Generalization of Singh's Common Fixed Point Theorem

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Abstract— In this article, I generalize Singh's common fixed point theorem for compatible mappings in fuzzy metric spaces. Examples are given to support the results proved herein.

Index Terms— fuzzy metric space; compatible mapping; common fixed point.

I. INTRODUCTION

The author defined intuitionistic (ψ,η) contractive mapping in [6]. Using the definition of ψ, we gave a common fixed point theorem. The generalization of the commuting mapping concept is compatible mapping which is introduced by Gerald Jungck[3]. This concept was generalized to fuzzy metric spaces by Mishra et al.[7]. Vasuki[13] proved a fuzzier version of the result of Pant[8]. She proved a common fixed point theorem using R-weakly commuting. Further, some Mathematicians proved common fixed point theorem for compatible mappings[11],[12],[10]. In the year 2000, Singh[1] proved a common fixed point result for two compatible pairs of maps in fuzzy metric spaces as follows:

Let A,B,S and T be self-mappings of a complete fuzzy metric space $(X,M,*)$ with $a * b = \min(a, b)$ satisfy the following conditions:

(I) $BX \subset SX, AX \subset TX$,

(II) A,S and B,T are compatible,

(III) S and T are continuous,

(IV) $M(Au,Bv,kt) \geq \min\{M(Su,Tv,t), M(Au,Su,t), M(Bv,Tv,t), M(Su,Bv,2t), M(Au,Tv,t)\}$, for all u, v ∈ X, t > 0 and k ∈ (0,1).

Then A,B,S and T have a unique common fixed point.

In our paper[6], ψ is defined as follows,

Let Ψ be the class of all mappings $\psi : [0,1] \to [0,1]$ such that

(i) ψ is non-decreasing and $\lim_{n\to\infty} \psi^n(s) = 1, \forall s \in (0,1]$;

(ii) $\psi(s) > s, \forall s \in (0,1)$;

(iii) $\psi(1) = 1$;

Example 1.1. [6] Define $\psi : [0,1] \to [0,1]$ by

$$\psi(s) = \frac{2s}{s+1}, \forall s \in [0,1].$$

$$\psi^2(s) = \frac{4s}{3s+1}, \psi^3(s) = \frac{8s}{7s+1}, \ldots,$$

$$\psi^n(s) = \frac{2^n s}{(2^n-1)s+1}, \forall s \in [0,1].$$

$$\lim_{n\to\infty} \psi^n(s) = \lim_{n\to\infty} \frac{2^n s}{(2^n-1)s+1} = 1, \forall s \in (0,1].$$

Clearly, $\psi(s) > s, \forall s \in (0,1)$ and $\psi(1) = 1$.

II. PRELIMINARIES

Definition 2.1. [9] A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is called t-norm if the following conditions hold:

(a) $*$ is associative and commutative;

(b) $a * 1 = a, \forall a \in [0,1]$;

(c) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, $\forall a,b,c,d \in [0,1]$.

If $*$ is continuous then it is called a continuous t-norm.

Definition 2.2.[5] Let X be an arbitrary set, $*$ be a continuous t-norm, and M be fuzzy sets on $X^2 \times (0,\infty)$. Consider the following conditions ∀ u, v, w ∈ X and t > 0,

(i) $M(u,v,0) = 0$;

(ii) $M(u,v,t) = 1$ if and only if $u = v$;

(iii) $M(u,v,t) = M(v,u,t)$;

(iv) $M(u,w,t + s) \geq M(u,v,t) * M(v,w,s)$;

(v) $M(u,v,t) : (0,\infty) \to [0,1]$ is left continuous;

The pair $(M,*)$ is called fuzzy metric on X. The triple $(X,M,*)$ is called a fuzzy metric space.

