Fuzzy M-open and Fuzzy M-closed Mappings in $\hat{S}$ostak's Fuzzy Topological Spaces

S. Bamini, M. Saraswathi, B. Vijayalakshmi, A. Vadivel

Abstract: *We introduce and investigate some new classes of mappings called fuzzy M-open map and fuzzy M-closed map to the fuzzy topological spaces in $\hat{S}$ostak's sense. Also, some of their fundamental properties are studied. Moreover, we investigate the relationships between fuzzy open, fuzzy θ-semiopen, fuzzy θ-open, fuzzy δ-semiopen, fuzzy δ-preopen, fuzzy a-open, fuzzy M-open, fuzzy e-open and fuzzy e^*-open mappings.*

Keywords and phrases: *fuzzy open, fuzzy θ-semiopen, fuzzy θ-open, fuzzy δ-semiopen, fuzzy δ-preopen, fuzzy a-open, fuzzy M-open, fuzzy e-open and fuzzy e^*-open mappings.*

I. INTRODUCTION

$\hat{S}$ostak [23] introduced the fuzzy topology as an extension of Chang's fuzzy topology [1]. It has been developed in many directions [6, 7, 22]. Ganguly and Saha [5] introduced the notions of fuzzy δ-cluster points in fuzzy topological spaces in the sense of Chang [1]. Kim and Park [8] introduced r-δ-cluster points and δ-closure operators in fuzzy topological spaces in view of the definition of $\hat{S}$ostak. In 2008, the initiations of e-open sets, e^*-open sets and a-open sets in topological spaces are due to Erdal Ekici[[3], [4]]. Sobana et.al [25] defined r-fuzzy e-open sets, fuzzy e-continuity, fuzzy e-open map and fuzzy e-closed map in a smooth topological space.

Throughout this paper, nonempty sets will be denoted by X, Y etc., $I = [0,1]$ and $I_0 = (0,1]$. For $\alpha \in I, \overline{\alpha}(x) = \alpha$ for all $x \in X$. A fuzzy point x_t for $t \in I_0$ is an element of I^X such that $x_t(y) = \begin{cases} t & if\ y = x \\ 0 & if\ y \notin x. \end{cases}$ The set of all fuzzy points in X is denoted by $P_t(X)$. A fuzzy point $x_t \in \lambda$ iff $t < \lambda(x)$. A fuzzy set λ is quasi-coincident with μ, denoted by $\lambda q \mu$, if there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$. If λ is not quasi-coincident with μ, we denoted $\lambda \bar{q} \mu$. If $A \subset X$, we define the characteristic function χ_A on X by $\chi_A(x) = \begin{cases} 1 & if\ x \in A, \\ 0 & if\ x \notin A. \end{cases}$ All other notations and definitions are standard, for all in the fuzzy set theory.

II. PRELIMINARIES

Lemma 1.1 [23] Let X be a nonempty set and $\lambda, \mu \in I^X$. Then

1. $\lambda q \mu$ iff there exists $x_t \in \lambda$ such that $x_t q \mu$.
2. $\lambda q \mu$, then $\lambda \wedge \mu \neq \underline{0}$.
3. $\lambda \bar{q} \mu$ iff $\lambda \leq \underline{1} - \mu$.
4. $\lambda \leq \mu$ iff $x_t \in \lambda$ implies $x_t \in \mu$ iff $x_t q \lambda$ implies $x_t q \mu$ implies $x_t \bar{q} \lambda$.
5. $x_t \bar{q} \bigvee_{i \in \Lambda} \mu_i$ iff there exists $i_0 \in \Lambda$ such that $x_t \bar{q} \mu_{i_0}$.

Definition 1.1 [23] A function $\tau : I^X \to I$ is called a fuzzy topology on X if it satisfies the following conditions:

1. $\tau(\underline{0}) = \tau(\underline{1}) = 1$,
2. $\tau(\vee_{i \in \Gamma} \mu_i) \geq \wedge_{i \in \Gamma} \tau(\mu_i)$, for any $\{\mu_i\}_{i \in \Gamma} \subset I^X$,
3. $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$, for any $\mu_1, \mu_2 \in I^X$.

The pair (X, τ) is called a fuzzy topological space (for short, sfts).

Remark 1.1 [20] Let (X, τ) be a fuzzy topological space. Then, for each $r \in I_0$, $\tau_r = \{\mu \in I^X : \tau(\mu) \geq r\}$ is a Change's fuzzy topology on X.

Theorem 1.1 [22] Let (X, τ) be a sfts. Then for each $\lambda \in I^X, r \in I_0$ we define an operator $C_\tau : I^X \times I_0 \to I^X$ as follows:

$C_\tau(\lambda, r) = \wedge \{\mu \in I^X : \lambda \leq \mu, \tau(\underline{1} - \mu) \geq r\}$.

For $\lambda, \mu \in I^X$ and $r, s \in I_0$, the operator C_τ satisfies the following conditions:

1. $C_\tau(\underline{0}, r) = \underline{0}$,
2. $\lambda \leq C_\tau(\lambda, r)$,
3.

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$C_\tau(\lambda,r) \vee C_\tau(\mu,r) = C_\tau(\lambda \vee \mu,r)$,

4. $C_\tau(\lambda,r) \le C_\tau(\lambda,s)$ if $r \le s$,
5. $C_\tau(C_\tau(\lambda,r),r) = C_\tau(\lambda,r)$.

Theorem 1.2 [22] Let (X,τ) be a sfts. Then for each $r \in I_0, \lambda \in I^X$ we define an operator $I_\tau : I^X \times I_0 \to I^X$ as $I_\tau(\lambda,r) = \vee \{\mu \in I^X : \lambda \ge \mu, \tau(\mu) \ge r\}$.

