

# A Research Algorithm to Sentence the Lots for Costly or Destructive Products in Mixed Quality Characteristics

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**Abstract**—This paper deals with the new operating procedure of Acceptance Sampling Plans for costly or destructive products when the incoming lots have mixed quality characteristics. The Operating Characteristic function and other associated measures of the plan are derived and provided. The procedure is given and designing of sampling plan are indexed through standard quality levels. Tables are constructed for easy selection of the plan. Illustrations are also provided.

**Keywords**— Mixed Sampling, Operating Characteristic function, AQL, LQL.

## INTRODUCTION:

Mixed Sampling Plan was introduced by Dodge and later Schilling (1967) has given a methodology to determine the Operating characteristics function and its associated measures. Mixed Sampling Plan has two stages. The first stage is considered with variable criteria and the second stage is considered with attribute quality characteristics. In a Mixed Sampling Plans, the second stage of attribute testing becomes more important to discriminate the lot, if the first stage variable inspection fails to accept the lot. In the second stage, acceptance number of zero plans is more emphasized for practical reason. However the OC curve of acceptance number with zero plans does not discriminate between good and bad lots. Hence Chain Sampling is recommended in the second stage of mixed plans. But Chain sampling plans in the second stage does not guarantee small sample sizes. Therefore Two Sided Complete chain sampling plans are recommended in the second stage. The resulting plans yield small samples in both the stages.

Dodge (1955) has designed Chain Sampling Inspection Plan. Clark (1960) has developed OC curves for ChSP – 1 Chain Sampling Plans. David Muse and Robert. (1982) have contributed to an approximate method for evaluating mixed sampling plans. Deva Arul Joyce (2010) have designed selection of Mixed Sampling Plans for Second Quality Lots. Deva Arul S and Edna (2010) developed Dependent Multi-dimensional Mixed Sampling Plans. Govindaraju (1990) has made, selection of Chsp -1 Chain sampling plans for given AQL and LQL. Govindaraju and Lai (1998) have developed modified chain sampling plans with very small sample sizes.

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## FORMULATION:

The development of mixed plans and the subsequent discussion are limited only to the upper specification limit. By symmetry a parallel discussion can be made use for lower specification limits. It is suggested that the mixed plans with two chain sampling as attribute plan in the case of single sided specification (U), standard deviation known can be formulated by the four parameters  $n_1$ ,  $n_2$ ,  $k$  and  $i$ .

Where,

$n_1$  = First Sample Size

$k$  = Variable Factor such that a lot is accepted if  $\bar{x} \leq U - k\sigma$

$U$  = Upper Specification Limit.

$n_2$  = Second Sample Size.

$N$  = Lot size.

$\bar{x}$  = Sample Mean

$\sigma$  = Population Standard Deviation.

## CONDITIONS FOR APPLICATIONS:

1. The Production Process should be steady and continuous.
2. Inspection is done due to variable Quality Characteristics in the first stage and Attribute quality characteristics in the second stage.
3. Single sampling inspection in the first stage and Modified chain sampling plans in the second stage.

## ALGORITHM FOR SENTENCING A LOT:

Let the two stages be independent.

Step (1) Determine the four parameters usually with reference to ASN & OC curves

Step (2) Take a random sample of size  $n_1$  from the lot.

Step (3) If the sample average  $\bar{x} \leq A = U - k\sigma$ , accept the lot.

Step (4) If the sample average  $\bar{x} > A$ , take a second sample of size  $n_2$ .

Step (5) Inspect and find the number of defectives ( $d$ ) in the second sample.

(i) Accept the lot, if  $d$  (the observed number of defectives) is zero in the sample of  $n_2$  units and reject if  $d > 1$ .

(ii) Accept the lot, if  $d = 1$  and if no defectives are found in the immediately preceding 'i' samples and succeeding 'j' samples of size  $n_2$ .

