Vague Seperation and Connectedness

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Abstract: In this paper we blessing dark standard space, questionable run of the mill space,vague entire common zone and besides deduce a couple of speculations on those. also mean cloud connectedness, dicey unequivocally connectedness, unsure insistently disconnectedness, questionable CS detached, part and additionally choose a couple of theories.

Keywords: Vague regular space, Vague normal space, Vague connectedness, Vague strongly connectedness

I. INTRODUCTION

Zadeh provided the idea of fleecy units in 1965[12]. A vague Set is an also theory of a Fuzzy set. questionable set transformed into given by methods for Gau and Burherer and referred the properties of uncertain units. instead of using point-based cooperation as in fuzzy sets, period in the middle of basically based enlistment is connected in an undefined set. The period in the middle of based investment in undefined set is more noteworthy exressivein getting uncertainty of records.The theory of cously topology become made by means of C.L Chang[2] in 1967:primarily dependent on those thoughts C.L. Chang gave the soft topological space in 1968. He present cousy open set and fleecy close sets. those two foresee imperative action in soft topology. various makers provided a first rate arrangement of wears down cousy set and fleecy topological space.

by and by a days such enormous quantities of researchers developed their investigation on far fetched theory. T.Anitha and V.Amarendra Babu [9]evolved the theory of questionable LI-convictions in 2015 by utilizing connected the dark hypothesis and LI-perfect hypothesis of go area thought algebras.V.Amarendra babu and ok.V.Rama Rao[10,11]brought and developed the vague algebralike as uncertain presented substance social events and questionable topological get-togethers in 2017.

The speculation of hazy topology become introduced by Mariapresenti.L and Arockia Rani [5] .She developed the speculation of ill defined generalized alpha close sets in far fetched topological space and discussed thier properties.

In our past work we blessing some indispensable properties ofVague Topol and called attention to thier habitations. further we speak to darken minimization, vague segment adages and called attention to their homes.

on this paper we're offering obscure customary area,vague complete ordinary space, vague connectedness, vague solidly connectedness, ill defined disconnectedness and tried their living arrangements.

II. PRELIMINARIES

Definition 2.1[4]: A dubious set P known to man of talk L is portrayed by two participation capacities given by, a genuine enrollment work $f_p : L \rightarrow [0,1]$ and a false membership function $\mu : L \rightarrow [0,1]$. The grade of membership of y in the vague set P is bounded by a sub interval \([T_r(y), 1-f_r(y)]\) of [0,1]. This indicates that if the actual grade of membership \(\mu(y), then T_r(y) \leq \mu(y) \leq 1-f_r(y)\). The vague set P is written as \(P = \{y | T_r(y) \leq f_r(y) \} / y \in X\), where the interval \([T_r(y), 1-f_r(y)]\) is called the vague value of y in P and is denoted by \(V(y)\).

The zero vague set of P and denoted by \(P\) and defined as \(P = \{<x,[0,0]> / x \in P\}\).

The unit vague set of P and denoted by \(P\) and defined as \(P = \{<x, [1,1]> / x \in P\}\).

Definition 2.2[1]: An ambiguous topologeny P is a family \(\sigma\) of uncertain sets on P fulfilling the accompanying conditions: \(0, 1 \in \sigma\).

1. \(A_1 \cap A_2, \) for any \(A_1, A_2 \in \sigma\).

2. \(\bigcup A_i \in \sigma, \) for any arbitrary family \(\{A_i \in \sigma, i \in I\}\). The pair \((P, \sigma)\) is called a vague topological space. The elements of \(\sigma\) are called vague open sets.

Definition 2.3[7]: A vague topological space \((P, \sigma)\) is calledVT1 space if all pair of distinct vague points \(p_{(a,b)}^{(a,b)}\) , \(P_{(a,b)}\) of X there exist \(U, V \in \tau\) suchthat \(p_{(a,b)}^{(a,b)} \in U, p_{(a,b)}^{(a,b)} \in V\).