For $\lambda,\mu \in I^X$ and $r,s \in I_0$, the operator I_τ satisfies the following conditions:

1. $I_\tau(\underline{1},r) = \underline{1}$,
2. $\lambda \ge I_\tau(\lambda,r)$,
3. $I_\tau(\lambda,r) \wedge I_\tau(\mu,r) = I_\tau(\lambda \wedge \mu,r)$,
4. $I_\tau(\lambda,r) \le I_\tau(\lambda,s)$ if $s \le r$,
5. $I_\tau(I_\tau(\lambda,r),r) = I_\tau(\lambda,r)$,
6. $I_\tau(\underline{1} - \lambda,r) = \underline{1} - C_\tau(\lambda,r)$ and $C_\tau(\underline{1} - \lambda,r) = \underline{1} - I_\tau(\lambda,r)$

Definition 1.2 [10] Let (X,τ) be a sfts. Then for each $\mu \in I^X, x_t \in P_t(X)$ and $r \in I_0$,

1. μ is called r-open Q_τ-neighbourhood of x_t if $x_t q \mu$ with $\tau(\mu) \ge r$.
2. μ is called r-open R_τ-neighbourhood of x_t if $x_t q \mu$ with $\mu = I_\tau(C_\tau(\mu,r),r)$. We denote $Q_\tau(x_t,r) = \{\mu \in I^X : x_t q\mu, \tau(\mu) \ge r\}$, $R_\tau(x_t,r) = \{\mu \in I^X : x_t q \mu = I_\tau(C_\tau(\mu,r),r)\}$.

Definition 1.3 [10] Let (X,τ) be a sfts. Then for each $\lambda \in I^X, x_t \in P_t(X)$ and $r \in I_0$,

1. x_t is called r-τ cluster point of λ if for every $\mu \in Q_\tau(x_t,r)$, we have $\mu q \lambda$.
2. x_t is called r-δ cluster point of λ if for every $\mu \in R_\tau(x_t,r)$, we have $\mu q \lambda$.
3. An δ-closure operator is a mapping $D_\tau : I^X \times I \to I^X$ defined as follows: $\delta\_C_\tau(\lambda,r)$ or $D_\tau(\lambda,r) = \vee \{x_t \in P_t(X) : x_t$ is r-δ-cluster point of $\lambda\}$

Definition 1.4 Let (X,τ) be a sfts. For $\lambda,\mu \in I^X$ and $r \in I_0$, λ is called an

1. r-fuzzy δ-semiopen (resp. r-fuzzy δ-semiclosed)[25] set if $\lambda \le C_\tau(\delta I_\tau(\lambda,r),r)$ (resp. $I_\tau(\delta C_\tau(\lambda,r),r) \le \lambda$).
2. r-fuzzy δ-preopen (resp. r-fuzzy δ-preclosed)[25] set if $\lambda \le I_\tau(\delta C_\tau(\lambda,r),r)$ (resp. $C_\tau(\delta I_\tau(\lambda,r),r) \le \lambda$).
3. r-fuzzy a-open (resp. r-fuzzy a-closed)[25] set if $\lambda \le I_\tau(C_\tau(\delta I_\tau(\lambda,r),r),r)$ (resp. $C_\tau(I_\tau(\delta C_\tau(\lambda,r),r),r) \le \lambda$).
4. r-fuzzy e-open (resp. r-fuzzy e-closed)[25] set if $\lambda \le C_\tau(\delta\_I_\tau(\lambda,r),r) \vee I_\tau(\delta\_C_\tau(\lambda,r),r)$ (resp. $C_\tau(\delta I_\tau(\lambda,r),r) \wedge I_\tau(\delta C_\tau(\lambda,r),r) \le \lambda$).
5. r-fuzzy e^*-open (resp. r-fuzzy e^*-closed)[19] set if $\lambda \le C_\tau(I_\tau(\delta\_C_\tau(\lambda,r),r),r)$ (resp. $I_\tau(C_\tau(\delta\_I_\tau(\lambda,r),r),r) \le \lambda$).
6. λ is called an r-fuzzy semiopen (resp. r-fuzzy semi-closed) [11] set if $\lambda \le C_\tau(I_\tau(\lambda,r),r)$ (resp. $I_\tau(C_\tau(\lambda,r),r) \le \lambda$).

Definition 1.5 [12] Let (X,τ) and (Y,η) be fts's. Let $f : X \to Y$ be a mapping. Then f is called

1. fuzzy continuous iff $f^{-1}(\mu)$ is r-fuzzy open set of X for each $\mu \in I^Y$ with $\eta(\mu) \ge r$.
2. fuzzy open map iff $f(\mu)$ is r-fuzzy open set of Y for each $\mu \in I^X$ with $\tau(\mu) \ge r$.
3. fuzzy closed map iff $f(\mu)$ is r-fuzzy closed set of Y for each $\mu \in I^X$ with $\tau(\bar{1} - \mu) \ge r$.

Definition 1.6 [25] Let (X,τ_1) and (Y,τ_2) be sfts's and $f : (X,\tau_1) \to (Y,\tau_2)$ be a mapping. Then f is called

1. fuzzy δ-semiopen (resp. fuzzy δ-preopen, fuzzy a-open and fuzzy e-open) mapping iff $f(\lambda)$ is r-fδso (resp. r-fδpo, r-fao and r-feo) set of Y for each $\lambda \in I^X, r \in I_0$ with $\tau_1(\lambda) \ge r$.
2. fuzzy δ-semiclosed (resp. fuzzy δ-preclosed, fuzzy a-closed and fuzzy e-closed) mapping iff $f(\lambda)$ is $r\_f\delta sc$ (resp. $r\_f\delta pc$, r_fac and r_fec) set of Y for each $\lambda \in I^X, r \in I_0$ with $\tau_1(1-\lambda) \ge r$.

