Mathematical Modelling and Simulation of Non-Linear PID Controller for Spherical Tank Level Control in oil and Gas Industry

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Abstract: *In process industry specifically in oil and Gas industries, Spherical tank is preferred for storage of high-pressure fluids because of its strong structure and its even distribution of stresses on the sphere's surfaces, both internally and externally. Geometrically speaking its structure naturally highly nonlinear in nature. In oil and gas industries, every operation is done by the automation with the help of field sensors, controllers and final control elements (Actuators). Controlling any process variable is commonly difficult in oil and gas process industries. The level control of spherical tank is highly difficult job because of its geometrical structure. The area of the spherical tank is continuously varying with respect to its height. So, it required a special type of controllers needs to control it. This article presents the controller tuning methods of PID controller to control the level of Spherical Tank Control system.*

Index Terms: *Spherical Tank, Level Control, Process Automation, Non-linear System, PID Controller, ZN Controller.*

I. INTRODUCTION

A. OIL & GAS COMPANIES IN INDIA - A SURVEY

Table 1. Types of Control System used in various industry in India

Company Name	Production	No. of Employee	Revenues /Year	Control Technique
Adani Welspun Ltd	oil and gas	24,000	₹66.2671 billion	Semi-automatic with DCS
Reliance Petroleum Limited	Petroleum and Natural Gas	187,729	₹36,075 crore	Semi-automatic
GAIL	Energy	4,355	₹50,059.26 crore	Semi-automatic
Oil and Natural Gas Corporation (ONGC)	Oil and gas	33,560	₹362,246.96 crore	Semi-automatic
BPCL	Oil and gas	12,567	₹244,648.50 crore	Semi-automatic

Table 2. Types of Control System used in various industry in Overseas

Company Name	Country	No. of Employees	Revenue	Control Technique
Saudi Aramco (1933)	Saudi Arabia	65,266	₹465.49 crore	Semi-automatic
Sinopec Group (1998)	China	2,49,142	₹459.25 crore	Semi-automatic
China National Petroleum Corporation (1988)	China	1,636,532	₹428.62 crore	Semi-automatic
Exxon Mobil (1999)	USA	69,600	₹268.9 crore	Semi-automatic
Royal Dutch Shell (1890)	Netherlands	92,000	₹265 crore	Semi-automatic
Kuwait Petroleum Corporation (1980)	Asia	96,000	₹251.94 crore	Semi-automatic

From the survey Table 1& 2, it is identified that the process automation isn't intelligent in automation. Process automation like pressure, Temperature, Level controls are in semi-automatic systems are employed till now. It is highly represented here to implement a fully automatic system is needed to control the process automation. In this present manuscript the nonlinear controller is implemented with real-time small-scale setup and simulated in MATLAB Simulink.

B. SPHERICAL TANK SYSTEM IN OIL AND GAS INDUSTRY

Spherical tank systems are used to hold liquids, compressed gases which is not typically labelled or regulated as a storage tank. Spherical tank could withstand internal pressure easily because of its geometrical structure. LPG is stored under pressure, and the tank must keep this storage in it. Spheres are the strongest shape so they can hold the highest pressures. Also, they have the lowest possible surface area to volume ratio (which minimizes the amount of heat that gets inside through the tank wall).

Revised Manuscript Received on May 10, 21019.
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There are five basic types of LPG storage,
1. Horton sphere
2. Mounded storage vessel
3. Refrigerated LPG storage vessel
4. Cavern storage

C. Horton Sphere Tank system

Horton spheres tank system is used to safely hold liquefied natural gas (LNG), which is produced by cooling natural gas at atmospheric pressure to minus 260 degrees Fahrenheit, at which point it liquefies. Sphere is the geometrically perfect shape for a system that resists internal pressure.

D. Mounded ground vessel Tank

Mounded tanks (horizontal) are majorly used for storage of Propane / Liquid Petroleum Gas which is generally safer than methods of storing the highly inflammable LPG. This type of LPG Bullets are Large, Horizontal cylindrical steel tanks with of size ranging about 3.5 to 8.5 diameter and lengths about 35 to 70 meters.

E. Refrigerated LPG storage vessel

This type of vessel system is used to store refrigerated and cryogenic gas under higher pressure level. Refrigerated LPG storage vessel type of tanks and their insulation systems must work together to ensure optimum performance for low temperature and cryogenic storage for safe operation.

