On Square Binormaland square n-binormal Operators

K Meenambika, C V Seshaiah, N Sivamani

Abstract: ***In this paper, a new class of operator called square binormal and square n-binormal are introduced.The objective of this paper is to study conditions on B which imply square binormal[sqBN] and square n-binormal[sqnBN].A result is proved stating that every binormal operator is square binormal and an example is given stating that the converse is not true.It is also proved that square n-binormal operator and n-isometry operators are independent classes.***

Index Terms: ***Hilbertspace, Normaloperator, Binormaloperator, Isometry operator.***

I. INTRODUCTION

Throughoutthis paper H is a Hilbert space and L(H) is defined as the algebra of all bounded linear operators acting on a Hilbert space H. An operator $B \in L(H)$ is called normalif $B^*B = BB^*$,n-normal if $B^*B^n = B^nB^*$ .In [4]Panayappan S and Sivamani N introduced a new class of operators calledn-binormalandit is defined as $B^*B^nB^nB^* = B^nB^*B^*B^n$,Meenambika K et al.,[1]have defined and analysed some of the properties of a new class of operator called Skew binormal operator and defined as $(B^*BBB^*)B = B(BB^*B^*B)$ .In [8] MahmoodKamilShihab defined an operator called square normal operator defined as $B^2(B^*)^2 = (B^*)^2B^2$ and an example is given to show that the square normal operator is not normal operator.In this paper, we investigate some basic properties of square n-binormal operator.

II. SQUARE BINORMAL AND SQUARE N-BINORMAL OPERATOR

An operator $B \in L(H)$ is called square binormal if $(B^*)^2B^2B^2(B^*)^2 = B^2(B^*)^2(B^*)^2B^2$ and square n-binormal if $(B^*)^2B^{2n}B^{2n}(B^*)^2 = B^{2n}(B^*)^2(B^*)^2B^{2n}$

Theorem 1.1

If B is a square n-binormal operator and β is any scalar which is real then βB is also square n-binormal operator.

Proof:

Since B is a square n-binormal operator,we have

$$(B^*)^2B^{2n}B^{2n}(B^*)^2 = B^{2n}(B^*)^2(B^*)^2B^{2n} \quad (1.1)$$

If β is any scalar,then $(\beta B)^* = \beta B^*$ also we have, $((\beta B)^*)^n = \beta^n(B^*)^2$

Using the results in (1.1),

$$[(\beta B)^*]^2[\beta B]^{2n}[\beta B]^{2n}[(\beta B)^*]^2 = \beta^2(B^*)^2\beta^{2n}B^{2n}\beta^{2n}B^{2n}\beta^2(B^*)^2 = \beta^4\beta^{4n}B^{4n}(B^*)^4 \quad (1.2)$$

$$[\beta B]^{2n}[(\beta B)^*]^2[(\beta B)^*]^2[\beta B]^{2n} = \beta^{2n}B^{2n}\beta^2(B^*)^2\beta^2(B^*)^2\beta^{2n}B^{2n} = \beta^{4n}B^{4n}\beta^4(B^*)^4 \quad (1.3)$$

From (1.1),(1.2) and (1.3) we see that βB is also square n-binormal operator.

Theorem 1.2

Every binormal operator is square binormal.

Proof:

An operator $B \in L(H)$ is binormal if $B^*BBB^* = BB^*B^*B$

$$\begin{aligned} B^{*2}B^2B^2B^{*2} &= (B^*BBB^*)^2 \\ &= B^*BBB^*B^*BBB^* \\ &= BB^*B^*BBB^*B^*B \\ &= (BB^*B^*B)^2 \\ &= B^2B^{*2}B^{*2}B^2 \end{aligned}$$

Hence B is square binormal operator.

The following example shows that square binormal operator need not be binormal.

Example 1.3

Let $B = \begin{pmatrix} i & i \\ 0 & -i \end{pmatrix}$ be an operator acting on a two dimensional Hilbert space. Then B is square binormal but not binormal.