Definition 1.7 [27] Let (X,τ) be a sfts. $\lambda,\mu \in I^X$ and $r \in I_0$, then

1. The r-fuzzy θ-interior (resp. r-fuzzy θ-closure) of λ is $\theta I_\tau(\lambda,r) = \vee \{I_\tau(\mu) : \lambda \ge \mu, \tau(\bar{1} - \mu) \ge r\}$ (resp. $\theta C_\tau(\lambda,r) = \wedge \{C_\tau(\mu) : \lambda \le \mu, \tau(\mu) \ge r\}$.)
2. The r-fuzzy θ-semi-interior (resp. r-fuzzy θ-semi-closure) of λ is $\theta sI_\tau(\lambda,r) = \vee \{sI_\tau(\mu) : \lambda \ge \mu, \mu$ is r-fsc$\}$ (resp. $\theta sC_\tau(\lambda,r) = \wedge \{sC_\tau(\mu) : \lambda \le \mu, \mu$ is r-fso$\}$.)
3. The r-fuzzy θ-pre-interior (resp. r-fuzzy θ-pre-closure) of λ is $\theta pI_\tau(\lambda,r) = \vee \{pI_\tau(\mu) : \lambda \ge \mu, \mu$ is r-fpc $\}$ (resp. $\theta pC_\tau(\lambda,r) = \wedge \{pC_\tau(\mu) : \lambda \le \mu, \mu$ is r-fpo$\}$.)

Definition 1.8 [27] Let (X,τ) be a sfts. For $\lambda,\mu \in I^X$ and $r \in I_0$, λ is called an

1. r-fuzzy θ-open (resp. r-fuzzy θ-closed) set if $\lambda = \theta I_\tau(\lambda,r)$ (resp. $\lambda = \theta C_\tau(\lambda,r)$).

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2. r-fuzzy θ-semiopen (resp. r-fuzzy θ-semiclosed) set if $\lambda \le C_\tau(\theta I_\tau(\lambda,r),r)$ (resp. $I_\tau(\theta C_\tau(\lambda,r),r) \le \lambda$).

3. r-fuzzy θ-preopen (resp. r-fuzzy θ-preclosed) set if $\lambda \le I_\tau(\theta C_\tau(\lambda,r),r)$ (resp. $C_\tau(\theta I_\tau(\lambda,r),r) \le \lambda$).

Definition 1.9 [27] Let (X,τ) be a fuzzy topological space. For $\lambda \in I^X$ and $r \in I_0$, λ is called an r-fuzzy

1. M-open set if $\lambda \le C_\tau(\theta I_\tau(\lambda,r),r) \vee I_\tau(\delta C_\tau(\lambda,r),r)$.

2. M-closed set if $\lambda \ge C_\tau(\delta I_\tau(\lambda,r),r) \wedge I_\tau(\theta C_\tau(\lambda,r),r)$.

Definition 1.10 [27] Let (X,τ) be a fuzzy topological space. $\lambda,\mu \in I^X$ and $r \in I_0$, then

1. $MI_\tau(\lambda,r) = \vee\ \{\mu \in I^X : \lambda \ge \mu, \mu$ is a r-fM_o set$\}$ is called the r-fuzzy M-interior of λ.

2. $MC_\tau(\lambda,r) = \wedge\ \{\mu \in I^X : \lambda \le \mu, \mu$ is a r-fM_c set$\}$ is called the r-fuzzy M-closure of λ.

Proposition 1.1 [27] If λ is an r-fuzzy M-open subset of a sfts (X,τ) and $\theta I_\tau(\lambda,r) = \overline{0}$, then λ is r-fuzzy δ-preopen.

Theorem 1.3 [27] Let (X,τ) be a sfts. Let $\lambda \in I^X$ and $r \in I_o$.

1. λ is r-fM_o iff $\lambda = MI_\tau(\lambda,r)$.
2. λ is r-fM_c iff $\lambda = MC_\tau(\lambda,r)$.

Theorem 1.4 [27] Let (X,τ) be a sfts. For $\lambda \in I^X$ and $r \in I_0$ we have

1. $MI_\tau(1-\lambda,r) = 1-(MC_\tau(\lambda,r))$.
2. $MC_\tau(1-\lambda,r) = 1-(MI_\tau(\lambda,r))$.

Theorem 1.5 [27] Let (X,τ) be a sfts. Let $\lambda \in I^X$ and $r \in I_o$, the following statements hold:

1. $MC_\tau(0,r) = 0$ and $MI_\tau(1,r) = 1$.
2. $I_\tau(\lambda,r) \le MI_\tau(\lambda,r) \le \lambda \le MC_\tau(\lambda,r) \le C_\tau(\lambda,r)$.
3. $\lambda \le \mu \Rightarrow MI_\tau(\lambda,r) \le MI_\tau(\mu,r)$ and $MC_\tau(\lambda,r) \le MC_\tau(\mu,r)$.
4. $MC_\tau(MC_\tau(\lambda,r),r) = MC_\tau(\lambda,r)$ and $MI_\tau(MI_\tau(\lambda,r),r) = MI_\tau(\lambda,r)$.
5. $MC_\tau(\lambda,r) \vee MC_\tau(\mu,r) < MC_\tau(\lambda \vee \mu,r)$ and $MI_\tau(\lambda,r) \vee MI_\tau(\mu,r) < MI_\tau(\lambda \vee \mu,r)$.
6. $MC_\tau(\lambda \wedge \mu,r) < MC_\tau(\lambda,r) \wedge MC_\tau(\mu,r)$ and $MI_\tau(\lambda \wedge \mu,r) < MI_\tau(\lambda,r) \wedge MI_\tau(\mu,r)$.

Theorem 1.6 [27] Let (X,τ) be a sfts. For $\lambda,\mu \in I^X$ and $r \in I_0$.

1. λ is r-fM_o iff $1-\lambda$ is r-fM_c.
2. If $\tau(\lambda) \ge r$, then λ is r-fM_o set.
3. $I_\tau(\lambda,r)$ is an r-fM_o set.
4. $C_\tau(\lambda,r)$ is an r-fM_c set.

