Fuzzy $M$-open and Fuzzy $M$-closed Mappings in Šostak’s Fuzzy Topological Spaces

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Abstract: We introduce and investigate some new classes of mappings called fuzzy $M$-open map and fuzzy $M$-closed map to the fuzzy topological spaces in Šostak’s sense. Also, some of their fundamental properties are studied. Moreover, we investigate the relationships between fuzzy open, fuzzy $\Theta$-semiopen, fuzzy $\Theta$-open, fuzzy $\delta$-semiopen, fuzzy $\delta$-preopen, fuzzy $\alpha$-open, fuzzy $M$-open, fuzzy $e$-open and fuzzy $e^*$-open mappings.

Keywords and phrases: fuzzy open, fuzzy $\Theta$-semiopen, fuzzy $\Theta$-open, fuzzy $\delta$-semiopen, fuzzy $\delta$-preopen, fuzzy $\alpha$-open, fuzzy $M$-open, fuzzy $e$-open and fuzzy $e^*$-open mappings.

1. Introduction

Šostak [23] introduced the fuzzy topology as an extension of Chang’s fuzzy topology [1]. It has been developed in many directions [6, 7, 22]. Ganguly and Saha [5] introduced the notions of fuzzy $\delta$-cluster points in fuzzy topological spaces in the sense of Chang [1]. Kim and Park [8] introduced $r$-$\delta$-cluster points and $\delta$-closure operators in fuzzy topological spaces in view of the definition of Šostak.

In 2008, the initiations of $e$-open sets, $e^*$-open sets and $\alpha$-open sets in topological spaces are due to Erdal Ekici [3], [4]. Sobana et.al [25] defined $T$-fuzzy $e$-open sets, fuzzy $e$-continuity, fuzzy $e$-open map and fuzzy $e^*$-closed map in a smooth topological space.

Throughout this paper, nonempty sets will be denoted by $X, Y$, etc., $I = [0, 1]$ and $I_0 = (0, 1]$. For $\alpha \in I, \alpha(x) = \alpha$ for all $x \in X$. A fuzzy point $x_\alpha$ for $t \in I_0$ is an element of $I^X$ such that

$$x_\alpha(y) = \begin{cases} t & \text{if } y = x \\ 0 & \text{if } y \notin x. \end{cases}$$

The set of all fuzzy points in $X$ is denoted by $P_\alpha(X)$. A fuzzy point $x_\alpha \in \lambda$ iff $t < \lambda(x)$. A fuzzy set $\lambda$ is quasi-coincident with $\mu$, denoted by $\lambda \mu q$, if there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$. If $\lambda$ is not quasi-coincident with $\mu$, we denoted $\lambda \mu$. If $A \subseteq X$, we define the characteristic function $\chi_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases}$ All other notations and definitions are standard, for all in the fuzzy set theory.

II. Preliminaries

Lemma 1.1 [23] Let $X$ be a nonempty set and $\lambda, \mu \in I^X$. Then
1. $\lambda q \mu$ iff there exists $x, \alpha \in \lambda$ such that $x \alpha q \mu$.
2. $\lambda q \mu$, then $\lambda \alpha \mu \neq 0$.
3. $\lambda q \mu$ iff $\lambda \leq 1 - \mu$.
4. $\lambda \leq \mu$ iff $x \alpha \in \lambda$ implies $x \alpha \in \mu$ iff $x \alpha q \lambda \mu$ implies $x \alpha q \lambda \mu$.
5. $\lambda \mu$ iff there exists $i_0 \in \Lambda$ such that $x \alpha q \mu i_0$.

Definition 1.1 [23] A function $\tau: I^X \rightarrow I$ is called a fuzzy topology on $X$ if it satisfies the following conditions:
1. $\tau(0) = \tau(1) = 1$.
2. $\tau(\bigvee_{i \in I} \mu_i) \geq \bigvee_{i \in I} \tau(\mu_i)$, for any $\{\mu_i\}_{i \in I} \subseteq I^X$.
3. $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$, for any $\mu_1, \mu_2 \in I^X$.

The pair $(X, \tau)$ is called a fuzzy topological space (for short, sfts).