Theorem 1.4

If B is a square n-binormal operator which is a self-adjoint operator,then B^* is also square n-binormal operator.

Proof:

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K Meenambika, Assistant Professor,Department of Mathematics, Sri Shanmugha College of Engineering and Technology, Sankari -637304, Tamilnadu, India.

C V Seshaiah, Professor,Department of Basic Science and Humanities, GMR Institute of Technology, GMR Nagar, Rajam-532127, Andhra Pradesh, India.

N Sivamani, Associate Professor,Department of Mathematics, Tamilnadu College of Engineering, Coimbatore-641659, Tamilnadu, India

If B is a square n-binormal operator,we have

$$\left(B^*\right)^2 B^{2n} B^{2n} \left(B^*\right)^2 = B^{2n} \left(B^*\right)^2 \left(B^*\right)^2 B^{2n} \quad (1.4)$$

Since B is self-adjoint, $B^* = B$

Replace T by T^* in (6.4),we get

$$\left(\left(B^*\right)^*\right)^2 \left(B^*\right)^{2n} \left(B^*\right)^{2n} \left(\left(B^*\right)^*\right)^2 = \left(B^{**}\right)^2 \left(B^*\right)^{2n} \left(B^*\right)^{2n} \left(B^{**}\right)^2$$
$$= B^2 \left(B^*\right)^{2n} \left(B^*\right)^{2n} B^2$$
$$= \left(B^*\right)^2 (B)^{2n} (B)^{2n} \left(B^*\right)^2 \quad (1.5)$$

Also

$$\left(B^*\right)^{2n} \left(\left(B^*\right)^*\right)^2 \left(\left(B^*\right)^*\right)^2 \left(B^*\right)^{2n} = \left(B^*\right)^{2n} \left(B^{**}\right)^2 \left(B^{**}\right)^2 \left(B^*\right)^{2n}$$
$$= \left(B^*\right)^{2n} B^2 B^2 \left(B^*\right)^{2n}$$
$$= (B)^{2n} \left(B^*\right)^2 \left(B^*\right)^2 (B)^{2n} \quad (1.6)$$

,

From (1.5) and (1.6), B^* is also square n-binormal.

Theorem 1.5:

If B is self adjoint operator,then B is square n-binormal operator.

Proof:

Since B is self-adjoint, $B^* = B$

Now,

$$\left(B^*\right)^2 B^{2n} B^{2n} \left(B^*\right)^2 = B^2 B^{2n} B^{2n} B^2$$
$$= B^4 B^{4n}$$
$$= B^{4+4n} \quad (1.7)$$

$$B^{2n} \left(B^{*^*}\right)^2 \left(B^*\right)^2 B^{2n} = B^{2n} B^2 B^2 B^{2n}$$
$$= B^4 B^{4n}$$
$$= B^{4+4n} \quad (1.8)$$

$\therefore$ B is square n- binormal operator.

Theorem 1.6

If B is any operator on a Hilbert space H.Then

i) $\left(B + B^*\right)$ is square n-binormal.

ii) BB^* is square n-binormal.

iii) B^*B is square n-binormal.

iv) $I + B^*B, I + BB^*$ are square n-binormal.

Proof:

i)Let $N = B + B^*$

$N^* = \left(B + B^*\right)^* = B^* + B = N$

Hence, Nis a self-adjoint operator.

We know that every self-adjoint operator is square n-binormal

Therefore, $N = B + B^*$ is square n-binormal operator.

ii) $\left(BB^*\right)^* = B^{**}B^* = BB^*$

Hence, BB^* is self-adjoint operator , so BB^* is square n-binormal operator.

iii) $\left(B^*B\right)^* = B^*B^{**} = B^*B$

Hence, B^*B is self-adjoint operator , so B^*B is square n-binormal operator.

iv) $\left(I + B^*B\right)^* = \left(I^* + B^*B^{**}\right) = \left(I + B^*B\right)$

$\left(I + BB^*\right)^* = \left(I^* + B^{**}B^*\right) = \left(I + BB^*\right)$

Hence, $I + B^*B, I + BB^*$ are self-adjointoperator,so $I + B^*B, I + BB^*$ are square n-binormal operators.