Definition 1.11 [28] Let (X,τ_1) and (X,τ_2) be sfts's and $f:(X,\tau_1) \to (Y,\tau_2)$ be a mapping. Then f is called

1. fuzzy M-continuous iff $f^{-1}(\mu)$ is r-fM_o for each $\mu \in I^X, r \in I_0$ with $\tau_2(\mu) \ge r$.
2. fuzzy θ-continuous iff $f^{-1}(\mu)$ is r-fθ_o for each $\mu \in I^X, r \in I_0$ with $\tau_2(\mu) \ge r$.
3. fuzzy θ-semicontinuous iff $f^{-1}(\mu)$ is r-fθ_{so} for each $\mu \in I^X, r \in I_0$ with $\tau_2(\mu) \ge r$.

Definition 1.12 [28] Let (X,τ_1) and (X,τ_2) be sfts's and $f:(X,\tau_1) \to (Y,\tau_2)$ be a mapping. Then f is called fuzzy θ-open map if the image of every r-fuzzy open set of (X,τ_1) is r-fuzzy θ-open set in (Y,τ_2).

III. RESULTS

Definition 2.1 Let (X,τ_1) and (Y,τ_2) be sfts's and $f:(X,\tau_1) \to (Y,\tau_2)$ be a mapping. Then f is called

1. fuzzy M-open mapping iff $f(\lambda)$ is r-fMo set of Y for each $\lambda \in I^X, r \in I_0$ with $\tau_1(\lambda) \ge r$.
2. fuzzy M-closed mapping iff $f(\lambda)$ is r-fMc set of Y for each $\lambda \in I^X, r \in I_0$ with $\tau_1(1-\lambda) \ge r$.

Remark 2.1 From the above definitions, it is clear that the following implications are true for

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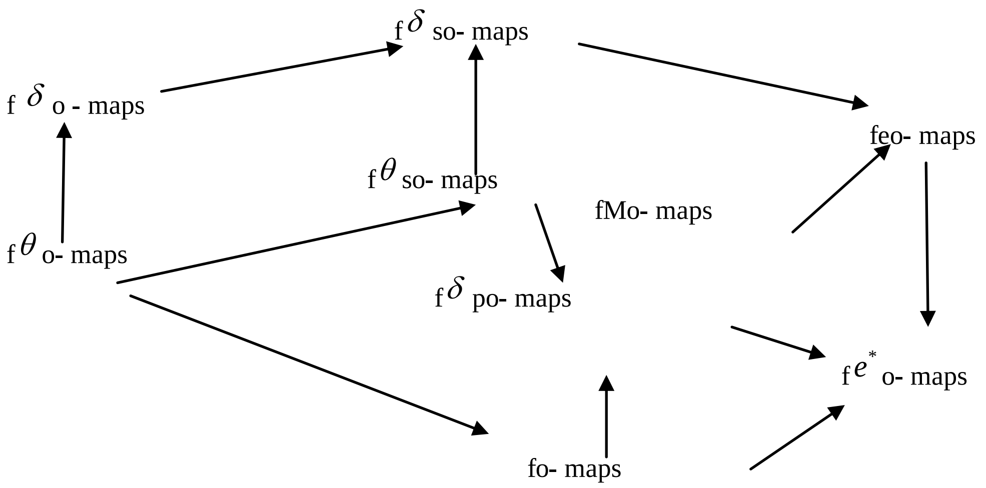

where fo, fθso, fθo, fso, fpo, fαo, f$\mathcal{M}$o, feo and fe^*o maps are abbreviated by fuzzy open, fuzzy θ-semiopen, fuzzy θ-open, fuzzy δ-semiopen, fuzzy δ-preopen, fuzzy α-open, fuzzy $\mathcal{M}$- open, fuzzy e-open and fuzzy e^*-open maps respectively.

From the above definitions, it is clear that every fuzzy δ-preopen map is fuzzy $\mathcal{M}$-open map and every fuzzy θ-semiopen map is fuzzy $\mathcal{M}$- open map. Also, it is clear that every fuzzy $\mathcal{M}$-open map is fuzzy e-open map and fuzzy e^*-open map. Also, every fuzzy θ-open map, fuzzy δ-open map, fuzzy α-open map is fuzzy $\mathcal{M}$-open map. The converses need not be true in general.

The converses of the above implications are not true as the following examples show:

Example 2.1 Let λ and μ be fuzzy subsets of $X = Y = \{a,b,c\}$ defined as follows $\lambda(a) = 0.5$, $\lambda(b) = 0.4$, $\lambda(c) = 0.7$; $\mu(a) = 0.4$, $\mu(b) = 0.5$, $\mu(c) = 0.2$. Then $\tau,\eta : I^X \to I$ defined as

$$\tau(\lambda)=\begin{cases}1, & \text{if } \lambda=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \lambda=\lambda,\\ 0, & \text{otherwise},\end{cases}\qquad \eta(\mu)=\begin{cases}1, & \text{if } \mu=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \mu=\mu,\\ 0, & \text{otherwise},\end{cases}$$

are fuzzy topologies on X and Y. Consider the identity mapping $f:(X,\tau)\to(Y,\eta)$. Take $r=\frac{1}{2}$. For any $\frac{1}{2}$-fuzzy

are fuzzy topologies on X and Y. Consider the identity mapping $f:(X,\tau)\to(Y,\eta)$. Take $r=\frac{1}{2}$. For any $\frac{1}{2}$-fuzzy
open set λ in (X,τ), $f(\lambda)=\lambda$ is $\frac{1}{2}$-fMo set in (Y,η). Then f is f$\mathcal{M}$o-map, but f is not fuzzy fδpo, fδo and fαo map, since $f(\lambda)=\lambda$ is not $\frac{1}{2}$-fδpo, $\frac{1}{2}$-fδo and $\frac{1}{2}$-fαo sets in (Y,η).