Remark 1.1 [20] Let $(X, \tau)$ be a fuzzy topological space. Then, for each $r \in I_0$, $\tau_r = \{\mu \in I^X : \tau(\mu) \geq r\}$ is a Chang’s fuzzy topology on $X$.

Theorem 1.1 [22] Let $(X, \tau)$ be a sfts. Then for each $\lambda \in I^X$, $r \in I_0$ we define an operator $C_\tau: I^X \times I_0 \rightarrow I^X$ as follows:

$$C_\tau(\lambda, r) = \lambda \mu \leq \mu, \tau(1 - \mu) \geq r.$$ 

For $\lambda, \mu \in I^X$ and $r \in I_0$, the operator $C_\tau$ satisfies the following conditions:
1. $C_\tau(0, r) = 0$.
2. $\lambda \leq C_\tau(\lambda, r)$.
C_\tau(\lambda, r) \lor C_\tau(\mu, r) = C_\tau(\lambda \lor \mu, r).
4. C_\tau(\lambda, r) \leq C_\tau(\lambda, s) \text{ if } r \leq s,
5. C_\tau C_\tau(\lambda, r), r) = C_\tau(\lambda, r).

Theorem 1.2 [22] Let \( (X, \tau) \) be a sfts. Then for each \( r \in I_0, \lambda \in I^X \) we define an operator \( I_\tau: I^X \times I_0 \to I^X \) as
\[ I_\tau(\lambda, r) = \{ \mu \in I^X : \lambda \geq \mu, \tau(\mu) \geq r \}. \]

For \( \lambda, \mu \in I^X \) and \( r, s \in I_0 \), the operator \( I_\tau \) satisfies the following conditions:
1. \( I_\tau(1, r) = 1 \)
2. \( \lambda \geq I_\tau(\lambda, r) \)
3. \( I_\tau(\lambda, r) \land I_\tau(\mu, r) = I_\tau(\lambda \land \mu, r) \)
4. \( I_\tau(\lambda, s) \leq I_\tau(\lambda, r) \) if \( s \leq r \)
5. \( I_\tau(\lambda, r), r) = I_\tau(\lambda, r) \)
6. \( I_\tau(1 - \lambda, r) = 1 - I_\tau(\lambda, r) \)

Definition 1.2 [10] Let \( (X, \tau) \) be a sfts. Then for each \( \mu \in I^X, x_\mu \in P_\tau(X) \) and \( r \in I_0 \):
1. \( \mu \) is called \( r \)-open \( \tau \)-neighbourhood of \( x_\tau \) if \( x_\tau \mu \) with \( \tau(\mu) \geq r \).
2. \( \mu \) is called \( r \)-closed \( \tau \)-neighbourhood of \( x_\tau \) if \( x_\tau \mu \) with \( \mu = I_\tau(\tau(\mu), r) \).
We denote \( Q_\tau(x_\tau, r) = \{ \mu \in I^X : x_\tau \mu, \mu \geq r \} \), \( R_\tau(x_\tau, r) = \{ \mu \in I^X : x_\tau \mu = I_\tau(\tau(\mu), r) \} \).

Definition 1.3 [10] Let \( (X, \tau) \) be a sfts. Then for each \( \lambda \in I^X, x_\tau \in P_\tau(X) \) and \( r \in I_0 \):
1. \( x_\tau \) is called \( r \)-open \( \tau \)-cluster point of \( \lambda \) if for every \( \mu \in Q_\tau(x_\tau, r) \), we have \( \mu \cap \lambda \).
2. \( x_\tau \) is called \( r \)-closed \( \tau \)-cluster point of \( \lambda \) if for every \( \mu \in R_\tau(x_\tau, r) \), we have \( \mu \cap \lambda \).
3. \( \delta \)-cluster operator is a mapping \( D_\tau: I^X \times I_0 \to I^X \) defined as follows: \( \delta C_\tau(\lambda, r) \) and \( D_\tau(\lambda, r) = \{ x_\tau \in P_\tau(X) : x_\tau \text{ is } \tau \text{-}C_\tau \text{-cluster point of } \lambda \} \)