Theorem 1.7

Let B be a square n-binormal operator on a Hilbert space H.Let L be self adjoint operator for which B and L commute,then LB is also square n-binormal operator.

Proof:

Since L is self-adjoint operator,we have $L^* = L$ and also since L and B are doubly commuting, we get

$$LB = BL$$
$$L^*B = BL^*$$
$$LB^* = B^*L$$

Hence $(LB)^* = (BL)^*$

Since B is a square n-binormal operator,we have

$\left(B^*\right)^2 B^{2n} B^{2n} \left(B^*\right)^2 = B^{2n} \left(B^*\right)^2 \left(B^*\right)^2 B^{2n}$.

From LB=BL it is easily seen that

$BL^n = L^n B$, $B^n L^* = L^* B^n$, $L^n B^* = B^* L^n$, $(LB)^n = B^n L^n$ which implies $(LB)^{2n} = B^{2n} L^{2n}$

Consider,

$$\left((LB)^*\right)^2 (LB)^{2n} (LB)^{2n} \left((LB)^*\right)^2 = \left(B^*L^*\right)^2 B^{2n} L^{2n} B^{2n} L^{2n} \left(B^*L^*\right)^2$$
$$= \left(B^*\right)^2 \left(L^*\right)^2 L^{2n} B^{2n} L^{2n} B^{2n} \left(L^*\right)^2 \left(B^*\right)^2$$
$$= \left(B^*\right)^2 \left(L^*\right)^2 L^{2n} L^{2n} B^{2n} B^{2n} \left(L^*\right)^2 \left(B^*\right)^2$$
$$= \left(B^*\right)^2 \left(L^*\right)^2 L^{2n} L^{2n} B^{2n} \left(L^*\right)^2 B^{2n} \left(B^*\right)^2$$
$$= \left(B^*\right)^2 L^{2n} \left(L^*\right)^2 \left(L^*\right)^2 L^{2n} L^{2n} L^{2n} \left(B^*\right)^2$$
$$= L^{2n} \left(B^*\right)^2 \left(L^*\right)^2 L^{2n} \left(L^*\right)^2 B^{2n} B^{2n} \left(B^*\right)^2$$
$$= L^{2n} \left(B^*\right)^2 \left(L^*\right)^2 L^{2n} B^{2n} \left(L^*\right)^2 B^{2n} \left(B^*\right)^2 \qquad (\Theta\ L\ is\ sqBN)$$
$$= L^{2n} \left(B^*\right)^2 \left(L^*\right)^2 B^{2n} L^{2n} \left(L^*\right)^2 B^{2n} \left(B^*\right)^2$$

$$= L^{2n} \left(B^*\right)^2 B^{2n} \left(L^*\right)^2 L^{2n} \left(L^*\right)^2 B^{2n} \left(B^*\right)^2$$
$$= L^{2n} B^{2n} \left(B^*\right)^2 \left(L^*\right)^2 L^{2n} \left(L^*\right)^2 L^{2n} \left(B^*\right)^2$$
$$= L^{2n} B^{2n} \left(B^*\right)^2 \left(L^*\right)^2 \left(L^*\right)^2 L^{2n} \left(B^*\right)^2 B^{2n}$$
$$= L^{2n} B^{2n} \left(B^*\right)^2 \left(L^*\right)^2 \left(L^*\right)^2 \left(B^*\right)^2 L^{2n} B^{2n}$$
$$= B^{2n} L^{2n} \left(B^*\right)^2 \left(L^*\right)^2 \left(B^*\right)^2 \left(L^*\right)^2 B^{2n} L^{2n}$$
$$= (LB)^{2n} \left((LB)^*\right)^2 \left((LB)^*\right)^2 (LB)^{2n}$$

Hence LB is square n-binormal operator.

Theorem 1.8

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If B is a self-adjoint operator,then B^{-1} is also square n-binormal operator.