Example 2.3. [2] Let (X,d) be a metric space. Denote $a*b = ab, \forall a,b \in [0,1]$ and let M_d be fuzzy set on $X \times X \times (0,+\infty)$ defined as follows:

$M_d(u,v,t) = \dfrac{t}{t+d(u,v)}, \forall t > 0$, then $(X,M_d,*)$ is a fuzzy metric space.

Definition 2.4. [4] Let $(X,M,*)$ be a fuzzy metric space. A sequence $\{u_n\}$ in X is called

(a) convergent to a point u ∈ X if and only if $\lim_{n\to\infty} M(u_n,u,t) = 1, \forall t > 0$,

(b) Cauchy if $\lim_{n\to\infty} M(u_n,u_{n+p},t) = 1, \forall t > 0$ and $p > 0$.

Definition 2.5. [4] A fuzzy metric space is said to be complete if every Cauchy sequence in X is convergent.

Definition 2.6. [7] In a fuzzy metric space $(X,M,*)$, two self mappings A and B are said to be compatible if $\lim_{n\to\infty} M(ABu_n,BAu_n,t) = 1$ whenever u_n is a sequence in X such that $\lim_{n\to\infty} Au_n = \lim_{n\to\infty} Bu_n = w$ for some w ∈ X.

Lemma 2.7. [8] If A and B are compatible mappings on a fuzzy metric space X and $Au_n, Bu_n \to w$ for some w in X (u_n being a sequence in X) then $ABu_n \to Bw$ provided B is continuous (at w).

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III. MAIN RESULTS

Proposition 3.1. Let A and B be compatible mappings of a fuzzy metric space $(X,M,*)$ into itself. If Aw = Bw for some

$w \in X$, then ABw = BAw.

Proof. Suppose that $\{u_n\}$ is a sequence in X defined by

$u_n = w_n = 1,2,\ldots$ for some $w \in X$ and Aw = Bw.

Then we have $Au_n, Bu_n \to Aw$ as $n \to \infty$.

Since A and B are compatible mapping,

$M(ABw,BAw,t) = \lim_{n\to\infty} M(ABu_n,BAu_n,t) = 1.$

Hence, we have ABw = BAw.

Since Aw = Bw, we have ABw = BAw.

Theorem 3.2. Let A,B,S and T be self-mappings of a complete fuzzy metric space $(X,M,*)$ with $a * b = \min(a, b)$ satisfy the following conditions:

(I) $BX \subset SX, AX \subset TX$,

(II) A,S and B,T are compatible,

(III) One of A,B,S and T is continuous,

(IV) $M(Au,Bv,t) \geq \psi[\min\{M(Su,Tv,t),M(Au,Su,t), M(Bv,Tv,t), M(Su,Bv,2t),M(Au,Tv,t)\}]$,

for all $u,v \in X$ and $t > 0$.

Then A,B,S and T have a unique common fixed point.

Proof. Consider a point $u_0 \in X$.

Since $BX \subset SX$ and $AX \subset TX$, We can define a sequence $\{v_n\}$ in X as follows:

there exists $u_1 \in X$ such that $Au_0 = Tu_1 = v_0$.

there exists $u_2 \in X$ such that $Bu_1 = Su_2 = v_1$.

there exists $u_{2n+1} \in X$ such that $Au_{2n} = Tu_{2n+1} = v_{2n}$.

there exists $u_{2n+2} \in X$ such that $Bu_{2n+1} = Su_{2n+2} = v_{2n+1}$.