Definition 2.4[7]: A vague topological space \((P, \sigma)\) is calledVT2 space or Vague Haussdorff space if all pair of distinct vague points \(p_{(a,b)}^{(a,b)}\) , \(P_{(a,b)}\) of X there exist \(U, V \in \tau\)

suchthat \(U \cap V = \emptyset\) and \(p_{(a,b)}^{(a,b)} \in U, p_{(a,b)}^{(a,b)} \in V\).

Definition 2.5[6]: An associated area is a topological space X which cannot be spoken to because the affiliation of two disjoint non-void open units.If the topological space isn't related is referred to as indifferent.
Definition 2.6 [6]: A completely disengaged space is a topological space X where each pair of particular focuses can be isolated by a detachment of X.

Definition 2.7 [7]: Let (X,τ), (Y,σ) are two vague topological spaces. Then a mapping f : (X,τ) →(Y,σ) is called a vague α generalised open mapping if f(A) is a vague α generalised open set in Y for each vague open set A in X.

III. VAGUE REGULAR SPACES AND NORMAL SPACES

In this stage we plot ill defined ordinary territory, unclear typical space, vague complete normal region and moreover determine a few hypotheses on those

Definition 3.1: If A is a vague closed set of a vague topological space (X,τ) and x is any point in X such that x does not belongs to A, there exist open sets Uα and Uβ ∈ τ containing A and x respectively. Then (x,τ) is called vague regular space.

Definition 3.2: If A and B are disjoint vague closed sets in a vague topological space (X,τ) there exist vague open sets Uα and Uβ ∈ τ containing A and B respectively. Then (X,τ) is called vague normal space.

Definition 3.3: If A and B are vague separated sets in a vague topological space (X,τ) there exist vague open sets Uα and Uβ ∈ τ containing A and B respectively. Then (X,τ) is called vague completely normal space.

Proposition 3.4: Every subspace of a vague regular space is regular.

Proof: Let A be subspace of a vague regular space X.

Now we have to show that A is a vague regular space of X.

Let P is a point in A implies p ∈ X. Since X is regular space, there exist vague closed sets F such that there exist vague open sets Uα and Uβ ∈ τ containing A and F respectively.

Therefore A is vague regular.

Proposition 3.5: If every point pα(U,B) of a vague topological space X has a vague closed neighbourhood which is a regular space of X then X is vague regular.

Proof: Let (X,τ) be a vague topological space. Now show that X is a vague regular space.

Let F be a vague closed set and pα(U,B) is a point in X this vague point pα(U,B) has a vague closed neighbourhood Fx , pα(U,B) ∈ F x which is a regular space of X.

There exist vague open sets Uα and Uβ containing F and x respectively such that pα(U,B) ∈ F then there exist vague open sets Uα and Uβ containing F and x respectively.

Remark 3.6: Every vague completely normal space is a vague normal space.

Remark 3.7: Every vague normal space is a vague regular space.

Corollary 3.8: deliver f and g a hazard to be two ceaseless mappings of a topological space X into a haussdorff space Y then the association of all x ∈ X with the give up intention that f(x) =g(x) is close in X.

Verification: permit f and g be two steady mappings of a topological area X right into a haussdorff area Y.

give x and y a chance to be two points in X with the cease purpose that f(x) =g(x),f(y)=g(y) wherein f(x),f(y) are additives in Y. due to the fact Y is a haussdorff area there exist obscure open units Ux and Uy with the give up purpose that f(x) ∈ Ux and f(y) ∈ Uy and Ux ∩ Uy =0.

IV. VAGUE CONNECTED SPACES

on this place wedefine uncertain connectedness, ambiguous emphatically connectedness, doubtful unequivocally disconnectedness, obscure C5 separated, part and moreover infer a few hypotheses.