Proof:

Since B is considered to be a self-adjoint operator, $B^* = B$

$$\Rightarrow \left(B^{-1}\right)^* = \left(B^*\right)^{-1} = B^{-1}$$

$\Rightarrow B^{-1}$ is self-adjoint

$\Rightarrow B^{-1}$ is square n-binormal.

Proof verification:

Since B^{-1} is self-adjoint, $\left(B^{-1}\right)^* = B^{-1}$

Now consider,

$$\left[\left(B^{-1}\right)^*\right]^2\left(B^{-1}\right)^{2n}\left(B^{-1}\right)^{2n}\left[\left(B^{-1}\right)^*\right]^2 = \left(B^{-1}\right)^2\left(B^{-1}\right)^{2n}\left(B^{-1}\right)^{2n}\left(B^{-1}\right)^2 = \left(B^{-1}\right)^{4+4n} \qquad (1.9)$$

$$\left(B^{-1}\right)^{2n}\left[\left(B^{-1}\right)^*\right]^2\left[\left(B^{-1}\right)^*\right]^2\left(B^{-1}\right)^{2n} = \left(B^{-1}\right)^{2n}\left(B^{-1}\right)^2\left(B^{-1}\right)^2\left(B^{-1}\right)^{2n} = \left(B^{-1}\right)^{4+4n} \qquad (1.10)$$

From (1.9) and (1.10) B^{-1} is also square n-binormal operator.

Theorem 1.9

Let B be a self adjoint operator on a Hilbert space H and L be any operator on H, then L^*BL is square n-binormal operator.

Proof:

Since B is considered to be a self-adjoint operator, $B^* = B$

Consider $\left(L^*BL\right)^* = L^*B^*L = L^*BL$

Therefore L^*BL self-adjoint which implies L^*BL is square n-binormal.

Proof verification:

Consider,

$$\left[\left(L^*BL\right)^*\right]^2\left(S^*TS\right)^{2n}\left(S^*TS\right)^{2n}\left[\left(S^*TS\right)^*\right]^2 = \left[\left(S^*TS\right)\right]^2\left(S^*TS\right)^{2n}\left(S^*TS\right)^{2n}\left[\left(S^*TS\right)\right]^2 = \left(S^*TS\right)^{4+4n} \qquad (1.11)$$

$$\left(L^*BL\right)^{2n}\left[\left(L^*BL\right)^*\right]^2\left[\left(L^*BL\right)^*\right]^2\left(L^*BL\right)^{2n} = \left(L^*BL\right)^{2n}\left[\left(L^*BL\right)\right]^2\left[\left(L^*BL\right)\right]^2\left(L^*BL\right)^{2n} = \left(L^*BL\right)^{4+4n} \qquad (1.12)$$

From (1.11) and (1.12),

L^*BL is square n-binormal.

Theorem 1.10

If B is a square binormal operator if and only if B^2 is binormal.

Proof:

Let B be a square binormaloperator,then

$$\left(B^*\right)^2 B^2 B^2\left(B^*\right)^2 = B^2\left(B^*\right)^2\left(B^*\right)^2 B^2$$

$$\Leftrightarrow \left(B^2\right)^* B^2 B^2\left(B^2\right)^* = B^2\left(B^2\right)^*\left(B^2\right)^* B^2$$

$\Leftrightarrow B^2$ isbinormal.

Theorem 1.11

If $B \in sqBN$,then so are,

i)any $L \in L(H)$ that is unitarily equivalent to B.

ii)the restriction B/M of B to any closed subspace M of H that reduces B.