Definition 1.4 Let \( (X, \tau) \) be a sfts. For \( \lambda, \mu \in I^X \) and \( r \in I_0 \), \( \lambda \) is called an
1. \( r \)-fuzzy \( \delta \)-semiopen (resp. \( r \)-fuzzy \( \delta \)-semiclosed) [25] set if \( \lambda \leq C_\tau(\delta I_\tau(\lambda, r), r) \) (resp. \( \lambda \leq C_\tau(\delta I_\tau(\lambda, r), r) \)).
2. \( r \)-fuzzy \( \tau \)-preopen (resp. \( r \)-fuzzy \( \tau \)-preclosed) [25] set if \( \lambda \leq C_\tau(\delta I_\tau(\lambda, r), r) \) (resp. \( \lambda \leq C_\tau(\delta I_\tau(\lambda, r), r) \)).
3. \( r \)-fuzzy \( \mu \)-open (resp. \( r \)-fuzzy \( \mu \)-closed) [25] set if \( \lambda \leq I_\tau(\delta C_\tau(\lambda, r), r) \) (resp. \( \lambda \leq I_\tau(\delta C_\tau(\lambda, r), r) \)).
4. \( r \)-fuzzy \( \mu \)-open (resp. \( r \)-fuzzy \( \mu \)-closed) [25] set if \( \lambda \leq I_\tau(\delta I_\tau(\lambda, r), r) \) (resp. \( \lambda \leq I_\tau(\delta I_\tau(\lambda, r), r) \)).
2. \( r \)-fuzzy \( \theta \)-semiopen (resp. \( r \)-fuzzy \( \theta \)-semiclosed) set if
\[
\lambda \leq C_{\tau l}(\lambda, r) \leq \lambda.
\]
3. \( r \)-fuzzy \( \theta \)-preopen (resp. \( r \)-fuzzy \( \theta \)-preclosed) set if
\[
\lambda \leq I_{\tau l}(\lambda, r) \leq \lambda.
\]
Definition 1.9 [27] Let \((X, \tau)\) be a fuzzy topological space. For \(\lambda \in I^X\) and \(r \in I_0\), \(\lambda\) is called an \(r\)-fuzzy \(M\)-open set if
\[
\lambda \leq C_{\tau l}(\lambda, r) \Rightarrow I_{\tau l}(\delta C_{\tau l}(\lambda, r), r).
\]

\(M\)-closed set if
\[
\lambda \geq C_{\tau l}(\delta I_{\tau l}(\lambda, r), r) \land I_{\tau l}(\theta C_{\tau l}(\lambda, r), r).
\]

Definition 1.10 [27] Let \((X, \tau)\) be a fuzzy topological space. For \(\lambda \in I^X\) and \(r \in I_0\),
1. \(M\)-open set if
\[
\lambda \leq C_{\tau l}(\lambda, r) \Rightarrow I_{\tau l}(\delta C_{\tau l}(\lambda, r), r).
\]
2. \(M\)-closed set if
\[
\lambda \geq C_{\tau l}(\delta I_{\tau l}(\lambda, r), r) \land I_{\tau l}(\theta C_{\tau l}(\lambda, r), r).
\]

Theorem 1.4 [27] Let \((X, \tau)\) be a sfts. Let \(f: (X, \tau_1) \rightarrow (Y, \tau_2)\) be a mapping. Then \(f\) is called
1. fuzzy \(M\)-continuous iff \(f^{-1}(\mu)\) is \(r \)-\(M\)-open for each \(\mu \in I^X\) and \(r \in I_0\).
2. \(\theta\)-continuous iff \(f^{-1}(\mu)\) is \(r \)-\(\theta\)-open for each \(\mu \in I^X\) and \(r \in I_0\).
3. \(\theta\)-semicontinuous iff \(f^{-1}(\mu)\) is \(r \)-\(\theta\)-closed for each \(\mu \in I^X\) and \(r \in I_0\).

III. RESULTS

Definition 2.1 Let \((X, \tau_1)\) and \((Y, \tau_2)\) be sfts’s and \(f: (X, \tau_1) \rightarrow (Y, \tau_2)\) be a mapping. Then \(f\) is called
1. fuzzy \(M\)-open mapping iff \(f(\lambda)\) is \(r \)-\(M\)-open set of \(Y\) for each \(\lambda \in I^X\) and \(r \in I_0\) with \(\tau_1(1 - \lambda) \geq r\).
2. fuzzy \(M\)-closed mapping iff \(f(\lambda)\) is \(r \)-\(M\)-closed set of \(Y\) for each \(\lambda \in I^X\) and \(r \in I_0\) with \(\tau_1(1 - \lambda) \geq r\).

