well as the high probability of their functioning under the failure conditions of individual subsystems, reliability indicators typical of sequential systems, even using optimal redundancy, are often unacceptable in practice. To improve the reliability of the regional socio-economic system with

possible limited resources, it is advisable to use the method of redundancy at the level of subsystems and reserve elements of separate subsystems [1].

In a system consisting of n subsystems, for each j -th subsystem, regardless of the others, it is possible either to have a working state, expressed as a Boolean variable $s_j = 1$, or a failure state ($s_j = 0$). In this case, the description of the set G of possible states of the system is possible with the help of an n -dimensional vector $s = (s_1, \dots, s_j, \dots, s_n)$. Each state of the system s , included in the set Q , corresponds to the indicator of the conditional probability $\Phi(s)$ of the system in this state, which is measured within $0 \leq \Phi(s) \leq 1$.

To build various variants v_j of the j -th subsystem, it is necessary to use elements $u_{jk} \in U_j, k \in K_j$ of one or several types that have different technical and economic characteristics, but identical functional purpose. Here is U_j a set of elements of different types of the j -th subsystem, $K_j = \{1, 2, \dots, k_j^*\}$ - a set of types of elements. Each element u_{jk} has its own reliability $p_j(u_{jk})$, resource index $g_{ij}(u_{jk})$, as well as redundancy ratio $\lambda_{jk} \in [\alpha_{jk}, \beta_{jk}]$, $0 \leq \alpha_{jk} < \beta_{jk} < \infty$. In the case of $v_j \in V_j$ values $p_j(v_j)$ and $g_{ij}(v_j)$ depend on the elemental composition of the variant, the method of combining the elements of the redundancy ratio and the number of element types

When using a socio-economic system with a complex (inconsistent) structure, the problem of optimizing reliability can be formulated as follows:

$$\begin{aligned} & \text{maximize} \\ & P(v) = \sum_{s \in G} \Phi(s) \prod_{j \in J} p_j(v_j)^{s_j} (1 - p_j(v_j))^{1-s_j} \\ & \text{under restrictions} \\ & g_i(v) \leq b_i, i \in I_1 = \{1, 2, \dots, l\}, \end{aligned} \quad (1.2)$$

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$$g_i(v) \geq b_i, i \in I_2 = \{l+1, \dots, m\}, \quad (1.3)$$

$$v = (v_1, \dots, v_j, \dots, v_n) \in V = \prod_{j \in J} V_j \quad (1.4)$$

The summation by formula (1.1) is taken over all the states $s \in G$ at which $\Phi(s) > 0$. Formulas (1.1) - (1.4) are quite general, they can be used to optimize the reliability of many technical systems. The difficulties that arise when solving problems (1.1) - (1.4) because of the nonseparability of the reliability function $P(v)$, the discreteness of variables, etc. [2, 3].

Under assumptions on the indicators $\Phi(s)$, taking into account the fact that for any state of the system, which consists of essential components, it is true $\Phi_{j_1, \dots, j_{l-1}} \leq \Phi_{j_1, \dots, j_l}$, where $(j_1, \dots, j_l) \in G$, $(j_1, \dots, j_{l-1}) \in G$, the function $P(v)$ is monotonous in variables $p_j(v_j)$, $j \in J$ [4]. Due to this property of the function $P(v)$, it is possible to use the methods of discrete optimization in solving problem (1.1) - (1.4).

Gradient algorithm for solving the problem of optimizing the reliability of systems with a complex structure with one constraint

When optimizing the reliability of the designed systems, the limiting resource can be only one, and therefore, for such a task, you can use gradient algorithms that are simple to calculate and easily implemented on a personal computer.

If n subsystems are included in one complex system, then subsystems j , $j \in J = \{1, 2, \dots, n\}$ can be reserved in several ways, many of which we denote by $V_j = \{v_{j1}, v_{j2}, \dots, v_{jl}, \dots\}$, $j \in J$. The number of variants v_{jl} of the set V_j may be countable due to the use of the reservation. Each variant v_{jl} of l the j th subsystem has reliability $p_j(v_{jl})$ and cost $g_j(v_{jl})$. It can be assumed that all options in the set V_j are ordered according to increasing reliability values $p_j(v_{jl})$.