Proof of (i):

Let $L \in L(H)$ be unitarily equivalent to B then there is a unitary operator $U \in L(H)$ such that

$$L^{2n} = U^*B^{2n}U,\ L^* = U^*B^*U$$

Consider,

$$\begin{aligned}\left(L^*\right)^2 L^{2n}L^{2n}\left(L^*\right)^2 &= \left(U^*B^*U\right)^2\left(U^*B^{2n}U\right)^2\left(U^*B^{2n}U\right)^2\left(U^*B^*U\right)^2 \\ &= \left(U^*\right)^2\left(B^*\right)^2U^2\left(U^*\right)^2B^{4n}U^2\left(U^*\right)^2B^{4n}U^2\left(U^*\right)^2\left(B^*\right)^2U^2 \\ &= \left(U^*\right)^2\left(B^*\right)^2B^{4n}B^{4n}\left(B^*\right)^2U^2 \\ &= \left(U^*\right)^2\left(B^*\right)^4B^{8n}U^2 \end{aligned} \qquad (1.13)$$

$$\begin{aligned}L^{2n}\left(L^*\right)^2\left(L^*\right)^2L^{2n} &= \left(U^*B^{2n}U\right)^2\left(U^*B^*U\right)^2\left(U^*B^*U\right)^2\left(U^*B^{2n}U\right)^2 \\ &= \left(U^*\right)^2B^{4n}U^2\left(U^*\right)^2\left(B^*\right)^2U^2\left(U^*\right)^2\left(B^*\right)^2U^2\left(U^*\right)^2B^{4n}U^2 \\ &= \left(U^*\right)^2B^{4n}\left(T^*\right)^2\left(T^*\right)^2B^{4n}U^2 \\ &= \left(U^*\right)^2\left(B^*\right)^4B^{8n}U^2 \end{aligned} \qquad (1.14)$$

From (1.13) and (1.14) $L \in sqBN$.

ii) If B is square n-binormaloperator,then

$$\left(B^*\right)^2B^{2n}B^{2n}\left(B^*\right)^2 = B^{2n}\left(B^*\right)^2\left(B^*\right)^2B^{2n}.$$

Consider,

$$\begin{aligned}\left(\left(B/M\right)^*\right)^2\left(B/M\right)^{2n}\left(B/M\right)^{2n}\left(\left(B/M\right)^*\right)^2 &= \left(B^*/M\right)^2\left(B^{2n}/M\right)\left(B^{2n}/M\right)\left(B^*/M\right)^2 \\ &= \left({B^*}^2/M\right)\left(B^{2n}/M\right)\left(B^{2n}/M\right)\left({B^*}^2/M\right) \\ &= {B^*}^2B^{2n}B^{2n}{B^*}^2/M \\ &= B^{2n}{B^*}^2{B^*}^2B^{2n}/M \qquad (\Theta T\ is\ sqnBN) \\ &= \left(B^{2n}/M\right)\left(B^*/M\right)^2\left(B^*/M\right)^2\left(B^{2n}/M\right) \\ &= \left(B/M\right)^{2n}\left(\left(B/M\right)^*\right)^2\left(\left(B/M\right)^*\right)^2\left(B/M\right)^{2n}\end{aligned}$$

$$\Rightarrow B/M \in [sqnBN].$$

The following examples show that square n-binormal and n-isometry operators are independent classes.

Example 1.12

Consider the operator $B = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ on R^2, which is square 3-binormal but not 3-isometry.

Example 1.13

Consider the operator $L = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ on R^2 , which is square 3-isometry but not square 3-binormal.

Theorem 1.14

Let $B_1, B_2 \ldots B_m$ be n-binormal operators in L(H).Then $(B_1 \oplus B_2 \oplus \ldots \oplus B_m)$ and $(B_1 \otimes B_2 \otimes \ldots \otimes B_m)$ are square n-binormal operators.