Now, for all $t > 0$,

$$\begin{aligned} M(v_{2n},v_{2n+1},t) &= M(Au_{2n},Bu_{2n+1}) \\ &\geq \psi[\min\{M(Su_{2n},Tu_{2n+1},t),M(Au_{2n},Su_{2n},t), M(Bu_{2n+1},Tu_{2n+1},t),M(Su_{2n},Bu_{2n+1},2t), M(Au_{2n},Tu_{2n+1},t)\}] \\ &= \psi[\min\{M(v_{2n-1},v_{2n},t),M(v_{2n},v_{2n-1},t), M(v_{2n+1},v_{2n},t),M(v_{2n-1},v_{2n+1},2t), M(v_{2n},v_{2n},t)\}] \\ &= \psi[\min\{M(v_{2n-1},v_{2n},t), M(v_{2n+1},v_{2n},t), [M(v_{2n-1},v_{2n},t)*M(v_{2n},v_{2n+1},t)],1\}] \\ &= \psi[M(v_{2n-1},v_{2n},t),M(v_{2n},v_{2n+1},t)] \\ &= \psi[\min\{M(v_{2n-1},v_{2n},t),M(v_{2n},v_{2n+1},t)\}]. \end{aligned}$$

Since $\psi(s) > s$, for all $s \in (0,1)$,

$M(v_{2n},v_{2n+1},t) \geq \psi(M(v_{2n},v_{2n+1},t)) > M(v_{2n},v_{2n+1},t)$ which is a contradiction.

Therefore, $M(v_{2n},v_{2n}+1,t) \geq \psi(M(v_{2n-1},v_{2n},t))$.

That is, $M(v_n,v_{n+1},t) \geq \psi(M(v_{n-1},v_n,t))$.

Continuing this process, we can get

$M(v_n,v_{n+1},t) \geq \psi(M(v_{n-1},v_n,t)) \geq \psi^2(M(v_{n-2},v_{n-1},t)) \geq \cdots \geq \psi^n(M(v_1,v_0,t))$.

That is, $M(v_n,v_{n+1},t) \geq \psi^n(M(v_1,v_0,t))$.

Taking limit as $n \to \infty$ and $\lim_{n\to\infty} \psi^n(s) = 1$, for all $s \in (0,1]$,

$\lim_{n\to\infty} M(v_n,v_{n+1},t) = 1.$

Similarly, we can prove

$M(v_{n+1},v_{n+2},t) \geq \psi^n(M(v_2,v_1,t))$.

$\lim_{n\to\infty} M(v_{n+1},v_{n+2},t) = 1.$

Now for all $p > 0$,

$$M(v_n,v_{n+p},t) \geq M(v_n,v_{n+1},\frac{t}{p}) * \ldots * M(v_{n+p-1},v_{n+p},\frac{t}{p}).$$

Taking limit $n \to \infty$, we have,

$$\begin{aligned} \lim_{n\to\infty} M(v_n,v_{n+p},t) &\geq \lim_{n\to\infty} M(v_n,v_{n+1},\frac{t}{p}) * \ldots * \lim_{n\to\infty} M(v_{n+p-1},v_{n+p},\frac{t}{p}) \\ &\geq 1 * \ldots * 1 \\ &= 1. \end{aligned}$$

That is, $\lim_{n\to\infty} M(v_n,v_{n+p},t) = 1$.

Hence, $\{v_n\}$ is a Cauchy sequence in X.

Since $(X,M,*)$ is a complete fuzzy metric space, there exists $w \in X$ such that $\lim_{n\to\infty} M(v_n,w,t) = 1$, for each $t > 0$.

Since $Au_{2n} = Tu_{2n+1} = v_{2n}$ and $Bu_{2n+1} = Su_{2n+2} = v_{2n+1}$ are subsequences of $\{v_n\}$,

$$\lim_{n\to\infty} Au_{2n} = \lim_{n\to\infty} Tu_{2n+1} = \lim_{n\to\infty} Bu_{2n+1} = \lim_{n\to\infty} Su_{2n+2} = w.$$

*Case 1.*Suppose A is continuous, since A and S are compatible and by Lemma 2.7, AAu_{2n} and SAu_{2n} converges to Aw as

$n \to \infty$.