F. Cavern Storage Tank

Until Fairway Energy commenced operations, all existing commercial crude oil storage in the Houston area has been in above ground tank storage. While above ground storage has its purpose, it is extremely wasteful in terms of space as compared to underground storage and is not protected from catastrophic events such as hurricanes and floods which have so adversely affected other types of oil storage facilities in the past [17], [18]. Highly economical in terms of land area utilization, underground storage has less impact on the environment and provides natural barriers from weather-related events.

II. MATHEMATICAL MODELLING FOR A SIMPLE SPHERICAL TANK SYSTEM

The spherical tank is basically a highly nonlinear system because of its geometrical shape. The level control the system is also highly complicated one. Hence PID controller is used in this process and the gain parameters were choosen by ZN tuning rule [1], [17]. The proportional integral- derivative (PID) controllers are widely used in many industrial control systems for several decades. S.Nithya, N.Sivakumaran, T.K.Radhakrishnan and N.Anantharaman [1]

The following CAD Model is used to model the entire spherical tank system with the all real-time parameters.

The proposed spherical tank system, identified as a nonlinear complex structure is shown in Fig. 1, where

H = Total height of the spherical tank,
f_1 = Input flow rate of spherical tank and
f_2 = Output flow rate of spherical tank.

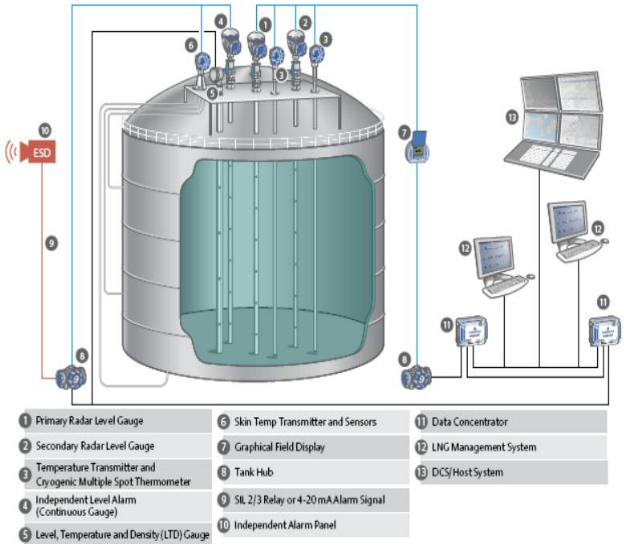

Fig.1. Spherical Tank and its controllers

Fig.1. Mathematical modeling of spherical tank system is derived based on the structure and the output transfer function is obtained.

The open loop system's performance is mainly based the PID gain parameters vales. Unfortunately, the values are chosen by trial and error method. Even though it yields good static response, it will take larger in time to tune the PID gain values. It is very difficult to develop the mathematical model of the process to adjust a particular control loop which time varying one [2]. Even though, it is possible to derive a transfer function-based model by conducting a open loop plant test. The open loop plant test is to bring a step change in the manipulated input, and observe the measured process output. Generally, the closed loop process is represented by a model as

$$G(s) = \frac{K}{\tau S+1} e^{-tdS} \quad (1)$$

The process plant basic parameters are the static gain (K), the time constant (τ), and the time delay (td) of the plant derived from the transient response.

Table 3. The spherical tank system's specification

Spherical Tank Company	Taishan Spherical storage tank
Maximum Pressure	2.5 Mpa
Inner diameter	10m
Max weight	1,000ton
Max capacity	10,000m³
Materials	Carbon Steel Alloy, Stainless Steel, Non-Ferrous Metals
Pump	ABB 3 Phase, 37 Kw, 50 Hp, 415 V, Induction motor
Control valve with Positioner	Siemens Valve Positioner Electro-Pneumatic

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Controller	Distributed Control System (DCS)
Controller Make	Yokogawa, India

Table 4. Specification of Sensors and Transducers

Flow Meter	**Magnetic Flowmeters,** Yokogawa's
Air regulator	**Parker, Electronic Proportional Regulator**
E/P converter	**Watson Smith I/P Converters**
Pressure gauge	EJA530E, In-Line Mount Gauge Pressure Transmitter, Yokogawa

Where,

Operating Parameters are:

F1 - rate of Inflow

F2 - rate of Outflow

H - height of the spherical tank.

R - radius of the spherical tank

H - Nominal level of the tank

r - Radius at nominal level

Mass balance equation could be written as [3], [4], [5],

Rate of accumulation in the tank = Mass flow in – Mass.