Example 2.4 Let λ and μ be fuzzy subsets of $X = Y = \{a,b,c\}$ defined as follows $\lambda(a) = 0.9$,

open set λ in (X,τ), $f(\lambda)=\lambda$ is $\frac{1}{2}$-fe*o set in (Y,η). Then f is fe^*o-map, but f is not f$\mathcal{M}$o-map, since $f(\lambda)=\lambda$ is not $\frac{1}{2}$-f$\mathcal{M}$o in (Y,η).

Example 2.2 Let λ and μ be fuzzy subsets of $X = Y = \{a,b,c\}$ defined as follows $\lambda(a) = 0.5$, $\lambda(b) = 0.4$, $\lambda(c) = 0.4$; $\mu(a) = 0.5$, $\mu(b) = 0.3$, $\mu(c) = 0.2$. Then $\tau,\eta : I^X \to I$ defined as

$$\tau(\lambda)=\begin{cases}1, & \text{if } \lambda=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \lambda=\lambda,\\ 0, & \text{otherwise},\end{cases}\qquad \eta(\mu)=\begin{cases}1, & \text{if } \mu=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \mu=\mu,\\ 0, & \text{otherwise},\end{cases}$$

are fuzzy topologies on X and Y. Consider the identity mapping $f:(X,\tau)\to(Y,\eta)$. Take $r=\frac{1}{2}$. For any $\frac{1}{2}$-fuzzy open set λ in (X,τ), $f(\lambda)=\lambda$ is $\frac{1}{2}$-feo set in (Y,η). Then f is feo-map, but f is not f$\mathcal{M}$o-map, since $f(\lambda)=\lambda$ is not $\frac{1}{2}$-f$\mathcal{M}$o in (Y,η).

Example 2.3 Let λ and μ be fuzzy subsets of $X = Y = \{a,b,c\}$ defined as follows $\lambda(a) = 0.9$, $\lambda(b) = 0.9$, $\lambda(c) = 0.9$; $\mu(a) = 0.1$, $\mu(b) = 0.1$, $\mu(c) = 0.1$. Then $\tau,\eta : I^X \to I$ defined as

$$\tau(\lambda)=\begin{cases}1, & \text{if } \lambda=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \lambda=\lambda,\\ 0, & \text{otherwise},\end{cases}\qquad \eta(\mu)=\begin{cases}1, & \text{if } \mu=\overline{0} \text{ or } \overline{1},\\ \frac{1}{2}, & \text{if } \mu=\mu,\\ 0, & \text{otherwise},\end{cases}$$

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$\lambda(b) = 0.9$, $\lambda(c) = 0.9$; $\mu(a) = 0.1$, $\mu(b) = 0.1$, $\mu(c) = 0.1$. Then $\tau, \eta : I^X \to I$ defined as

$$\tau(\lambda) = \begin{cases} 1, \text{if} \lambda = \overline{0} \text{or} \overline{1}, \\ \frac{1}{2}, \text{if} \lambda = \lambda, \\ 0, \text{otherwise}, \end{cases} \quad \eta(\mu) = \begin{cases} 1, \text{if} \mu = \overline{0} \text{or} \overline{1}, \\ \frac{1}{2}, \text{if} \mu = \mu, \\ 0, \text{otherwise}, \end{cases}$$

are fuzzy topologies on X and Y. Consider the identity mapping $f: (X,\tau) \to (Y,\eta)$. Take $r=\frac{1}{2}$. For any $\frac{1}{2}$-fuzzy open set λ in (X,τ), $f(\lambda) = \lambda$ is $\frac{1}{2}$-$f\delta so$ set in (Y,η). Then f is fδso-map, but f is not fδo map, since $f(\lambda) = \lambda$ is not $\frac{1}{2}$-$f\delta o$ set in (Y,η).

Example 2.5 Let λ, μ and ω be fuzzy subsets of $X = Y = \{a,b,c\}$ defined as follows
$\lambda(a) = 0.3$, $\lambda(b) = 0.4$, $\lambda(c) = 0.5$; $\mu(a) = 0.6$, $\mu(b) = 0.9$, $\mu(c) = 0.5$;
$\omega(a) = 0.7$, $\omega(b) = \overline{1}$, $\omega(c) = 0.5$. Then $\tau, \eta : I^X \to I$ defined as

$$\tau(\omega) = \begin{cases} 1, \text{if} \lambda = \overline{0} \text{or} \overline{1}, \\ \frac{1}{2}, \text{if} \omega = \omega, \\ 0, \text{otherwise}, \end{cases} \quad \eta(\lambda) = \begin{cases} 1, \text{if} \lambda = \overline{0} \text{or} \overline{1}, \\ \frac{1}{2}, \text{if} \lambda = \lambda, \mu, \\ 0, \text{otherwise}, \end{cases}$$

are fuzzy topologies on X and Y. Consider the identity mapping $f: (X,\tau) \to (Y,\eta)$. Take $r=\frac{1}{2}$. For any $\frac{1}{2}$-fuzzy open set ω in (Y,η), $f(\omega) = \omega$ is $\frac{1}{2}$-fMo set in (Y,η). Then f is fMo map, but f is neither fθso nor fδso map, since $f(\omega) = \omega$ is neither $\frac{1}{2}$-$f\theta so$ nor $\frac{1}{2}$-$f\delta so$ set in (Y,η).