Consider,

$$M(AAu_{2n},Bu_{2n+1},t) \geq \psi[\min\{M(SAu_{2n},Tu_{2n+1},t), M(AAu_{2n},SAu_{2n},t), M(Bu_{2n+1},Tu_{2n+1},t), M(SAu_{2n},Bu_{2n+1},2t),M(AAu_{2n},Tu_{2n+1},t)\}].$$

Taking limit as $n \to \infty$, we get

$$\begin{aligned} M(Aw,w,t) &\geq \psi[\min\{M(Aw,w,t), M(Aw,Aw,t), M(w,w,t), M(Aw,w,2t), M(Aw,w,t)\}] \\ &= \psi[\min\{M(Aw,w,t),M(Aw,w,2t),1\}] \\ &= \psi(M(Aw,w,t)). \end{aligned}$$

That is, $M(Aw,w,t) \geq \psi(M(Aw,w,t))$.

Since $\psi(s) \geq s$ for all $s \in (0,1]$, it is possible only when $M(Aw,w,t) = 1$.

That is, Aw = w.

Since $AX \subset TX$ and hence there exists a point $x \in X$ such that w = Aw = Tx.

We claim that w = Bx.

$$M(AAu_{2n},Bx,t) \geq \psi[\min\{M(SAu_{2n},Tx,t), M(AAu_{2n},SAu_{2n},t), M(Bx,Tx,t), M(SAu_{2n},Bx,2t), M(AAu_{2n},Tx,t)\}].$$

Takin limit as $n \to \infty$, we get

$M(Aw,Bx,t) \geq \psi[\min\{M(Aw,Aw,t),M(Aw,Aw,t),M(Bx,Aw,t), M(Aw,Bx,2t),M(Aw,Aw,t)\}]$.

That is, $M(w,Bx,t) = \psi[\min\{1,1,M(Bx,w,t),M(w,Bx,2t),1\}] = \psi[M(w,Bx,t)]$.

That is, $M(w,Bx,t) \geq \psi[M(w,Bx,t)]$.

It is possible only when $M(w,Bx,t) = 1$. That is, w = Bx.

Since B and T are compatible and Bx = Tx,

by Proposition 3.1, BTx = TBx and Tw = TBx = BTx = Bw.

Also we have,

$M(Au2n,Bw,t) \geq \psi[\min\{M(Su_{2n},Tw,t), M(Au_{2n},Su_{2n},t), M(Bw,Tw,t), M(Su_{2n},Bw,2t),M(Au_{2n},Tw,t)\}]$.

Taking limit as $n \to \infty$, we get

$M(w,Bw,t) \geq \psi[\min\{M(w,Bw,t), M(w,w,t),1, M(w,Bw,2t), M(w,Bw,t)\}]$
$= \psi[M(w,Bw,t)]$.

That is, $M(w,Bw,t) \geq \psi[M(w,Bw,t)]$.

It is possible only when $M(w,Bw,t) = 1$.

That is $w = Bw = Tw$.

Since $BX \subset SX$, there exists a point $y \in X$ such that $w = Bw = Sy$.

We claim that $w = Ay$.

$M(Ay,w,t) = M(Ay,Bw,t)$
$\geq \psi[\min\{M(Sy,Tw,t), M(Ay,Sy,t), M(Bw,Tw,t), M(Sy,Bw,2t), M(Ay,Tw,t)\}]$

Taking limit as $n \to \infty$, we get

$M(Ay,w,t) \geq \psi[\min\{M(w,w,t), M(Ay,w,t),1, M(w,w,2t), M(Ay,w,t)\}]$
$\geq \psi[M(Ay,w,t)]$
$= 1$.

Since $\psi(t) > t$ for all $t \in (0,1)$ and $\psi(1) = 1$, $M(Ay,w,t) = 1$. That is, $Ay = w$.

Since A and S are compatible and $Ay = Sy$, by Proposition 3.1, $ASy = Say$ and hence $Sw = SAy = ASy = Aw$.

Hence, $w = Bw = Tw = Aw = Sw$.