The dynamics of spherical tank system is described by the first order differential equation flow out.

$$\frac{dV}{dt} = F_1 - F_2 \tag{2}$$

Where F_1, F_2,

F_1-Oil Inlet Flow Rate Litres/Min

F_2-Oil Inlet Flow Rate Litres/Min

V- Tank volume

Mathematical Formulae for calculating the volume of the Spherical Tank.

$$V = \frac{4}{3}\pi r^3 \tag{3}$$

While allowing the Oil into the tank the rate of change of accumulation of oil is based on the volume of the nature of the Spherical Tank

$$\frac{dV}{dt} = F_1 - F_2 \tag{4}$$

The ratio of

$$\frac{R}{H} = \frac{r}{h} \tag{5}$$

$$\frac{R}{H}h = r \tag{6}$$

$$V = \frac{4}{3}\pi\frac{R^3h^3}{H^3} \tag{7}$$

Volume of Spherical Tank is varying with respect to time because if input and out flowrate is changing with respect to time.

Differentiate with respect to time,

The volume of tank becomes

$$\frac{dv}{dt} = 4\pi\frac{R^3h^2}{H^3}\frac{dh}{dt} \tag{8}$$

Cross sectional area of the spherical tank is that,

$$A = 4\pi\frac{R^3h^2}{H^3} \tag{9}$$

spherical tank volume is varying with liquid height also,

$$A\frac{dh}{dt} = F1 - F2 \tag{10}$$

We need to find out the transfer function, for that that we need the transient mass balance equation

$$A1\frac{dh1}{dt} = F1 - F2 \tag{11}$$

At steady state,

$$A1\frac{dh1s}{dt} = F1s - F2s \tag{12}$$

s-steady state

at steady state condition

inlet flow rate = equal to outlet flow rate

$$F_{1s}=F_{2s} \tag{13}$$

$$h_{1s}=0 \tag{14}$$

F_{1s} - Inlet flow rate at steady state

F_{2s} -Outlet flow rate steady state

To find out the outlet flow rate, Volumetric flow rate is defined as

$$F_2=v*a, \tag{15}$$

F_2 - flow rate

V - velocity of the fluid,

a - area of the cross section of the space

refer to Torricelli's law,

$$F2 = Cv\sqrt{2gh} \tag{16}$$

Where,

g Gravitational acceleration (9.81 m/s^2),

h Fluid's height

V Fluid speed,

Cv- Valve Coefficients

G. Flow Coefficient in Pipe line (Cv)

Table 5. Selection of Valve co-efficient factor

% Travel	Cv
0	2
10	2.96
20	4.37
30	6.47
40	9.56
50	14.14
60	20.91
70	30.92
80	45.73
90	67.62
100	100

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The flow coefficient or valve coefficient represented by "Cv". It is used to calculate the valve size that will govern to allow the valve to pass the required flow rate in it. The following table will provide the essential data to select the perfect "Cv" for the process fluid [9].

$$\frac{dh1}{dt} = \left[\frac{H^3}{4\pi R^3}\right] h1F1 - \left[\frac{H^3}{4\pi R^3}\right] h1\, Cv\sqrt{2gh} \qquad (17)$$

$$\alpha = [\frac{H^3}{4\pi R^3}]$$

$$\beta = \alpha Cv\sqrt{2gh}$$

$$\frac{dh1}{dt} = \alpha h1F1 - \beta h1s \qquad (18)$$

α, β- non linear element..

To convert linear to non linear we could use Taylor series.
Let we can consider for [10], [11],

H= H-H1s

F=F-F1s

dH1/dt=h1F1-βh1sH1 (19)

Apply Laplace Transform

SH1(s)=h1F1(s)-β(h1s)H1(s) (20)

SH1(s)=h1F1(s)-β(h1s)H1(s) (21)

SH1(s)+β(h1s)H1(s)= h1F1(s) (23)

(SH1(s)+β(h1s)H1(s))/(F1(s))=h1 (24)

H1(s)[(S+β(h1s))/F1(s)]=h1 (25)

(H1(s))/(F1(s))=h1/(S+β(h1s)) (26)

Where, β =α Cv√ 2gh,

α =[H^3/(4π R^3)]

The Transfer function for the Spherical tank system was derived as a first order with time delay.
General Transfer function for the Spherical tank system
(H1(s))/(F1(s))=A/(τS+1) (27)

Where,
K=2hs/F$_2$,
τ =4π Khs and F$_2$=Cv√ 2gh

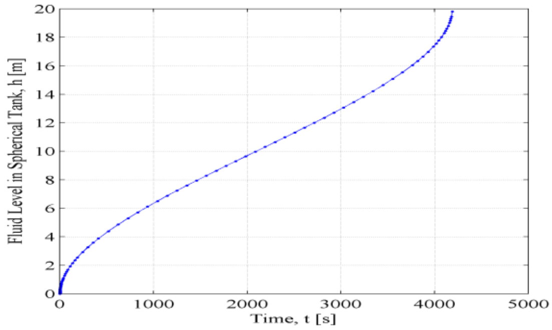

Fig. 2. Fluid level filling ratio

Fig.2. The results are plotted over the amount of time it takes to fill the sphere, which can be easily calculated.