Remark 2.1 From the above definitions, it is clear that the following implications are true for
where $fo, f^\delta o, f^\theta o, fso, fp o, fMo, f^\delta o$ and $f^e o$ maps are abbreviated by fuzzy open, fuzzy $\theta$-semiopen, fuzzy $\theta$-open, fuzzy $\delta$-semiopen, fuzzy $\delta$-preopen, fuzzy $a$-open, fuzzy $M$-open, fuzzy $e$-open and fuzzy $e^*$-open maps respectively.

From the above definitions, it is clear that every fuzzy $\delta$-preopen map is fuzzy $M$-open map and every fuzzy $\theta$-semiopen map is fuzzy $M$-open map. Also, it is clear that every fuzzy $M$-open map is fuzzy $e^*$-open map and fuzzy $e^*$-open map. Also, every fuzzy $\theta$-open map, fuzzy $\delta$-open map, fuzzy $a$-open map is fuzzy $M$-open map. The converses need not be true in general.

The converses of the above implications are not true as the following examples show:

Example 2.1 Let $\lambda$ and $\mu$ be fuzzy subsets of $X = Y = \{a, b, c\}$ defined as follows $\lambda(a) = 0.5, \lambda(b) = 0.4, \lambda(c) = 0.7$, $\mu(a) = 0.4, \mu(b) = 0.5$, $\mu(c) = 0.2$. Then $\tau, \eta: I^X \to I$ defined as

$$
\tau(\lambda) = \begin{cases} 
1, & \text{if } \lambda = \bar{0}o \bar{1}, \\
\frac{1}{2}, & \text{if } \lambda = \lambda, \\
0, & \text{otherwise,}
\end{cases}
\eta(\mu) = \begin{cases} 
1, & \text{if } \mu = \bar{0}o \bar{1}, \\
\frac{1}{2}, & \text{if } \mu = \mu, \\
0, & \text{otherwise,}
\end{cases}
$$

are fuzzy topologies on $X$ and $Y$. Consider the identity mapping $f: (X, \tau) \to (Y, \eta)$. Take $r = 2$. For any $2$-fuzzy open set $\lambda$ in $(X, \tau)$, $f(\lambda) = \lambda$ is $\frac{1}{2}fMo$ set in $(Y, \eta)$. Then $f$ is $fMo$-map, but $f$ is not $fMo$-map, since $f(\lambda) = \lambda$ is not $2fMo$ in $(Y, \eta)$.

Example 2.2 Let $\lambda$ and $\mu$ be fuzzy subsets of $X = Y = \{a, b, c\}$ defined as follows $\lambda(a) = 0.5, \lambda(b) = 0.4, \lambda(c) = 0.4$, $\mu(a) = 0.5, \mu(b) = 0.3$, $\mu(c) = 0.2$. Then $\tau, \eta: I^X \to I$ defined as

$$
\tau(\lambda) = \begin{cases} 
1, & \text{if } \lambda = \bar{0}o \bar{1}, \\
\frac{1}{2}, & \text{if } \lambda = \lambda, \\
0, & \text{otherwise,}
\end{cases}
\eta(\mu) = \begin{cases} 
1, & \text{if } \mu = \bar{0}o \bar{1}, \\
\frac{1}{2}, & \text{if } \mu = \mu, \\
0, & \text{otherwise,}
\end{cases}
$$