Example 2.6 Let λ, μ and ω be fuzzy subsets of $X = Y = \{a,b,c\}$ defined as follows
$\lambda(a) = 0.3$, $\lambda(b) = 0.4$, $\lambda(c) = 0.5$; $\mu(a) = 0.6$, $\mu(b) = 0.5$, $\mu(c) = 0.5$;
$\omega(a) = 0.7$, $\omega(b) = 0.6$, $\omega(c) = 0.5$. Then $\tau, \eta : I^X \to I$ defined as

$$\tau(\omega) = \begin{cases} 1, \text{if} \lambda = \overline{0} \text{or} \overline{1}, \\ \frac{1}{2}, \text{if} \omega = \omega, \\ 0, \text{otherwise}, \end{cases} \quad \eta(\lambda) = \begin{cases} 1, \text{if} \lambda = \overline{0} \text{or} \overline{1}, \\ \frac{1}{2}, \text{if} \lambda = \lambda, \mu, \\ 0, \text{otherwise}, \end{cases}$$

are fuzzy topologies on X and Y. Consider the identity mapping $f: (X,\tau) \to (Y,\eta)$. Take $r=\frac{1}{2}$. For any $\frac{1}{2}$-fuzzy open set ω in (Y,η), $f(\omega) = \omega$ is $\frac{1}{2}$-fMo and $\frac{1}{2}$-fθso set in (Y,η). Then f is fMo and fθso map, but f is not fθo map, since $f(\omega) = \omega$ is not $\frac{1}{2}$-$f\theta o$ set in (Y,η).

Example 2.7 Let λ, μ and ω be fuzzy subsets of $X = Y = \{a,b,c\}$ defined as follows
$\lambda(a) = 0.3$, $\lambda(b) = 0.5$, $\lambda(c) = 0.5$; $\mu(a) = 0.5$, $\mu(b) = 0.5$, $\mu(c) = 0.5$;
$\omega(a) = 0.7$, $\omega(b) = 0.6$, $\omega(c) = 0.5$. Then $\tau, \eta : I^X \to I$ defined as

$$\tau(\lambda) = \begin{cases} 1, \text{if} \lambda = \overline{0} \text{or} \overline{1}, \\ \frac{1}{2}, \text{if} \lambda = \lambda, \mu, \\ 0, \text{otherwise}, \end{cases} \quad \eta(\mu) = \begin{cases} 1, \text{if} \mu = \overline{0} \text{or} \overline{1}, \\ \frac{1}{2}, \text{if} \mu = \mu, \\ 0, \text{otherwise}, \end{cases}$$

are fuzzy topologies on X and Y. Consider the identity mapping $f: (X,\tau) \to (Y,\eta)$. Take $r=\frac{1}{2}$. For any $\frac{1}{2}$-fuzzy open set λ in (X,τ), $f(\lambda) = \lambda$ is $\frac{1}{2}$-fuzzy open in (Y,η). Then f is fuzzy open map, but f is not fθo and fδo map, since $f(\lambda) = \lambda$ is not $\frac{1}{2}$-$f\theta o$ and $\frac{1}{2}$-fδo set in (Y,η).

Theorem 2.1 Let (X,τ_1) and (Y,τ_2) be sfts's and $f: X \to Y$ be a mapping. Then the following statements are equivalent:

1. f is a fuzzy M-open mapping.
2. $f(I_{\tau_1}(\lambda,r)) \le MI_{\tau_2}(f(\lambda),r)$ for each $\lambda \in I^X$ and $r \in I_0$.
3. $I_{\tau_1}(f^{-1}(\mu),r) \le f^{-1}(MI_{\tau_2}(\mu,r))$ for each $\mu \in I^Y$ and $r \in I_0$.

Proof.It is obviously

Theorem 2.2 Let (X,τ_1) and (Y,τ_2) be sfts's and $f: (X,\tau_1) \to (Y,\tau_2)$ be a fuzzy M-open (resp. fuzzy δ-semiopen, fuzzy δ-preopen) mapping. If $\mu \in I^Y$ and $\lambda \in I^X, \tau_1(1-\lambda) \ge r, r \in I_0$ such that $f^{-1}(\mu) \le \lambda$, then there exists an r-fMc (resp. r-$f\delta sc$, r-$f\delta pc$) set ν of Y such that $\mu \le \nu, f^{-1}(\nu) \le \lambda$.

Theorem 2.3 If $f: (X,\tau_1) \to (Y,\tau_2)$ be a fuzzy M-open mapping. Then for each $\mu \in I^Y, r \in I_0$,

$$f^{-1}(C_{\tau_2}(\theta I_{\tau_2}(\mu,r),r) \wedge f^{-1}(I_{\tau_2}(\delta C_{\tau_2}(\mu,r),r)) \le C_{\tau_1}(f^{-1}(\mu),r)$$

.

$$\ge f^{-1}(C_{\tau_2}(\delta I_{\tau_2}(\mu,r),r)) \wedge f^{-1}(I_{\tau_2}(\theta C_{\tau_2}(\mu,r),r)).$$

Hence

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$f^{-1}(C_{\tau_2}(\delta I_{\tau_2}(\mu,r),r)\wedge f^{-1}(I_{\tau_2}(\theta C_{\tau_2}(\mu,r),r)\leq C_{\tau_2}(f^{-1}(\mu),r)$.

Theorem 2.4 If $f:(X,\tau_1)\to(Y,\tau_2)$ be a bijective mapping such that

$$f^{-1}(C_{\tau_2}(\delta I_{\tau_2}(\mu,r),r))\wedge f^{-1}(I_{\tau_2}(\theta C_{\tau_2}(\mu,r),r))\leq C_{\tau_1}(f^{-1}(\mu),r).$$

for each $\mu\in I^Y, r\in I_0$, then f is fuzzy M-open map.