Definition four.1: A dubious topological area (X,T) is said to be uncertain separated if there exist non-0 ambiguous open sets An and B in X with the give up purpose that A∪B=1 and A∩B=zero .If X is not difficult to understand disengaged then it’s far said to be ambiguous related.

Example 4.2: Let X ={a,b}, the τ = {0,1, A,B} where A={<a[0.3,0.6]>,<b[0.4,0.7]>},
B={<a[0.4,0.7]>,<b[0.4,0.8]>}. A∪B ={<a[0.4,0.7]>,<b[0.4,0.8]>} ≠ 1
A∩B ={<a[0.3,0.6]>,<b[0.4,0.7]>} ≠ 0
Hence X is connected.

Proposition 4.3: Let f: (X,τ) → (Y,σ) be a vague irresolute surjection, (X,τ) is vague connected then (Y,σ) is vague connected.

Proof: Assume that (Y,σ) is not vague connected then there exist non-empty vague open sets A and B in Y such that A∪B=1 and A∩B=0. Since f is vague irresolute map , C=f(A)= f(B) =0 which are vague opensets in

Thus X is vague disconnected, which is a contradiction to the hypothesis, hence Y is vague connected.

Definition 4.4: A vague topological space (X,τ) is vague strongly connected if there exist no empty vague closed sets A and B such that μA+μB≥1 and (1-μA)+(1-μB)≥1.

In another words

A vague topological space (X,τ) is vague strongly connected if there exist no empty vague closed sets A and B in X suchthat A∩B =0.

Example 4.5: Let X={a,b}, τ = {0,1, A,B} where A={<a[0.5,0.5]>,<b[0.5,0.5]>}, B={<a[0.6,0.8]>,<b[0.5,0.6]>}.
Here A∩B =A ≠ 0
Hence X is vague strongly connected.

Theorem 4.6: Let h: (X,τ) → (Y,σ) be a vague irresolute surjection, (X,τ) is vague strongly connected then (Y,σ) is also vague strongly connected.

Proof: Suppose that Y is not vague strongly connected then there exist vague closed sets F1 and F2 in Y such that F1≠0, F2≠0, F1∩F2=0. Since h is vague irresolute surjection h⁻¹(F1)∩h⁻¹(F2)=0, h⁻¹(F1) ≠ 0, h⁻¹(F2) ≠ 0.

Hence X is vague strongly disconnected, which is a contradiction to our hypothesis.

So our assumption is wrong.

So Y is vague strongly connected.
Theorem 4.7: Let $Y$ be a vague topological space. If $\{K_i\}$ is a nonempty class of vague connected subspaces of $Y$ such that $\cap K_i$ is nonempty then $K= \cup K_i$ is also a vague connected subspace of $Y$.

Proof: In a contrary way suppose that $K$ is vague disconnected subspace of $Y$.

So there exist two vague open sets $G$ and $H$ such that $GUH=\emptyset$ and $G\cap H=\emptyset$.

So, that union $GUH$ contains $K$ and whose intersection with $K$ are disjoint and non empty.

Since all the $K_i$’s are connected and each lies in $GUH$.

So each lies either in $G$ or $H$ and is disjoint from the other.

Since $\cap K_i$ is non empty, so all the $K_i$’s completely in $G$ and are disjoint from $H$ or all completely in $H$ and are disjoint from $G$, which is a contradiction.

So our assumption is wrong.

So $K= \cup K_i$ is a vague connected subspace of $Y$.

\textbf{Theorem 4.8:} A topological space $(Y, \tau)$ is disconnected iff there exist a continuous mapping of $Y$ onto the vague indiscrete space $\{0, 1\}$.

Proof: If $Y$ is disconnected, so there exist two vague open sets $G$ and $H$ such that $GUH=1, G\cap H=\emptyset$.

Now we define a mapping $f$ from $X \to \{0, 1\}$ as

If $p_{\alpha\beta}$ is a vague point of $X$ then this point is belongs $G$ or $H$.