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Example 2.3 Let $\lambda$ and $\mu$ be fuzzy subsets of $X = Y = \{a, b, c\}$ defined as follows $\lambda(a) = 0.9, \lambda(b) = 0.9, \lambda(c) = 0.9$, $\mu(a) = 0.1, \mu(b) = 0.1$, $\mu(c) = 0.1$. Then $\tau, \eta: I^X \to I$ defined as

$$
\tau(\lambda) = \begin{cases} 
1, & \text{if } \lambda = \bar{0}o \bar{1}, \\
\frac{1}{2}, & \text{if } \lambda = \lambda, \\
0, & \text{otherwise,}
\end{cases}
\eta(\mu) = \begin{cases} 
1, & \text{if } \mu = \bar{0}o \bar{1}, \\
\frac{1}{2}, & \text{if } \mu = \mu, \\
0, & \text{otherwise,}
\end{cases}
$$

are fuzzy topologies on $X$ and $Y$. Consider the identity mapping $f: (X, \tau) \to (Y, \eta)$. Take $r = 2$. For any $2$-fuzzy open set $\lambda$ in $(X, \tau)$, $f(\lambda) = \lambda$ is $\frac{1}{2}fMo$ set in $(Y, \eta)$. Then $f$ is $fMo$-map, but $f$ is not $fMo$-map, since $f(\lambda) = \lambda$ is not $2fMo$ in $(Y, \eta)$.

Example 2.4 Let $\lambda$ and $\mu$ be fuzzy subsets of $X = Y = \{a, b, c\}$ defined as follows $\lambda(a) = 0.9$,
\[ \lambda(b) = 0.9, \lambda(c) = 0.9, \mu(a) = 0.1, \mu(b) = 0.1, \mu(c) = 0.1. \]

Then \( \tau, \eta : I^X \to I \) defined as

\[
\tau(\lambda) = \begin{cases} 
1, & \text{if } \lambda = 0 \text{ or } 1, \\
\frac{1}{2}, & \text{if } \lambda = \lambda, \\
0, & \text{otherwise},
\end{cases}
\eta(\mu) = \begin{cases} 
1, & \text{if } \mu = 0 \text{ or } 1, \\
\frac{1}{2}, & \text{if } \mu = \lambda, \\
0, & \text{otherwise},
\end{cases}
\]

are fuzzy topologies on \( X \) and \( Y \). Consider the identity mapping \( f : (X, \tau) \to (Y, \eta) \). Take \( r = 2 \). For any 2-fuzzy open set \( \lambda \) in \((X, \tau), f(\lambda) = \frac{1}{2}f\Delta_o \) set in \((Y, \eta)\). Then \( f \) is \( f\Delta_o \)-map, but \( f \) is not \( f\Delta_o \)-map, since \( f(\lambda) = \lambda \) is \( f\Delta_o \)-set in \((Y, \eta)\).

**Example 2.5** Let \( \lambda, \mu \) and \( \omega \) be fuzzy subsets of \( X = Y = \{a, b, c\} \) defined as follows

\[
\lambda(a) = 0.3, \lambda(b) = 0.4, \lambda(c) = 0.5, \mu(a) = 0.6, \mu(b) = 0.5, \mu(c) = 0.5, \\
\omega(a) = 0.7, \omega(b) = 0.6, \omega(c) = 0.5.
\]

Then \( \tau, \eta : I^X \to I \) defined as

\[
\tau(\omega) = \begin{cases} 
1, & \text{if } \omega = 0 \text{ or } 1, \\
\frac{1}{2}, & \text{if } \omega = \lambda, \\
0, & \text{otherwise},
\end{cases}
\eta(\lambda) = \begin{cases} 
1, & \text{if } \lambda = 0 \text{ or } 1, \\
\frac{1}{2}, & \text{if } \lambda = \lambda, \mu, \\
0, & \text{otherwise},
\end{cases}
\]

are fuzzy topologies on \( X \) and \( Y \). Consider the identity mapping \( f : (X, \tau) \to (Y, \eta) \). Take \( r = 2 \). For any 2-fuzzy open set \( \lambda \) in \((X, \tau), f(\lambda) = \lambda \) is 2-fuzzy open in \((Y, \eta)\). Then \( f \) is fuzzy open map, but \( f \) is not \( f\Delta_0 \)-map, since \( f(\lambda) = \lambda \) is not \( f\Delta_0 \)-set in \((Y, \eta)\).

**Theorem 2.1** Let \((X, \tau_1)\) and \((Y, \tau_2)\) be sft's and \( f : X \to Y \) be a mapping. Then the following statements are equivalent:

1. \( f \) is a fuzzy \( M \)-open mapping.
2. \( f(\lambda, \tau_1) \leq M_{\tau_2}(f(\lambda), \tau_2) \) for each \( \lambda \in I^X \) and \( r \in I_0 \).
3. \( I_{\tau_2}(f^{-1}(\mu), \tau_2) \leq f^{-1}(M_{\tau_2}(\mu, \tau_2)) \) for each \( \mu \in I^Y \) and \( r \in I_0 \).

