

Execution Proportions of Multi Server Queuing Model with Pentagonal Fuzzy Number: DSW Algorithm Approach

K. Usha Prameela, Pavan Kumar

Abstract: - This paper displays a multi-server queuing miniature in fuzzy condition. We accept the inter-arrival (entry) time and service (overhauled) time as pentagon fuzzy numbers, and the arithmetic of interval numbers is enforced. In this proposed model, the idea of the α -cut through DSW calculation is connected to decide the execution proportions. The effectiveness of the proposed model is outlined by fathoming a numerical precedent by thinking about hypothetical information.

Keywords: Queuing Model, Pentagon Fuzzy Numbers, DSW Algorithm, α -cut, execution proportions.

I. INTRODUCTION

A queue contains at least one line or at least one administration offices under a lot of tenets. The parameters entry rate (λ) and administration rate (μ) are required to pursue appropriation in queuing hypothesis. In lining models for the most part, the information, landing rate and administration rate are uncertainly known, and this vulnerability is settled by utilizing fuzzy idea. This hypothesis gives significant and amazing portrayal of vulnerability. In this way, lining models have more applications by applying fuzzy nature. Fuzzy lining miniature was introduced by R. J. Lie and E.S. Lee [9] in 1989 and developed by J.J. Buckley [3] in 1990. R. S. Negi and E.S. Lee in 1992, and Chen [2005-2006] advanced (FM/FM/1): (∞ /FCFS) and (FM/FM/k): (∞ /FCFS) where FM indicates Fuzzified exponential time dependent on queuing hypothesis. Srinivasan [7] in 2014 proposed a fuzzy queuing model utilizing DSW calculation. S. Thamothen [4] in 2015 exhibited an investigation on multi server fuzzy queuing model in triangular and trapezoidal fuzzy numbers utilizing α cuts. S. Shanmugasundaram & Venkatesh [6] in 2015 anticipated fuzzy multi server queuing model through DSW calculation. K.U Madhuri & V. Ashok in 2017 have dissected fuzzy lines utilizing Zadeh's [2] augmentation rule. P. Selvam et. al. [12] in 2017 submitted a ranking of pentagonal fuzzy numbers applying incentre of centroids. Kao et al constructed the membership functions of the system characteristics for fuzzy queues utilizing parametric linear programming. Fuzzy set hypothesis is an augmentation of traditional set hypothesis. The crisp numbers are not adequate to manage vulnerability. So, the fuzzy numbers, for example, triangular, trapezoidal, are utilized to manage vulnerability. Triangular and trapezoidal fuzzy numbers are regularly utilized in suplications.

They are not reasonable for executions where vulnerabilities emerge in multiple focuses. In such cases, pentagonal fuzzy numbers i.e., five points can be utilized to take care of the issues. This paper depicts multi server lining model and FCFS discipline utilizing pentagonal fuzzy numbers under α cut portrayal through DSW calculation to manage unsure parameters. Here, Fuzzy set can be divided into permeate unmistakable focuses through α cut strategy. To characterize membership function of the execution proportions in multi-server fuzzy lining model, DSW calculation is utilized. In segment II, some essential ideas and definitions are presented. In segment III, the suspicions and inscriptions are portrayed. In segment IV, the proposed queuing model is given. In segment V, the resolution perspective to the present model is reported. In segment VI, numerical example is solved. In segment VII, results and discussions are conferred. In segment VIII, the model is concluded.

II. II. ESSENTIAL IDEAS & DEFINITIONS

Fuzzy Set [2]: A Fuzzy set can be characterized as:

$$\tilde{A} = \{ \langle x, \mu_{\tilde{A}}(x) \rangle : x \in X \}$$

and X be a non-void set, $\mu_{\tilde{A}}(x) \in [0, 1]$ is the membership of $x \in X$ in \tilde{A}

Fuzzy Number [3]: A fuzzy set \tilde{A} characterized on the set of real numbers R is said to be fuzzy number if it has the accompanying qualities such as \tilde{A} is normal, convex and the support of it is closed and bounded.