Therefore, w is a common fixed point of A,B,S and T.

Case 2. Similarly, we can prove when B is continuous.

Case 3. Suppose S is continuous, since A and S are compatible and by Lemma 2.7, SSu_{2n} and ASu_{2n} converges to Sw as $n \to \infty$.

Consider,

$M(ASu_{2n},Bu_{2n+1},t) \geq \psi[\min\{M(SSu_{2n},Tu_{2n+1},t), M(ASu_{2n},SSu_{2n},t), M(Bu_{2n+1},Tu_{2n+1},t), M(SSu_{2n},Bu_{2n+1},2t),M(ASu_{2n},Tu_{2n+1},t)\}]$

Taking limit as $n \to \infty$, we get

$M(Sw,w,t) \geq \psi[\min\{M(Sw,w,t), M(Sw,Sw,t), M(w,w,t), M(Sw,w,2t), M(Sw,w,t)\}]$
$= \psi[\min\{M(Sw,w,t),M(Sw,w,2t),1\}]$
$= \psi(M(Sw,w,t))$.

That is, $M(Sw,w,t) \geq \psi(M(Sw,w,t))$.

Since $\psi(s) \geq s$ for all $s \in (0,1]$, it is possible only when $M(Sw,w,t) = 1$. That is, $Sw = w$.

Now, we claim that $w = Aw$.

$M(Aw,Bu_{2n+1},t) \geq \psi[\min\{M(Sw,Tu_{2n+1},t), M(Aw,Sw,t), M(Bu_{2n+}1,Tu_{2n+1},t),M(Sw,Bu_{2n+}1,2t), M(Aw,Tu_{2n+1},t)\}]$

Taking limit as $n \to \infty$, we get

$M(Aw,w,t) \geq \psi[\min\{M(w,w,t), M(w,w,t), M(w,w,t), M(w,w,2t), M(w,w,t)\}]$
$\geq \psi[1]$
$= 1$.

Since $\psi(t) > t$ for all $t \in (0,1)$ and $\psi(1) = 1$, $M(w,Aw,t) = 1$. That is, $Aw = w$.

Since $AX \subset TX$ and hence there exists a point $z \in X$ such that $w = Aw = Tz$.

We claim that $w = Bz$.

$M(w,Bz,t) = M(Aw,Bz,t)$
$\geq \psi[\min\{M(Sw,Tz,t), M(Aw,Sw,t), M(Bz,Tz,t), M(Sw,Bz,2t),M(Aw,Tz,t)\}]$.
$\geq \psi[\min\{M(w,w,t),M(w,w,t),M(Bz,w,t), M(w,Bz,2t),1\}]$.
$= \psi[M(w,Bz,t)]$.

That is, $M(w,Bz,t) \geq \psi[M(w,Bz,t)]$.

It is possible only when $M(w,Bz,t) = 1$. That is, $w = Bz$.

Since B and T are compatible and $Bz = Tz$, by Proposition 3.1, $BTz = TBz$ and $Tw = TBz = BTz = Bw$.

Also, we claim that $w = Bw$.

$M(w,Bw,t) = M(Aw,Bw,t)$
$\geq \psi[\min\{M(Sw,Tw,t), M(Aw,Sw,t), M(Bw,Tw,t), M(Sw,Bw,2t),M(Aw,Tw,t)\}]$.
$\geq \psi[\min\{M(w,Bw,t),M(w,w,t),1,M(w,Bw,2t), M(w,Bw,t)\}]$.
$= \psi[M(w,Bw,t)]$.

That is, $M(w,Bw,t) \geq \psi[M(w,Bw,t)]$.

It is possible only when $M(w,Bw,t) = 1$. That is, $w = Bw$.

Hence $w = Aw = Sw = Bw = Tw$. Therefore, w is a common fixed point of A,B,S and T.

Case 4. Similarly, we can prove when T is continuous.