If this vague point is belongs to the vague open set $G$ then we define $f(p_{\alpha\beta})=1$, if this vague point is belongs to the vague open set $H$ then we define $f(p_{\alpha\beta})=0$.

So $f$ is vague continuous and onto mapping.

On the other hand if we have a vague continuous mapping $f: X \to \{0, 1\}$.

Now we have to show that $X$ is vague disconnected.

In a contrary way assume that $X$ is vague connected.

We have a statement continuous image of a vague connected space is vague connected.

So if $X$ is vague connected then $\{0, 1\}$ is vague connected, but this is a contradiction.

So our assumption is wrong.

So $X$ is vague connected.

Definition 4.9: A vague topological space $(X, \tau)$ is vague clopen disconnected if there exist vague open set $A$ such that $A \cap A^c = \emptyset$ and $A^c \cap A^c = \emptyset$.

Example 4.10: Let $X=\{a, b\}$, $\tau=\{0, 1, A\}$ where $A=\{a, b\}$.

Since $A$ is both vague closed and vague open set.

Example 4.11: Let $X=\{a, b\}$, $\tau=\{0, 1, A\}$ where $A=\{a, b\}$.

Since $A$ is both vaguely closed and vaguely open set.

Theorem 4.12: If $X$ is vague connected space then $X$ is vague connected space.

Proof: In a contrary way suppose that $X$ is vague disconnected space.

So there exist non empty vague open sets $G$ and $H$ such that $GUH=1$ and $G\cap H=\emptyset$.

Then $t_a \vee t_b = (1-t_a) \vee (1-t_b) = 1$ and $t_a A t_b = 0$.

Results & Discussions

Example 4.16: Let $X=\{a, b\}$, $\tau=\{0, 1, A, B\}$ where $A=\{a, b\}$ and $B=\{a, b\}$.

Here $A$ and $B$ are vague open sets in $X$ and $t_a + t_b \leq 1$.

Hence $X$ is vague strongly connected and vague clopen disconnected.

Lezione 4.14: An ill defined completely disengaged region is an indistinguishable topological region $X$ wherein each pair of great vague variables can be isolated by method for a separation of $X$.

That means for every pair of distinct vague points $p_{\alpha\beta}$ and $p_{\gamma\delta}$ in $X$ such that $x \neq y$ there exist a disconnection $X=AUW$ with $x \in A$ and $y \in B$.

Definition 4.17: An ambiguous associated space which isn’t appropriately contained in any bigger associated space is called dubious part of obscure associated space.

Theorem 4.19: The components of a vague totally disconnected space are its vague points.

Proof: Let $X$ be a vague totally disconnected space.

That is for every pair of distinct vague points can be separated by a disconnection of $X$.

This means $X=AUW$ with $x \in A$ and $y \in B$.

Now, we have to show that every subspace $Y$ of $X$ which contains more than point is connected.

Let $p_{\alpha\beta}$ and $p_{\gamma\delta}$ be distinct vague points in $Y$ and let $X=AUB$ is a disconnection of $X$ with $p_{\alpha\beta} \in A$ and $p_{\gamma\delta} \in B$. 

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Y = (Y ∩ A) U (Y ∩ B), since A ∩ B = ∅ that implies (Y ∩ A) ∩ (Y ∩ B) = ∅.
So Y is vague disconnected space.

Which is a contradiction.
So there is only one vague point in Y.

**Theorem 4.20**: Let X be a vague hausdorff space. If X has a vague open base whose sets are also vague closed then X is vague totally disconnected space.

**Proof**: Let X be a vague hausdorff space. Let px(α,β) and py(γ,δ) be distinct vague points in X.
That is there exist a vague open set G which contains px(α,β) which does not contain py(γ,δ).
By our assumption there exist a basic open set B which is also vague closed suchthat
px(α,β) ∈ B ⊆ G.
So X = B U B' is clearly a disconnection of X which separates px(α,β) and py(γ,δ).

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