B. Introduction to PID Controller in Oil and Gas Industries

Proportional-Integral-Derivative (PID) controllers are the controller commonly used in process control applications. In many industries it is easy to implement and control. In oil and gas Industry to regulate flow, temperature, pressure, level, and many other industrial process variables are very complex [12]. They date back to 1939, when the Taylor and Foxboro instrument company was introduced the first two PID controllers to automate the process in processing industries.

PID controllers are the base controllers of now a days modern process control systems, it will automatically regulate the process otherwise it has been tuned by manually by changing its gains values manually. The controller gain setting is playing a most important role in automation process [15],[16]. If the controller gains values are doesn't meet the requirement the automation process won't happen properly. So it needs to select such a way that it leads to minimize the process error. But it is a tedious process. Only by the experience operator can able to choose the gain values. Ziegler-Nichols tuning rule gives sometime better results but not suitable for all process. Because of the improper tuned controller parameters. This is one of the major disadvantages in Ziegler-Nichols controller.

III. Conventional Z-N PID for Spherical Tank System

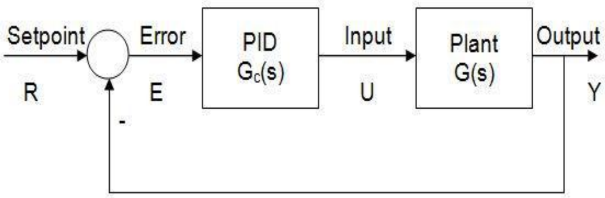

Fig. 3. Block diagram - Conventional Z-N PID controller of Spherical Tank System

Fig.3. shows the general Ziegler-Nichols PID controller with plant G(s) which is shown by figure.

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$$u(t) = K_P e(t) + K_i \int_0^t e(t)dt + K_d \frac{d}{dt} e(t) \qquad (28)$$

The following gains are,

Proportional gain , Kp
Integral gain, Ti = 1/Ki
Derivative gain, T_d = 1/Kd
Error = Set point value - Process variable value

The above gains formulae derived in John G. Ziegler and Nathaniel B. Nichols. Z-N rulebook and it was originally introduced by Ziegler-Nichols to tune Proportional – Integral - Derivative controller [13].

Conventional controller is the controller where the controller parameters Kp, Ki, Kd are all calculated by trial and error method. With the knowledge of field operator, the processes were automated i.e. known as open loop control system [14].

Table 6. Transfer function obtained for various levels

Region (Meter)	Process Gain K	Time Constant τ	Transfer function
0-4	0.945	4747.68	$\frac{H1(s)}{F1(s)} = \frac{0.945}{4747.68S+1}$
4-8	1.3367	13431.25	$\frac{H1(s)}{F1(s)} = \frac{1.3367}{13431.25S+1}$
8-12	1.6361	24659.33	$\frac{H1(s)}{F1(s)} = \frac{1.6361}{24659.33S+1}$
12-16	1.8892	37965.35	$\frac{H1(s)}{F1(s)} = \frac{1.8892}{37965.35S+1}$
16-20	2.1123	53061.5	$\frac{H1(s)}{F1(s)} = \frac{2.1123}{53061.5S+1}$

The following Table 7 shows the formulae to calculate the PID controller gains,

Table 7. PID Gain calculation Formulae

Controller Type	Ultimate Gain (Kc)	Integral Time (Ti)	Derivative time (Td)
P	Ku/2	-	-
PI	Ku/2.2	Pu/1.2	-
PID	Ku/1.7	Pu/2	Pu/8

IV. Simulation Results & Discussion

A. Open loop Response

The Simulation Results Fig.4 shows the open loop response for the regulator response of the level system at level 400 cm,

$$\frac{H1(s)}{F1(s)} = \frac{0.945}{4747.68S+1} \qquad (29)$$

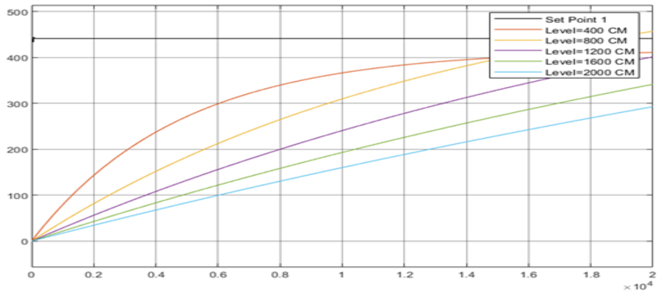

Fig.4. Open Loop response without controller

From the MATLAB simulation results it is identified that the level of the tank couldn't be sustained its setpoint.