Uniqueness:

Suppose w_1 is also a common fixed point of A,B,S and T.

$M(w,w_1,t) = M(Aw,Bw_1,t)$
$\geq \psi[\min\{M(Sw,Tw_1,t), M(Aw,Sw,t), M(Bw_1,Tw_1,t), M(Sw,Bw_1,2t),M(Aw,Tw_1,t)\}]$
$= \psi[\min\{M(w,w_1,t), M(w,w,t), M(w_1,w_1,t), M(w,w_1,2t), M(w,w_1,t)\}]$
$= \psi[M(w,w_1,t)]$.

That is, $M(w,w_1,t) \geq \psi[M(w,w_1,t)]$

This is possible only when $M(w,w_1,t) = 1$. That is, $w = w_1$. Hence the proof.

Example 3.3. Let $X = (-\infty,\infty)$ with the metric d defined by $d(u,v) = |u - v|$, define $M(u,v,t) = \frac{t}{t+d(u,v)}$, for all $u,v \in X$ and $t > 0$. Note that, $(X,M,*)$ where $a * b = \min(a,b)$ is a complete fuzzy metric space.

The maps $A,B,S,T : X \to X$ is defined by

$A(u) = \frac{2+u}{3}$, $B(u) = 3 - 2u$, $S(u) = 2 - u$ and $T(u) = u$.

Let $u_n = (1 - \frac{1}{n})$.

Now, we verify that A,S is compatible.

$\lim_{n\to\infty} M(ASu_n,SAu_n,t) = \lim_{n\to\infty} M(A2 - u_n, S2 + u_n 3,t)$

$= \lim_{n\to\infty} M(\frac{4-u_n}{3}, \frac{4-u_n}{3}, t)$

$= 1.$

$\lim_{n\to\infty} M(ASu_n,SAu_n,t) = 1.$

Also $\lim_{n\to\infty} Au_n = \lim_{n\to\infty} \frac{2+u_n}{3} = \lim_{n\to\infty} \frac{2+(1-\frac{1}{n})}{3} = 1.$

$\lim_{n\to\infty} Su_n = \lim_{n\to\infty} 2 - u_n = \lim_{n\to\infty} 2 - (1 - \frac{1}{n}) = 1.$

Therefore, A and S are compatible mapping.

Similarly, we can verify that B and T are compatible. Also

$BX \subset SX$ and $AX \subset TX$ and T is continuous.

Define the map $\psi : (0,1] \to (0,1]$ by $\psi(s) = \frac{2s}{s+1}$ for each $s \in (0,1]$. Now, we verify that

$M(Au,Bv,t) \geq \psi[\min\{M(Su,Tv,t),M(Au,Su,t),M(Bv,Tv,t), M(Su,Bv,2t),M(Au,Tv,t)\}]$, for all $u,v \in X$.

Consider the following:

Suppose u = 4 and v = 10.

$M(A4,B10,t) \geq \psi[\min\{M(S4,T10,t), M(A4,S4,t), M(B10,T10,t),M(S4,B10,2t), M(A4,T10,t)\}]$

if $M(2,-17,t) \geq \psi[\min\{M(-2,10,t), M(2,-2,t),M(-17,10,t), M(-2,-17,2t),M(2,10,t)\}]$

That is if $\frac{t}{t+15} \geq \psi[\min\{\frac{t}{t+12}, \frac{t}{t+4}, \frac{t}{t+27}, \frac{t}{t+7.5}, \frac{t}{t+15}\}]$

That is if $\frac{t}{t+15} \geq \psi[\frac{t}{t+27}]$

That is if $\frac{t}{t+15} \geq \frac{t}{t+13.5}$

That is if $15 \geq 13.5$.

Similarly, we can verify, for all $u,v \in (-\infty,\infty)$.

All the conditions of the previous theorem are verified.

Then, 1 is the unique fixed point.

Hence, A,B,S and T have the common fixed point in X.

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