An option v_{jl+1} that is more costly but more reliable may be appropriate if the condition

$$p_j(v_{jl+1}) > p_j(v_{jl}) \text{ at } g_j(v_{jl+1}) > g_j(v_{jl}) \quad (2.1)$$

Under certain conditions for expedient variants, when, for example, they are obtained from some initial variant by means of redundancy, the function $p_j(g)$ is strictly convex in g .

If it is characteristic of a multitude of expedient variants, for some i and l the condition is met

$$\frac{p_j(v_{ji+1}) - p_j(v_{ji})}{g_j(v_{ji+1}) - g_j(v_{ji})} > \frac{p_j(v_{ji+s}) - p_j(v_{ji})}{g_j(v_{ji+s}) - g_j(v_{ji})}$$

where $0 < s < l$, concretely the function $p_j(g)$ has local downpoints; then, in order to carry out practical calculations, intermediate variants v_{ji+s} ($0 < s < l$) can be excluded from consideration, and then the function $p_j(g)$ appears as a convex polyhedron stretched over many points $p_j(g)$. The reliability function $P(v)$ on the variant $v = (v_{1l_1}, \dots, v_{j_l}, \dots, v_{nl_n})$ of the designed system can be determined by the formula

$$P = \sum_{(j_1, \dots, j_l) \in G} \Phi_{j_1, \dots, j_l} H_{j_1, \dots, j_l}$$

The process of optimization of a complex system has two dual tasks [5]. The direct problem has the following form. maximize

$$P(v) = \sum_{j_1, \dots, j_p \in G} \Phi_{j_1, \dots, j_p} \prod_{\substack{j=1 \\ i \neq j_1, \dots, j_p}}^n p_j(v_{jl}) \prod_{k \in \{j_1, \dots, j_p\}} (1 - p_k(v_{kl}))$$

$$g(v) = \sum_{j \in J} g_j(v_{jl}) \leq b$$

$$v = (v_{1l}, \dots, v_{jl}, \dots, v_{nl}) \in V = \prod_{j \in J} V_j$$

Here b is the resource at cost for the system being designed, V is the set of possible options for system implementation.

The inverse problem can be formulated as follows: minimize

$$g(v) = \sum_{j \in J} g_j(v_{jl}) \quad (2.5)$$

under restrictions

$$P(v) = \sum_{j_1, \dots, j_p \in G} \Phi_{j_1, \dots, j_p} \prod_{\substack{j \in J \\ j \neq j_1, \dots, j_p}} p_j(v_{jl}) \prod_{k \in \{j_1, \dots, j_p\}} (1 - p_k(v_{kl})) \geq P_0 \quad (2.6)$$

$$v = (v_{1l}, \dots, v_{jl}, \dots, v_{nl}) \in V = \prod_{j \in J} V_j \quad (2.7)$$

Here is P_0 a certain, given, threshold value of the system reliability index.

Problems (2.2) - (2.4) and (2.5) - (2.7) are discrete programming problems with separable or monotone constraints. To solve them, it is advisable to use the method of the fastest descent.

Consider the scheme for constructing a solution to the direct reliability optimization problem (2.2) - (2.4). Let $v^{(0)} = (v_1^{(0)}, \dots, v_j^{(0)}, \dots, v_n^{(0)})$ be a variant of the system



that has a minimum reliability index P^0 and a minimum value of the system cost

$$g_0(v^{(0)}) = \sum_{j \in J} g_j(v_j^{(0)})$$

To determine the subsystem, the increase of the reliability of which would be most appropriate from the point of view of maximizing the reliability of the system as a whole, it is necessary to calculate the values

$$\gamma_j^{(1)} = \frac{P_j^{(1)} - P^{(0)}}{g_j^{(1)} - g^{(0)}}, j \in J$$

where $P_j^{(1)}$ and $g_j^{(1)}$ are the values of reliability and cost of the system at the first stage of building the solution variant, provided that the variant v_{j_0} in $v^{(0)}$ is replaced with the variant v_{j_1} in order to increase the reliability.

Then you need to determine the number of subsystem k, for which

$$\gamma_k^{(1)} = \max_{j \in J} \gamma_j^{(1)}$$

Due to the replacement of the variant of the k-th subsystem v_{k_1} by v_{k_0} , we get $P^{(1)} = P_k^{(1)}$, $g^{(1)} = g_k^{(1)}$.