Proof. Let $\lambda\in I^X, r\in I_0$ with $\tau_1(\lambda)\geq r$. Then, from the given condition,

$f^{-1}(C_{\tau_2}(\delta I_{\tau_2}(f(\bar{1}-\lambda),r),r))\wedge f^{-1}(I_{\tau_2}(\delta C_{\tau_2}(f(\bar{1}-\lambda),r),r))\leq C_{\tau_1}(f^{-1}(f(1-\lambda)),r)=C_{\tau_1}(\bar{1}-\lambda,r=\bar{1}-\lambda$

and so

$C_{\tau_2}(\delta I_{\tau_2}(f(\bar{1}-\lambda),r),r)\wedge I_{\tau_2}(\delta C_{\tau_2}(f(\bar{1}-\lambda),r),r)\leq f(\bar{1}-\lambda)$, which shows that $f(\bar{1}-\lambda)$ is an r-fMc set of Y. Since f is bijective, then $f(\lambda)$ is an r-fMo set of Y, therefore f is fuzzy M-open map.

Theorem 2.5 Let (X,τ) and (Y,η) be sfts's. Let $f:X\to Y$ be a fMc mapping. Then the following statements hold.

1. If f is a surjective map and $f^{-1}(\alpha)\bar{q}f^{-1}(\beta)$ in X, then there exists $\alpha,\beta\in I^Y$ such that $\alpha\bar{q}\beta$.
2. $MI_\eta(MC_\eta(f(\lambda),r),r)\leq f(C_\tau(\lambda,r))$, for each $\lambda\in I^X$ and $r\in I_0$.

Proof. (i) Let $\gamma_1,\gamma_2\in I^X$ such that $f^{-1}(\alpha)\leq\gamma_1$ and $f^{-1}(\beta)\leq\gamma_2$ such that $\gamma_1\bar{q}\gamma_2$. Then there exists two r-fMo sets μ_1 and μ_2 such that $f^{-1}(\alpha)\leq\mu_1\leq\gamma_1$, $f^{-1}(\beta)\leq\mu_2\leq\gamma_2$. But f is a surjective map, then $ff^{-1}(\alpha)=\alpha\leq f(\mu_1)\leq f(\gamma_1)$ and $ff^{-1}(\beta)=\beta\leq f(\mu_2)\leq f(\gamma_2)$. Since $\gamma_1\bar{q}\gamma_2$, then also $f(\gamma_1\wedge\gamma_2)=\bar{0}$. Hence $\alpha\wedge\beta\leq f(\mu_1\wedge\mu_2)\leq f(\gamma_1\wedge\gamma_2)=\bar{0}$. Therefore, $\alpha\bar{q}\beta$ in Y. that is $\alpha\wedge\beta=\bar{0}$.

(ii) Since $\lambda\leq C_\tau(\lambda,r)\leq\bar{1}$ and f is an fuzzy M-closed mapping, then $f(C_\tau(\lambda,r))$ is fuzzy M-closed set in Y. Hence 8$f(\lambda)\leq MC_\tau(\lambda,r)\leq f(C_\tau(\lambda,r))$. So $MI_\eta(MC_\eta(f(\lambda),r),r)\leq f(C_\tau(\lambda,r))$.

Theorem 2.6 Let (X,τ) and (Y,η) be sfts's. Let $f:X\to Y$ be a mapping. Then the following statements are equivalent:

1. f is called fuzzy M-closed map.
2. $MC_\eta(f(\lambda),r)\leq f(C_\tau(\lambda,r))$, for each $\lambda\in I^X$ and $r\in I_0$.
3. If f is surjective, then for each subset μ of Y and each r-fuzzy open set α in X containing $f^{-1}(\mu)$, there exists an r-fMo set β of Y containing μ such that $f^{-1}(\beta)\leq\alpha$.

Definition 2.2 A sfts (X,τ) is called

1. r-fuzzy M-T_1 (resp. r-fuzzy T_1) if for every two distinct fuzzy points x,y of X, there exists two r-fuzzy M-open sets (resp. r-fuzzy open sets) λ,μ such that $x\in\lambda$, $y\notin\lambda$ and $y\in\mu, x\notin\mu$.
2. r-fuzzy M-T_2 (resp. r-fuzzy T_2) if for every two distinct fuzzy points x,y of X, there exists two disjoint r-fuzzy M-open sets (resp. r-fuzzy open sets) λ,μ such that $x\in\lambda, y\in\mu$.
3. r-fuzzy M-connected (resp. r-fuzzy connected) if it cannot be expressed as the union of two disjoint non-empty r-fuzzy M-open sets (resp. r-fuzzy open sets) of X. If X is not r-fuzzy M-connected (resp. not r-fuzzy connected), then it is r-fuzzy M-disconnected (resp. r-fuzzy disconnected).
4. r-fuzzy M-lindeloff (r-fuzzy lindeloff) if every r-fuzzy M-open cover (resp. r-fuzzy open cover) of X has a countable subcover.
5. r-fuzzy M-compact (resp. r-fuzzy M-compact) if for every r-fuzzy M-open cover (resp. r-fuzzy open cover) of X has a finite subcover.

Definition 2.3 [9] Let (X,τ) be a sfts and $r\in I_0$. A fuzzy set $\mu\in I^X$ is called r-fuzzy compact in (X,τ) iff for each family $\{\lambda_i\in I^X|\tau(\lambda_i)\geq r, i\in\Gamma\}$ such that $\mu\leq\bigvee_{i\in\Gamma}\lambda_i$ there exists a finite index set $\Gamma_0\subset\Gamma$ such that $\mu\leq\bigvee_{i\in\Gamma_0}\lambda_i.(X,\tau)$ is called r-fuzzy compact iff $\bar{1}$ is r-fuzzy compact in (X,τ).

Definition 2.4 Let (X,τ) be a sfts and $r\in I_0$. A fuzzy set $\mu\in I^X$ is called r-fuzzy M-compact in (X,τ) iff for each family $\{\lambda_i\in I^X|\lambda_i$ is r-fMo, $i\in\Gamma\}$ such that $\mu\leq\bigvee_{i\in\Gamma}\lambda_i$ there exists a finite index set $\Gamma_0\subset\Gamma$ such that $\mu\leq\bigvee_{i\in\Gamma_0}\lambda_i$. (X,τ) is called r-fuzzy M-compact iff $\bar{1}$ is r-fuzzy M-compact in (X,τ).