**Proof.** It is obviously

**Theorem 2.2** Let \((X, \tau_1)\) and \((Y, \tau_2)\) be sft's and \( f : (X, \tau_1) \to (Y, \tau_2) \) be a fuzzy \( M \)-open (resp. fuzzy \( \delta \)-semiopen, fuzzy \( \delta \)-preopen) mapping. If \( \mu \in I^X \) and \( \lambda \in I^X, \tau_1(1 - \lambda) \geq r, r \in I_0 \), then there exists an \( r, f\Delta_0 \) set \( f\Delta_0 \) set \( \nu \) or \( Y \) such that \( \mu \leq \nu, f^{-1}(\nu) \leq \lambda \).

**Theorem 2.3** If \( f : (X, \tau_1) \to (Y, \tau_2) \) be a fuzzy \( M \)-open mapping. Then for each \( \mu \in I^X, r \in I_0, f^{-1}(C_{\tau_2}(\theta \tau_2(\mu, \tau_2), r) \wedge f^{-1}(I_{\tau_2}(\theta C_{\tau_2}(\mu, \tau_2), r)) \leq C_{\tau_1}(f^{-1}(\mu), r)) \).

Hence
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Fuzzy open set $\alpha$ in $X$ containing $f^{-1}(\mu)$, there exists an $r_iM_0$ set $\beta$ of $Y$ containing $\mu$ such that $f^{-1}(\beta) \leq \alpha$.

Definition 2.2 A sfts $(X, \tau)$ is called $r_{\text{fuzzy}} M T_1$ (resp. $r_{\text{fuzzy}} T_1$) if for every two distinct fuzzy points $x, y$ of $X$, there exists two $r_{\text{fuzzy}} M$ open sets (resp. $r_{\text{fuzzy}}$ open sets) $\lambda, \mu$ such that $x \in \lambda, y \in \mu \in \lambda \setminus y$. If $X$ is not $r_{\text{fuzzy}}$-connected (resp. not $r_{\text{fuzzy}}$ connected), then it is $r_{\text{fuzzy}}$- disconnected (resp. $r_{\text{fuzzy}}$ disconnected).

Theorem 2.4 If $f: (X, \tau_1) \to (Y, \tau_2)$ be a bijective mapping such that

$$f^{-1}(C_{\tau_2}(\delta I_{\tau_2}(\mu, r), r)) \land f^{-1}(I_{\tau_1}(\delta C_{\tau_1}(\mu, r), r)) \leq C_{\tau_1}(f^{-1}(\mu), r),$$

for each $\mu \in I^Y, r \in I_0$, then $f$ is fuzzy $M$-open map.

Proof. Let $\lambda \in I^X, r \in I_0$ with $\tau_1(\lambda) \geq r$. Then from the given condition,

$$f^{-1}(C_{\tau_2}(f^{-1}(I_{\tau_1}(\delta C_{\tau_1}(f^{-1}(\lambda), r), r))) \leq C_{\tau_1}(f^{-1}(f([1-\lambda], r)), r),$$

so there exists a finite index set $\{\tilde{I}_1, \tilde{I}_2, \ldots, \tilde{I}_n\}$ such that $f^{-1}(1 - \lambda) \in \tilde{I}_M$ set of $Y$. Since $f$ is bijective, then $f(\lambda)$ is an $r_iM_0$ set of $Y$, therefore $f$ is fuzzy $M$-open map.

Theorem 2.5 Let $(X, \tau)$ and $(Y, \eta)$ be sfts’s. Let $f: X \to Y$ be a $f M_0$ mapping. Then the following statements hold.

1. If $f$ is a surjective map and $f^{-1}(\alpha) \subseteq f^{-1}(f(\lambda))$ in $X$, then there exists $\alpha, \beta \in I^Y$ such that $\alpha \subseteq \beta$.