$P_1$	Values of $n_2$					Values of $k$			
	$i=j=1$	$i=j=2$	$i=j=3$	$i=j=4$	$i=j=5$	$n_1=5$	$n_1=10$	$n_1=15$	$n_1=20$
.001	121	98	78	69	63	2.8674	2.9356	2.9657	2.9837
.0015	80	61	52	46	42	2.7274	2.7956	2.8257	2.8437
.002	60	46	39	35	30	2.6474	2.7156	2.7457	2.7637
.0025	48	36	31	28	25	2.5774	2.6456	2.6757	2.6937
.003	40	30	26	23	21	2.5174	2.5856	2.6157	2.6337
.0035	34	26	22	20	18	2.4674	2.5356	2.5657	2.5837
.004	30	23	19	17	15	2.4174	2.4856	2.5157	2.5337
.0045	26	20	17	14	13	2.3774	2.4456	2.4757	2.4937
.005	24	18	15	13	12	2.3374	2.4056	2.4357	2.4537

**MEASURES OF INDEPENDENT PLAN:**

*I. Probability of Acceptance*

$$P_a(p) = P_{n_1}(\bar{x} \leq A) + P_{n_1}(\bar{x} > A)P_0^i P_1^j P_0^j$$

$$P_0 = e^{-n_2 p} \text{ and } P_1 = n_2 p e^{-n_2 p}$$

$$P_a(p) = e^{-np} \{1 + npe^{-2inp}\}, \text{ if } i = j$$

*II. Average sample number:*

$$ASN = n_1 + n_2 P(\bar{x} > A)$$

*III. Average Total Inspection*

$$ATI = ASN + (N - n_1 - n_2) (1 - P_a(p))$$

*IV. Average Outgoing Quality:*

$$AOQ = p \cdot P_a(p), \text{ if } n \text{ is large}$$

**DESIGNING MIXED SAMPLING PLANS INDEXED THROUGH AQL**

Designing the mixed sampling plan with variable Single sampling plans in the first stage and two-sided complete chain sampling plan as attribute plan in the second stage, when  $(p_1, \beta_1)$ ,  $i$  and the first sample size  $n_1$  are known. Assume that probability of acceptance assigned to the attribute plan associated with the second stage sample as the mixed plans are independent

1. Split the probability of acceptance that will be assigned to the first stage. Let it be  $\beta_1^1$ .
2. Decide the sample size  $n_1$ , which is to be used.
3. Calculate the acceptance limit as

$$A = U - \left[ Z(p_1) + \frac{Z(\beta_1^1)}{\sqrt{n_1}} \right] \sigma$$

4. Now determine  $\beta_1^2$ , the probability of acceptance assigned to the attribute plan associated with the second stage sample as,

$$\beta_1^2 = \frac{\beta_1 - \beta_1^1}{1 - \beta_1^1}$$

5. Determine the appropriate second stage sample of size  $n_2$  for the given index  $i$  from the relation.

$$e^{-n_2 p} \{1 + n_2 p e^{-2inp}\} = \beta_1^2, \text{ if } i = j$$

The above equations cannot be solved easily. Hence the solutions are obtained by using an iterative procedure. A computer program is written to solve the equations and to construct the tables. Using the above procedure tables are constructed to facilitate easy selection of mixed acceptance sampling plan with modified chain sampling as attribute plan.

**Table :1** Shows the values of the first stage variable criteria 'k' and the second stage sample size  $n_2$  for the known AQL (Using two-sided chain sampling plan as attribute plan in the second stage).  $\beta_1=0.99, \beta_1^1=0.70$

*Illustration :1*

Suppose a production process turns out 4% defective, obtain the corresponding mixed sampling plan with two-sided chain sampling as attribute plan with Probability of acceptance as 95% and the past lot index  $i=j=4$ .

*Solution:*

Let the first stage sample size  $n_1=15$ . It is given that the probability of acceptance is 95%. Let us bifurcate the 1<sup>st</sup> stage probability of acceptance.  $\beta_1^1=0.70$ . Then the second stage probability of acceptance will be  $\beta_1^2 = 0.97$ . If the first stage sample size  $n_1 = 15$  then from table 1,  $n_2=17$

Hence, the parameters of mixed sampling plan with two-sided chain sampling are  $n_1=15, n_2=17, i=j=4$  and  $k=2.5157$ .

*Algorithm for sentencing a lot*

Step (i) Take a random sample of size  $n_1 = 15$  from the lot.

Step (iii) If the sample average  $\bar{x} \leq A = U - 2.5157\sigma$ , accept the lot.

Step (iv) If the sample average  $\bar{x} > A = U - 2.5157\sigma$  take a second sample of size  $n_2=17$

Step (v) Inspect and find the number of defectives (d) in the second sample..

- (1) Accept the lot if d (the observed number of defectives) is zero in the sample of  $n_2=17$  units and reject if  $d > 1$ .