Proof:

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$$\left[(B_1\oplus B_2\oplus\ldots\oplus B_m)^*\right]^2(B_1\oplus B_2\oplus\ldots\oplus B_m)^{2n}(B_1\oplus B_2\oplus\ldots\oplus B_m)^{2n}\left[(B_1\oplus B_2\oplus\ldots\oplus B_m)^*\right]^2$$

$$=\left[(B_1\oplus B_2\oplus\ldots\oplus B_m)^*(B_1\oplus B_2\oplus\ldots\oplus B_m)^n(B_1\oplus B_2\oplus\ldots\oplus B_m)^n(B_1\oplus B_2\oplus\ldots\oplus B_m)^*\right]^2$$

$$=(B_1\oplus B_2\oplus\ldots\oplus B_m)^*(B_1\oplus B_2\oplus\ldots\oplus B_m)^n(B_1\oplus B_2\oplus\ldots\oplus B_m)^n(B_1\oplus B_2\oplus\ldots\oplus B_m)^*$$
$$(B_1\oplus B_2\oplus\ldots\oplus B_m)^*(B_1\oplus B_2\oplus\ldots\oplus B_m)^n(B_1\oplus B_2\oplus\ldots\oplus B_m)^n(B_1\oplus B_2\oplus\ldots\oplus B_m)^*$$

$$=(B_1\oplus B_2\oplus\ldots\oplus B_m)^n(B_1\oplus B_2\oplus\ldots\oplus B_m)^*(B_1\oplus B_2\oplus\ldots\oplus B_m)^*(B_1\oplus B_2\oplus\ldots\oplus B_m)^n$$
$$(B_1\oplus B_2\oplus\ldots\oplus B_m)^n(B_1\oplus B_2\oplus\ldots\oplus B_m)^*(B_1\oplus B_2\oplus\ldots\oplus B_m)^*(B_1\oplus B_2\oplus\ldots\oplus B_m)^n$$

$$=\left[(B_1\oplus B_2\oplus\ldots\oplus B_m)^n(B_1\oplus B_2\oplus\ldots\oplus B_m)^*(B_1\oplus B_2\oplus\ldots\oplus B_m)^*(B_1\oplus B_2\oplus\ldots\oplus B_m)^n\right]^2$$

$$=(B_1\oplus B_2\oplus\ldots\oplus B_m)^{2n}\left[(B_1\oplus B_2\oplus\ldots\oplus B_m)^*\right]^2\left[(B_1\oplus B_2\oplus\ldots\oplus B_m)^*\right]^2(B_1\oplus B_2\oplus\ldots\oplus B_m)^{2n}$$

Hence $(B_1 \oplus B_2 \oplus \ldots \oplus B_m)$ is square n-binormal operator.

Now $x_1, x_2 \ldots x_m \in H$,

$$=\left[(B_1\otimes B_2\otimes\ldots\otimes B_m)^*\right]^2(B_1\otimes B_2\otimes\ldots\otimes B_m)^{2n}(B_1\otimes B_2\otimes\ldots\otimes B_m)^{2n}\left[(B_1\otimes B_2\otimes\ldots\otimes B_m)^*\right]^2(x_1\otimes x_2\otimes\ldots\otimes x_m)$$

$$=\left(B_1^*\otimes B_2^*\otimes\ldots\otimes B_m^*\right)^2\left(B_1^{2n}\otimes B_2^{2n}\otimes\ldots\otimes B_m^{2n}\right)\left(B_1^{2n}\otimes B_2^{2n}\otimes\ldots\otimes B_m^{2n}\right)\left(B_1^*\otimes B_2^*\otimes\ldots\otimes B_m^*\right)^2(x_1\otimes x_2\otimes\ldots\otimes x_m)$$

$$=\left(B_1^{*^2}\otimes B_2^{*^2}\otimes\ldots\otimes B_m^{*^2}\right)\left(B_1^{2n}\otimes B_2^{2n}\otimes\ldots\otimes B_m^{2n}\right)\left(B_1^{2n}\otimes B_2^{2n}\otimes\ldots\otimes B_m^{2n}\right)\left(B_1^{*^2}\otimes B_2^{*^2}\otimes\ldots\otimes B_m^{*^2}\right)(x_1\otimes x_2\otimes\ldots\otimes x_m)$$

$$=\left(B_1^{*^2}B_1^{2n}B_1^{2n}B_1^{*^2}x_1\otimes B_2^{*^2}B_2^{2n}B_2^{2n}B_2^{*^2}x_2\otimes\ldots\otimes B_m^{*^2}B_m^{2n}B_m^{2n}B_m^{*^2}x_m\right)$$

$$=\left(B_1^{2n}B_1^{*^2}B_1^{*^2}B_1^{2n}x_1\otimes B_2^{2n}B_2^{*^2}B_2^{*^2}B_2^{2n}x_2\otimes\ldots\otimes B_m^{2n}B_m^{*^2}B_m^{*^2}B_{m1}^{2n}x_m\right)$$

Hence $(B_1 \otimes B_2 \otimes \ldots \otimes B_m)$ is square n-binormal operator.