2. $M_{\eta}(MC_{\eta}(f(\lambda), r), r) \leq f(C_{\tau}(\lambda, r))$ for each $\lambda \in I^X$ and $r \in I_0$.

Proof. (i) Let $Y_1, Y_2 \subseteq I^X$ such that $f^{-1}(\alpha) \subseteq Y_1$ and $f^{-1}(\beta) \subseteq Y_2$ such that $Y_1 \cap Y_2$. Then there exist two $r_{\text{fuzzy}} M_0$ sets $\mu_1$ and $\mu_2$ such that $f^{-1}(\alpha) \leq \mu_1 \leq \mu_2 \leq f^{-1}(\beta)$. But $f$ is a surjective map, then $f^{-1}(\alpha) = \alpha \leq f^{-1}(\mu_1) = f^{-1}(Y_1)$ and $f^{-1}(\beta) = \beta \leq f^{-1}(\mu_2) = f^{-1}(Y_2)$. Since $Y_1 \cap Y_2$, then also $f(Y_1 \cap Y_2) = \beta$. Hence $\alpha \leq \beta$ in $Y_2$. Therefore, $\alpha \subseteq \beta$.

(ii) Since $\lambda \subseteq C_{\tau}(\lambda, r) \subseteq 1$ and $f$ is an fuzzy $M$-closed mapping, then $f(C_{\tau}(\lambda, r))$ is fuzzy $M$-closed set in $Y$. Hence $f(C_{\tau}(\lambda, r)) \subseteq MC_{\tau}(f(\lambda), r), r) \leq f(C_{\tau}(\lambda, r)) < M_{\eta}(MC_{\eta}(f(\lambda), r), r) \leq f(C_{\tau}(\lambda, r))$.

Theorem 2.6 Let $(X, \tau)$ and $(Y, \eta)$ be sfts’s. Let $f: X \to Y$ be a mapping. Then the following statements are equivalent:

1. $f$ is called fuzzy $M$-closed map.

2. $MC_{\eta}(f(\lambda), r) \leq f(C_{\tau}(\lambda, r))$, for each $\lambda \in I^X$ and $r \in I_0$.

3. If $f$ is surjective, then for each subset $\mu$ of $Y$ and each $r$.
Lindeloff space, then $X$ is $r^*$-fuzzy compact (resp. $r^*$-fuzzy Lindeloff).

Theorem 2.8 Let $(X, \tau_1)$ and $(Y, \tau_2)$ be sfts. If $f: X \to Y$ is a surjective fuzzy $\text{M}_0$-open mapping and $Y$ is $r^*$-fuzzy $\text{M}_0$-connected space, then $X$ is $r^*$-fuzzy connected.

Remark 2.2 Let $(X, \tau_1)$ and $(Y, \tau_2)$ be sfts’s and $f: X \to Y$ be a mapping. The composition of two fuzzy $\text{M}_0$-open mappings need not be fuzzy $\text{M}$-open map as shown by the following example.

Example 2.8 Let $\lambda$, $\omega$ and $\mu$ be fuzzy subsets of $X = Y = Z = \{a, b, c\}$ defined as follows

$\lambda(a) = 0.4$, $\lambda(b) = 0.5$, $\lambda(c) = 0.2$, $\omega(a) = 0.7$, $\omega(b) = 0.1$, $\omega(c) = 0.5$,

$\mu(a) = 0.5$, $\mu(b) = 0.3$, $\mu(c) = 0.2$.

Then $\tau_1$, $\tau_2$ and $\tau_3: X \to I$ defined as

$$
\tau_1(\lambda) = \begin{cases} 
1, \text{if } \lambda = \bar{0} \lor \bar{1}, \\
\frac{1}{2}, \text{if } 0 < \lambda < 1, \\
0, \text{otherwise,}
\end{cases} \quad \tau_1(\omega) = \begin{cases} 
1, \text{if } \omega = \bar{0} \lor \bar{1}, \\
\frac{1}{2}, \text{if } 0 < \omega < 1, \\
0, \text{otherwise,}
\end{cases} \\
\tau_2(\omega) = \begin{cases} 
1, \text{if } \omega = \bar{0} \lor \bar{1}, \\
\frac{1}{2}, \text{if } 0 < \omega < 1, \\
0, \text{otherwise,}
\end{cases} \quad \tau_3(\mu) = \begin{cases} 
1, \text{if } \mu = \bar{0} \lor \bar{1}, \\
\frac{1}{2}, \text{if } 0 < \mu < 1, \\
0, \text{otherwise,}
\end{cases}
$$

are fuzzy topologies on $X$, $Y$ and $Z$. Consider the identity mapping $f: (X, \tau_1) \to (Y, \tau_2)$ and