(2) Accept the lot if  $d = 1$  and if no defectives are found in the immediately preceding  $i=4$  samples and succeeding  $j=4$  samples of size

**DESIGNING MIXED SAMPLING PLAN INDEXED THROUGH LQL & RESULTS**

Designing the mixed sampling plan with variable Single sampling plans in the first stage the two-sided complete chain sampling plan as attribute plan in the second stage, when  $(p_2, \beta_2)$ , and the first sample size  $n_1$  are known. Let the probability of acceptance assigned to the attribute plan associated with the second stage sample as the mixed plans are independent

1. Split the probability of acceptance that will be assigned to the first stage. Let it be  $\beta_2'$ .
2. Decide the sample size  $n_1$ , which is to be used.
3. Calculate the acceptance limit as

$$A = U - \left[ Z(p_2) + \frac{Z(\beta_2')}{\sqrt{n_1}} \right] \sigma$$

$p_2$	Values of $n_2$					Values of $k$			
	$i=j=1$	$i=j=2$	$i=j=3$	$i=j=4$	$i=j=5$	$n_1=5$	$n_1=10$	$n_1=15$	$n_1=20$
.01	>300	>300	>300	>300	>300	4.2225	3.8937	3.7481	3.6613
.015	>300	>300	>300	>300	>300	4.0825	3.7537	3.6081	3.5213
.02	276	276	276	276	276	4.0025	3.6737	3.5281	3.4413
.025	220	220	220	220	220	3.9325	3.6037	3.4581	3.3513
.03	184	184	184	184	184	3.8725	3.5437	3.3981	3.3113
.035	157	157	157	157	157	3.8225	3.4937	3.3481	3.2613
.04	138	138	138	138	138	3.7725	3.4437	3.2981	3.2113
.045	122	122	122	122	122	3.7325	3.4037	3.2581	3.1713
.05	110	110	110	110	110	3.6925	3.3637	3.2181	3.1313

**Table :2** Values of the first stage variable criteria 'k' and the second stage sample size  $n_2$  for the known LQL (Using two-sided chain sampling plan as attribute plan in the second stage).  $\beta_2=0.01, \beta_2'=0.006$

*Illustration : 2*

If  $n_1=10, p_2=.05$  is a fraction defective corresponds to LQL and attribute index  $i=j=3$ , obtain the corresponding mixed sampling plan.

Solution : Let  $\beta_2' = .006$  be the probability of acceptance in the 1<sup>st</sup> stage and assume  $n_1=10$

From the table 2,  $n_2=110$ . Hence the parameters are  $n_1=10, n_2=110, i=j=3$  and  $k=3.3637$

*Algorithm for sentencing a lot*

Step (i) Take a random sample of size  $n_1 = 10$  from the lot.

Step (iii) If the sample average  $\bar{x} \leq A = U - 3.3637\sigma$ , accept the lot.

Step (iv) If the sample average  $\bar{x} > A = U - 3.3637\sigma$  take a second sample of size  $n_2=110$

Step (v) Inspect and find the number of defectives (d) in the second sample..

- (1) Accept the lot if d (the observed number of defectives) is zero in the sample of  $n_2=110$  units and reject if  $d > 1$ .

4. Now determine  $\beta_2''$  the probability of acceptance assigned to the attribute plan associated with the second stage sample as,

$$\beta_2'' = \frac{\beta_2 - \beta_2'}{1 - \beta_2'}$$

5. Determine the appropriate second stage sample of size  $n_2$  for the known index  $i$  from the relation.

$$e^{-n_2 p} \{1 + n_2 p e^{-in_2 p}\} = \beta_2'', \text{ if } i = j$$

The above equations cannot be solved easily. Hence, the solutions are obtained by using an iterative procedure. A computer program is written to solve the equations and tables are constructed to facilitate easy application in industries.

- (2) Accept the lot if  $d = 1$  and if no defectives are found in the immediately preceding  $i=3$  samples and succeeding  $j=3$  samples of size  $n_2=110$

**CONCLUSION:**

Hence in this paper Mixed Sampling Plan with Two Sided Complete Chain Sampling as attribute plan is developed by considering the results of past as well as future lots if the current sample does not lead to acceptance. There are possibilities in many production industries for considering past, current and future samples. Hence this proposed plan can be used to inspect very costly or destructive items. Tables are constructed for the easy selection of the plan.

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