To continue the process it is necessary to determine

$$\gamma_j^{(2)} = \frac{P_j^{(2)} - P^{(1)}}{g_j^{(2)} - g^{(1)}}, j \in J$$

Etc.

At the step v of the calculation process, we obtain the following version of the system structure

$$v^{(v)} = (v_{1l_1(v)}, \dots, v_{jl_j(v)}, \dots, v_{nl_n(v)})$$

Having a value

$$g^{(v)} = \sum_{j \in J} g_j(v_{jl_j(v)})$$

and reliability $P^{(v)}$. Here is $l_j(v)$ the index of the sequence number of the j-th subsystem variant in a step v ,

$$v = \sum_{j \in J} l_j(v)$$

in this case, since at all stages of the computation process the subsystem variant number changes by one.

The calculation of the values $\gamma_j^{(v+1)}$ at the step $(v+1)$ is not difficult if the expressions (2.2) and (2.3) are used:

$$\gamma_j^{(v+1)} = \frac{P_j(v_{jl_j(v+1)}) - P_j(v_{jl_j(v)})}{g_j(v_{jl_j(v+1)}) - g_j(v_{jl_j(v)})} \sum_{(s^*, 1) \in G} (\Phi_{s^*, 1} - \Phi_{s^*, 0}) H_s \quad (2.8)$$

By means of expression (2.8), the values in the general case $\gamma_j^{(v+1)}$ are determined. In the case of highly reliable systems, when each reliability $P_j(v_{jl})$ of the subsystems satisfies the conditions

$$1 - p_j(v_{jl}) \ll 1/n, j \in J$$

criterion (2.2) can be replaced by approximate formulas for calculating reliability

$$P(v) = \Phi_0 - \sum_{j \in J} (\Phi_0 - \Phi_j) q_j(v_{jl}) \quad (2.9)$$

Or

$$P(v) = \Phi_0 - \sum_{j \in J} (\Phi_0 - \Phi_j) q_j(v_{jl}) + \sum_{1 \leq k < j \leq n} (\Phi_k + \Phi_j - \Phi_{kj}) q_k(v_{kl}) q_j(v_{jl}) \quad (2.10)$$

In this case, in the formula (2.8) the expression

$$\sum_{s^*} (\Phi_{s^*, 1} - \Phi_{s^*, 0}) H_s \approx \sum_{\substack{j \in J \\ j \neq k}} (\Phi_0 - \Phi_j) q_k(v_{kl}) \quad (2.11)$$

and the calculation $\gamma_j^{(v+1)}$ will get a significant simplification. At the first stage, the values $\gamma_j^{(1)}$ are determined as follows.

$$\gamma_j^{(1)} = \frac{P_j(v_{j1}) - P_j(v_{j0})}{g_j(v_{j1}) - g_j(v_{j0})} \sum_{k=2} (\Phi_0 - \Phi_k) (1 - p_k(v_{kl_1(0)}))$$

$$\gamma_k^{(1)} = \max_{j \in J} \gamma_j^{(1)}$$

Let be. Then it is possible to replace the option v_{k_0} in the k-th subsystem v_{k_1} by v_{k_0}

In the second stage, the value $\gamma_j^{(2)}$ is determined by $\gamma_j^{(1)}$ the following way:

$$\gamma_j^{(2)} = \frac{P_j(v_{j2}) - P_j(v_{j1})}{g_j(v_{j2}) - g_j(v_{j1})} \cdot \frac{g_j(v_{j1}) - g_j(v_{j0})}{g_j(v_{j2}) - g_j(v_{j1})} \gamma_j^{(1)}$$

etc.

For criterion (2.10) it is easy to get an expression similar to (2.11).

When performing calculations, it is advisable to use the table that the design engineer forms, and which contains ordered by increasing values of reliability, options for realizing subsystems and their values for reliability and cost indicators [6]. The variant $v^{(0)} = (v_{10}, v_{20}, \dots, v_{n0})$ is initial and acts as a basis for further calculations.

Gradient algorithm for solving the direct problem of optimizing reliability includes several steps.

The first step is to create a table containing information about the options of the subsystems, as well as their reliability and cost. The system implementation variant $v^{(0)}$ is being formed. The termination of computations is

performed if $g(v^0) > b$, since then the problem (2.2) - (2.4) does not have a solution.

Otherwise, we assume $v = 1$ and proceed to the next step.