B. Closed Loop Response

At Level 400cm, Derived Transfer Function is follows as,

$$\frac{H1(s)}{F1(s)} = \frac{0.945}{4747.68S+1} \qquad (30)$$

The following Simulation Results shows at level of 400 cm. The closed loop response for the level system static response parameters. the closed response of the system is also doesn't meet the setpoint requirement. Because the system is basically a nonlinear system one. So, it is highly recommended to implement a nonlinear controller to that system.

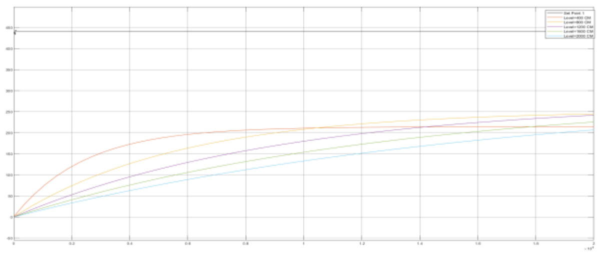

Fig.5. Closed Loop response without controller

C. Conventional ZN Method

The following Simulation Results shows the PID controller response for the regulator response of the level system at level 400 cm

$$\frac{H1(s)}{F1(s)} = \frac{0.945}{4747.68S+1} \qquad (31)$$

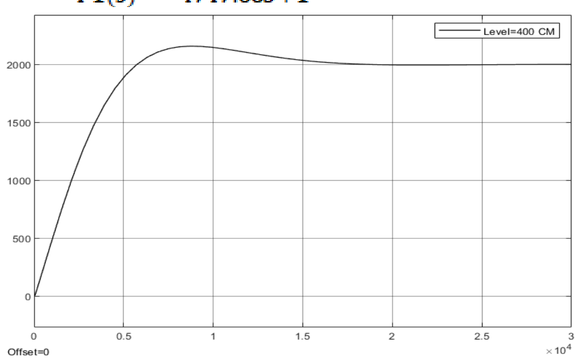

Fig.6. Response of Non-Linear PID controller

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Fig.6. shows the PID controller tuned level control simulation MATLAB. It shows the efficient error rejection between its output to setpoint. It has better error rejection capability, it yields the better regulatory response in offline. Even though it was proved the good results, it has some draw back during dynamic transient period of the system.

D. Comparison Results

At Level 400cm, the Derived Transfer Function,

$$\frac{H1(s)}{F1(s)} = \frac{0.945}{4747.68S+1} \qquad (32)$$

The following Fig.7. Simulation Results shows the overall comparison of the open & closed loop and Controller response for the regulator response of the level system at level 400 CM

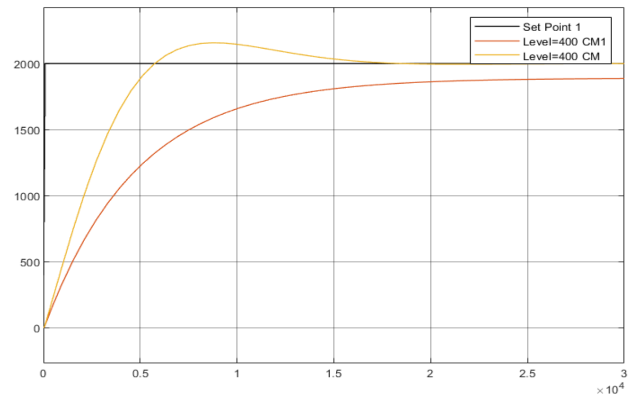

Fig.7. Response of closed loop vs Non-Linear PID controller

V. CONCLUSION

In this research article, the mathematical modelling of spherical tank is derived as transfer function with the help of mass balance equation and differential equation. The simulation results were obtained for open and closed for various level liquid. The results are compared with open, closed, ZN PID tuned controller. The gains were chosen by trial and Error method with the help of field operator. ZN based PID controller yields the good response over closed loop response in offline mode. The response of the open, closed, PID controller response are simulated. But here the chosen of gain parameter are made by trial and error method. Due to that Dynamic response of the system doesn't meet the requirement. In future the PID gain values are trained and chosen by evolutionary algorithm.

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