Theorem 2.7 Let (X,τ) and (Y,η) be sfts's. Let $f:X\to Y$ be a bijective fMo mapping. Then the following statements hold.

1. If X is a r-fuzzy T_i-space, then Y is r-fuzzy M-T_i where i=1,2.
2. If Y is an r-fuzzy M-compact (resp. r-fuzzy M-

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Lindeloff) space, then X is r-fuzzy compact(resp. r-fuzzy Lindeloff).

Theorem 2.8 Let (X,τ_1) and (Y,τ_2) be sfts's. If $f: X \to Y$ is a surjective fuzzy M-open mapping and Y is r-fuzzy M-connected space, then X is r-fuzzy connected.

Remark 2.2 Let (X,τ_1) and (Y,τ_2) be sfts's and $f: X \to Y$ be a mapping. The composition of two fuzzy M-open mappings need not be fuzzy M-open map as shown by the following example.

Example 2.8 Let $\lambda,\ \omega$ and μ be fuzzy subsets of $X = Y = Z = \{a,b,c\}$ defined as follows

$\lambda(a) = 0.4,\ \lambda(b) = 0.5,\ \lambda(c) = 0.2$; $\omega(a) = 0.7$, $\omega(b) = \overline{1}, \omega(c) = 0.5$.

$\mu(a) = 0.5,\quad \mu(b) = 0.3,\quad \mu(c) = 0.2$; Then $\tau_1, \tau_2\, and\, \tau_3 : I^X \to I$ defined as

$$\tau_1(\lambda) = \begin{cases} 1, \text{if}\lambda = \overline{0} \text{or} \overline{1}, \\ \frac{1}{2}, \text{if}\lambda = \lambda, \\ 0, \text{otherwise}, \end{cases} \quad \tau_2(\omega) = \begin{cases} 1, \text{if}\mu = \overline{0} \text{or} \overline{1}, \\ \frac{1}{2}, \text{if}\omega = \omega, \\ 0, \text{otherwise}, \end{cases}$$

$$\tau_2(\omega) = \begin{cases} 1, \text{if}\lambda = \overline{0} \text{or} \overline{1}, \\ \frac{1}{2}, \text{if}\omega = \omega, \\ 0, \text{otherwise}, \end{cases} \quad \tau_3(\mu) = \begin{cases} 1, \text{if}\mu = \overline{0} \text{or} \overline{1}, \\ \frac{1}{2}, \text{if}\mu = \mu, \\ 0, \text{otherwise}, \end{cases}$$

are fuzzy topologies on X, Y and Z. Consider the identity mapping $f: (X,\tau_1) \to (Y,\tau_2)$ and $g: (Y,\tau_2) \to (Z,\tau_3)$. Take $r = \frac{1}{2}$. For any $\frac{1}{2}$-fuzzy open set λ in (X,τ_1), $f(\lambda) = \lambda$ is $\frac{1}{2}$-fMo set in (Y,τ_2). Also, for any $\frac{1}{2}$-fuzzy open set ω in (Y,τ_2), $g(\omega) = \omega$ is $\frac{1}{2}$-fMo in (Z,τ_3). Thus f is fuzzy M-open map and g is fuzzy M-open map. But $g \circ f$ is not fuzzy M-open map, as λ is $\frac{1}{2}$-fuzzy open set in (X,τ_1), $(g \circ f)(\lambda) = g(f(\lambda)) = \lambda$ is not $\frac{1}{2}$-fMo in (Z,τ_3).

Theorem 2.9 Let (X,τ_1), (Y,τ_2) and (Z,τ_3) be sfts's. If $f: (X,\tau_1) \to (Y,\tau_2)$ and $g: (Y,\tau_2) \to (Z,\tau_3)$ are mappings, then

1. If f is fuzzy open map and g is fuzzy M-open map, then $g \circ f$ is fuzzy M-open mapping.
2. If $g \circ f$ is fuzzy M-open mapping and f is a surjective continuous map, then g is fuzzy M-open map.
3. If $g \circ f$ is fuzzy open mapping and g is an injective M-continuous map, then f is fuzzy M-open map.

Proof. (i) Let $\mu \in \tau_1$. Since f is fuzzy open map, then $f(\mu)$ is an r-fuzzy open set in (Y,τ_2). Since g is fuzzy M-open map, then $g(f(\mu)) = (g \circ f)(\mu)$ is r-fMo set in (Z,τ_3). Hence $g \circ f$ is fuzzy M-open map.

(ii) Let $\mu \in \tau_2$. Since f is fuzzy continuous, then $f^{-1}(\mu)$ is an r-fuzzy open set in (X,τ_1). But $g \circ f$ is r-fMo map, then $(g \circ f)(f^{-1}(\mu))$ is r-fMo set in (Z,τ_3). Hence by surjective of f, we have $g(\mu)$ is r-fMo set of (Z,τ_3). Hence, g is fuzzy M-open map.

(iii) Let $\mu \in \tau_1$ and $g \circ f$ be an fuzzy open map. Then $(g \circ f)(\mu) = g(f(\mu)) \in \tau_3$. Since g is an injective fuzzy M-continuous map, hence $f(\mu)$ is fuzzy M-open map in (Y,τ_2). Therefore f is fuzzy M-open.

IV. Conclusion:

In this paper, we introduce and investigate some new classes of mappings called fuzzy M-open map and fuzzy M-closed map to the fuzzy topological spaces in $\hat{S}$ostak's sense. Also, some of their fundamental properties are studied. Moreover, we investigate the relationships between fuzzy open, fuzzy θ-semiopen, fuzzy θ-open, fuzzy δ-semiopen, fuzzy δ-preopen, fuzzy a-open, fuzzy M-open, fuzzy e-open and fuzzy e^*-open mappings.

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