III. RESULTS

The following results are proved in this article.

1. If B is a square n-binormal operator and β is any scalar which is real then βB is also square n-binormal operator.
2. Every binormal operator is square binormal.
3. Let $B=\begin{pmatrix} i & i \\ 0 & -i \end{pmatrix}$ be an operator acting on a two dimensional Hilbert space. Then B is square binormal but not binormal.
4. If B is a square n-binormal operator which is a self-adjoint operator, then B^* is also square n-binormal operator.
5. If B is self adjoint operator, then B is square n-binormal operator.
6. If B is any operator on a Hilbert space H. Then
   i) $(B+B^*)$ is square n-binormal.
   ii) BB^* is square n-binormal.
   iii) B^*B is square n-binormal.
   iv) $I+B^*B, I+BB^*$ are square n-binormal.
7. Let B be a square n-binormal operator on a Hilbert space H. Let L be self adjoint operator for which B and L commute, then LB is also square n-binormal operator.
8. If B is a self-adjoint operator, then B^{-1} is also square n-binormal operator
9. Let B be a self adjoint operator on a Hilbert space H and L be any operator on H, then L^*BL is square n-binormal operator.
10. If B is a square binormal operator if and only if B^2 is binormal.

IV. CONCLUSION

This paper has presented a deserving class of operators called Squarebinormaland square n-binormaloperators. Some of the characters of square n-binormal operators were studied. The described work is focused on relationship between self- adjoint and squaren-binormal operators.

REFERENCES

1. Meenambika K et al.2018 Skew Binormal operators acting on a Hilbert SpaceJournal of Physics1139 pp 1-8.
2. Meenambika K et al.2018 Skew Normal operators acting on a Hilbert space International Journal of Statistics and Applied Mathematics 3pp 128-133.
3. Alzuraiqi SA and Patel AB 2010 On n-normal operators General Mathematics Notes2 pp61-73.
4. Panayappan S and Sivamani N 2012 On n-binormal operators General Mathematics Notes10 pp1-8.
5. LaithKShaakir and Elaf S Abdulwahid 2014 Skew N-normal operators Aust.J.Basic&Appl.Sci16pp340-44.
6. Seshaiah CV and Meenambika K 2016 A generalization of aluthge transformation using semi-hyponormaloperators International Journal for Research in Mathematics And Statistics2pp18-28.
7. Veluchamy T and Manikandan KM 2016 n-Power quasi normal operators on the hilbert space IOSR Journal of Mathematics12pp06-09
8. MahmoodKamilShihab 2016 Square-normal operator British Journal of Mathematics & Computer Science 3pp 1-7
9. Seshaiah CV and Meenambika K 2018 Factorization of semi-hyponormal operators Journal of faculty of agronomy34pp306-09
10. Anuradha Gupta et al. 2016Skew n-normal composition and weighted composition operators on l2 (µ)International Journal of Pure and Applied Mathematics107pp625-634.

AUTHORS PROFILE

K Meenambika, Assistant Professor,Department of Mathematics, Sri Shanmugha College of Engineering and Technology, Sankari -637304, Tamilnadu, India.

C V Seshaiah, Professor,Department of Basic Science and Humanities, GMR Institute of Technology, GMR Nagar, Rajam-532127, Andhra Pradesh, India.

N Sivamani, Associate Professor,Department of Mathematics, Tamilnadu College of Engineering, Coimbatore-641659, Tamilnadu, India.

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