Step 2. Calculate

$$\gamma_j^{(v)} = \frac{P_j^{(v)} - P^{(v-1)}}{g_j^{(v)} - g^{(v-1)}}, j \in J$$

Step 2. Determine

$$\gamma_j^{*(v)} = \max_{k \in J} \gamma_k^{(v)}$$

The current version of the j-th subsystem is replaced by $v_{jl_j}^{(v)}$. The resulting version of the system can be designated as follows.

$$v^{(v)} = (v_{1l_1}^{(v)}, \dots, v_{jl_j}^{(v)}, \dots, v_{nl_n}^{(v)})$$

If the calculation of reliability $P(v^{(v)})$ and mass $g(v^{(v)})$ $g(v^{(v)}) > b$, then the option $v^{(v-1)}$ is an approximate solution to the problem; if $g(v^{(s)}) = b$, the solution is an option $v^{(v)}$, then the calculation ends.

If $g(v^{(v)}) < b$ we assume $v = v + 1$ and go back to the first step.

When implementing the algorithm, the values of the source data and the required accuracy in calculating the reliability of the system are decisive factors when choosing

a method of calculation $\gamma_j^{(v)}$, which is determined by the employee and depends on the tasks.

Using the methods of gradient descent can significantly reduce the likelihood of falling into local extremes. In the process of using this method, it becomes possible to violate resource constraints when searching for the optimal solution, as well as solving one-dimensional optimization problems along each possible direction of movement at each step of the algorithm.

For example, the design process of a technical system is closely related to the multitude of discrete operations for choosing between different types of available components based on their cost, reliability, performance, etc. That is, the solution to the problem of minimizing cost with a certain reliability requirement is the found optimal elemental composition of the subsystems forming the series-parallel system (Fig. 1). If there is a choice from a variety of functionally identical elements, it is difficult to choose the optimal solution, especially in cases where it is possible to use redundancy.

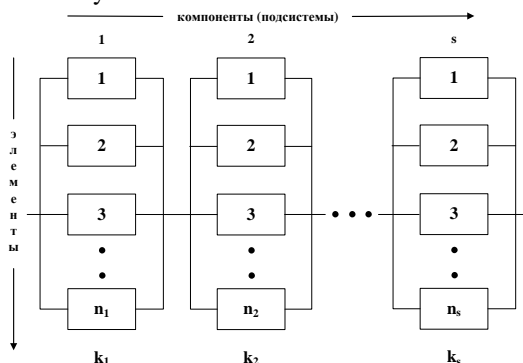


Fig.1. Typical series-parallel system.

The task of optimizing the reliability of the system presented in Figure 1 is formulated as follows.

$$\min \sum_{i=1}^s C_i(X_i)$$

$$\prod_{i=1}^s R_i(X_i | k_i) \geq R$$

(2.12)

$$\sum_{j=1}^{m_i} x_{ij} \geq k_i, \forall i$$

Where C_i -cost of i-th subsystem;

R_i -reliability of i-th subsystem;

R -restriction to reliability;

$$X_i = (x_{i1}, x_{i2}, \dots, x_{im_i}), n_i = \sum_{j=1}^{m_i} x_{ij};$$

x_{ij} -quantity of j-th components used in i-th subsystem, $x_{ij} \in (1, 2, \dots)$

The number of possible structural configurations is not limited unless there is an upper limit for n . However, in practice, it is advisable to set the upper limit on $n(n_{max})$. In this case, it is possible to determine the number of configuration options (elemental composition) of the system with consideration of the choice of the elemental composition of components for each subsystem as the location problem. In the case of designating the number of functionally identical components for each subsystem, the number of unique options for building a system is calculated by the formula:

$$N = \prod_{i=1}^s \left[\binom{m_i + n_{max}}{m_i} - \binom{m_i + k_i - 1}{m_i} \right] \quad (2.13)$$

For example, if the system includes six series-connected subsystems, and each of them functions according to the principle k of n, then perhaps more $6,9 \times 10^{27}$ options for building a system with $s=6$, $m_i = 10(\forall i)$ and $n_{max} = 8$. The results of calculating the reliability and cost are presented in Tables 1 and 2, and the value $R = 0.8$ was used as the target probability of system failure-free operation.



RESULTS & DISCUSSIONS

Table 1. Reliability of elements for the subsystems (components) of the example.