$g: (Y, \tau_2) \to (Z, \tau_3)$. Take $\tau = \tau_2$. For any 2-fuzzy open set $\lambda$ in $(X, \tau_1)$, $f(\lambda) = \lambda$ is not 2-$\text{M}_0$-open in $(Y, \tau_2)$. Also, for any 2-fuzzy open set $\omega$ in $(Y, \tau_2)$, $g(\omega) = \omega$ is not 2-$\text{M}_0$-open in $(Z, \tau_3)$. Thus $f$ is fuzzy $\text{M}_0$-open map and $g$ is fuzzy $\text{M}_0$-open map. But $g \circ f$ is not fuzzy $\text{M}_0$-open map, as $\lambda$ is 2-fuzzy open set in $(X, \tau_1)$, $(g \circ f)(\lambda) = g(f(\lambda)) = \lambda$ is not 2-$\text{M}_0$-open in $(Z, \tau_3)$.

Theorem 2.9 Let $(X, \tau_1), (Y, \tau_2)$ and $(Z, \tau_3)$ be sfts mappings, then

1. If $f$ is fuzzy open map and $g$ is fuzzy $\text{M}_0$-open map, then $g \circ f$ is fuzzy $\text{M}_0$-open mapping.
2. If $g \circ f$ is fuzzy $\text{M}_0$-open mapping and $f$ is a surjective continuous map, then $g$ is fuzzy $\text{M}_0$-open map.
3. If $g \circ f$ is fuzzy open mapping and $g$ is an injective $\text{M}_0$-continuous map, then $f$ is fuzzy $\text{M}_0$-open map.

Proof. (i) Let $\mu \in \tau_1$. Since $f$ is fuzzy open map, then $f(\mu)$ is an $r^*$-fuzzy open set in $(Y, \tau_2)$. Since $g$ is fuzzy $\text{M}_0$-open map, then $g(f(\mu)) = (g \circ f)(\mu)$ is $r^*$-$\text{M}_0$-set in $(Z, \tau_3)$. Hence $g \circ f$ is fuzzy $\text{M}_0$-open.

(ii) Let $\mu \in \tau_2$. Since $f$ is fuzzy continuous, then $f^{-1}(\mu)$ is an $r^*$-fuzzy open set in $(X, \tau_1)$. But $g \circ f$ is $r^*$-$\text{M}_0$ map, then $(g \circ f)(f^{-1}(\mu))$ is $r^*$-$\text{M}_0$ set in $(Z, \tau_3)$. Hence by surjective of $f$, we have $g(\mu)$ is $r^*$-$\text{M}_0$ set of $(Z, \tau_3)$. Hence, $g$ is fuzzy $\text{M}_0$-open.

(iii) Let $\mu \in \tau_1$ and $g \circ f$ be an fuzzy open map. Then $g(f(\mu)) = (g \circ f)(\mu) \in \tau_3$. Since $g$ is an injective fuzzy $\text{M}_0$-continuous map, hence $f(\mu)$ is fuzzy $\text{M}_0$-open map in $(Y, \tau_2)$. Therefore $f$ is fuzzy $\text{M}_0$-open.

IV. CONCLUSION:

In this paper, we introduce and investigate some new classes of mappings called fuzzy $\text{M}_0$-open map and fuzzy $\text{M}_0$-closed map to the fuzzy topological spaces in Ostak’s sense. Also, some of their fundamental properties are studied. Moreover, we investigate the relationships between fuzzy open, fuzzy $\theta$-semiopen, fuzzy $\theta$-open, fuzzy $\delta$-semiopen, fuzzy $\delta$- preopen, fuzzy $\alpha$-open, fuzzy $\text{M}_0$-open, fuzzy $\theta$-open and fuzzy $\vartheta^*$-open mappings.

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