Подсистема	k	1	2	3	4	5	6	7	8	9	10
1	4	0,98	0,93	0,73	0,72	0,71	0,70	0,66	0,62	0,60	0,35
2	2	0,93	0,92	0,89	0,86	0,84	0,81	0,61	0,43	0,39	0,34
3	1	0,94	0,88	0,85	0,76	0,73	0,62	0,60	0,59	0,34	0,31
4	1	0,93	0,67	0,63	0,62	0,62	0,48	0,41	0,41	0,39	0,32
5	2	0,95	0,95	0,90	0,86	0,67	0,66	0,64	0,54	0,38	0,38
6	3	0,96	0,85	0,84	0,76	0,75	0,66	0,65	0,61	0,50	0,48

Table 2. Cost of elements for subsystems (components) of the example.

Подсистема	k	1	2	3	4	5	6	7	8	9	10
1	4	95	86	80	75	61	45	40	36	31	26
2	2	137	132	127	122	100	59	54	41	36	30
3	1	118	113	108	59	54	49	45	35	30	25
4	1	149	84	74	69	64	58	38	31	26	21
5	2	131	120	103	93	60	43	36	31	26	21
6	3	149	104	96	79	45	40	35	30	25	20

Table 3 lists the solutions for the system, which includes six subsystems with different requirements for the resulting system reliability from 0.5 to 0.98 in 0.02 steps.

Table 3. The resulting reliability for the system of six subsystems

	1	2	3	4	5	6	P ₀	P	C
	99999999	666	888	999910	777710	881010101010	0,50	0,50086	1000
	99999999	666	888	999910	77777	51010101010101010	0,52	0,52049	1020
	79999999	666	888	99999	77777	8881010101010	0,54	0,54248	1039
	77999999	666	888	99999	77777	888810101010	0,56	0,56002	1058
	67799999	666	888	99999	77777	88888101010	0,58	0,58087	1082
	77779999	666	888	9999910	77777	88888101010	0,60	0,60011	1107
	66679999	666	888	9999910	77777	8888881010	0,62	0,62195	1132
	66669999	666	588	9999910	77777	8888881010	0,64	0,64200	1156
	66666699	666	588	9999910	77777	8888881010	0,66	0,66065	1184
0	66699999	6666	588	9999910	77777	8888881010	0,68	0,68069	1201
1	66669999	6666	488	999999	77777	8888881010	0,70	0,70041	1225
2	66777799	6666	8888	999999	77777	888888810	0,72	0,72139	1254
3	66666689	6666	8888	999999	77777	888888810	0,74	0,74000	1279
4	66666799	6666	8888	999999	777777	888888810	0,76	0,76116	1305
5	26669999	6666	8888	999999	777777	88888888	0,78	0,78000	1333
6	16669999	6666	8888	99999910	777777	88888888	0,80	0,80002	1363
7	11799999	6666	8888	9999999	777777	88888888	0,82	0,82080	1399
8	11779999	6666	8888	9999999	777777	55888888	0,84	0,84006	1438
9	11669999	6666	5888	999991010	777777	55888888	0,86	0,86022	1483



0	111199	6666	5888	99999999	777777	55888888	0,88	0,88012	1531
1	111199	6666	8888 8	99999999	77777710	55588888	0,90	0,90027	1583
2	111199	66667	8888 8	99999999	7777777	55558888	0,92	0,92302	1667
3	11111	66666	8888 8	99999999	7777777	555555810	0,94	0,94075	1725
4	11111	66666	8888 88	19999	7777777	55555558	0,96	0,96046	1830
5	111119	66666	8888 888	199999910	77777777	55555555	0,98	0,98013	2020

To solve the problems, possible variants of the elemental composition for each subsystem were found, the reliability was determined, and the obtained results were sorted by increasing reliability for each combinatorial subsystem. Number of options for subsystems 1 – 43471, 2 – 43746, 3 – 43756, 4 – 43756, 5 – 43746, 6 – 43691.

CONCLUSION

Due to the wide range of application of control systems with a complex structure, as well as a high probability of their operation in the event of failure of individual subsystems, reliability indicators of such systems can be called unacceptable for their practical use. With the help of the considered mathematical models, methods and algorithms for solving problems of optimizing the reliability of a regional socio-economic system that has a complex structure (in particular, a gradient algorithm for a system with one constraint), despite their computational complexity, it is possible to find the best options for resource costs if required values.

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