α -cut [2]: An α -cut of a fuzzy set is a crisp set \tilde{A}_{α} that contains all the elements of the universal set X that have a participation grade in \tilde{A} greater than or equal to determined estimation of α . Therefore

$$\tilde{A}_{\alpha} = \{ x \in X : \mu_{\tilde{A}}(x) \geq \alpha, 0 \leq \alpha \leq 1 \}$$

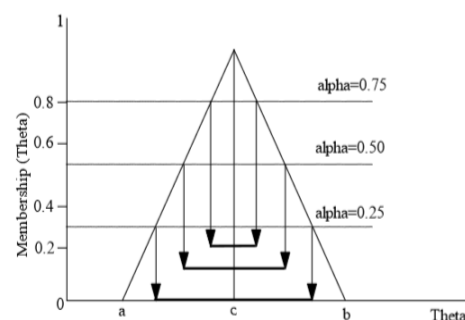


Figure 1: α – cut in Triangular Fuzzy Parameters

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Pentagon Fuzzy Number [9]

A fuzzy number $\tilde{A} = [a, b, c, d, e]$ where $a \leq b \leq c \leq d \leq e$ is said to be a pentagon fuzzy number if its membership function is given by

$$\mu_{P(A, S)}(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{x-b}{c-b}, & b \leq x \leq c \\ 1, & x = c \\ \frac{d-x}{d-c}, & c \leq x \leq d \\ \frac{e-x}{e-d}, & d \leq x \leq e \\ 0, & x > e \end{cases}$$

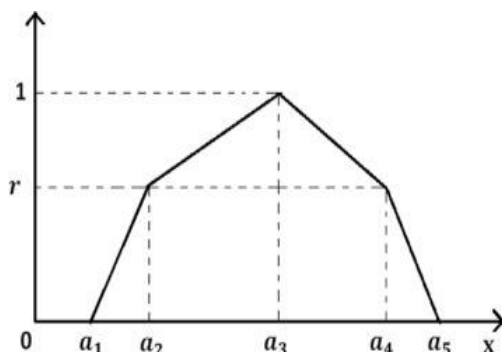


Figure 2: Geometrical Portrayal of Pentagon Fuzzy Number

Arithmetic operations on Pentagonal Fuzzy Number (PFN) [11]

For $A = (a_1, a_2, a_3, a_4, a_5)$ and $B = (b_1, b_2, b_3, b_4, b_5)$, we have

Addition:

$$A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5).$$

Subtraction:

$$A - B = (a_1 - b_5, a_2 - b_4, a_3 - b_3, a_4 - b_2, a_5 - b_1).$$

Multiplication:

$$AB = (a_1 \cdot b_1, a_2 \cdot b_2, a_3 \cdot b_3, a_4 \cdot b_4, a_5 \cdot b_5).$$

Inverse: The inverse of a PFN is characterized when all its components are non-zero. For $A = (a_1, a_2, a_3, a_4, a_5)$, we have

$$A^{-1} = \frac{1}{A} = \left(\frac{1}{a_5}, \frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1} \right)$$

If one of the components of a PFN ends up zero, at that point we cannot find its inverse.

Development of PFN.

The pentagonal fuzzy number is spoken to by the five parameters a_1, a_2, a_3, a_4 and a_5 where a_1 and a_2 signify the smallest conceivable qualities, a_3 the most encouraging worth and a_4, a_5 the biggest conceivable esteem. The pentagonal fuzzy number can be created by utilizing the accompanying equation.

$$A = (a - 2, a - 1, a, a + 1, a + 2), \text{ for each of } a = 3, 4, 5, 6, 7$$

APPRAISAL OF INTERVALS [9]

The two interval numbers designated by ordered pairs of real numbers with lower and upper limits be

$$G = [a_1, a_2], a_1 \leq a_2; \text{ and } H = [b_1, b_2], b_1 \leq b_2$$

The math property is signified generally with the symbol $*$, where $*$ = $[+, -, \times, \div]$ and the activity is characterized by

$$G * H = [a_1, a_2] * [b_1, b_2]$$

Where $[a_1, a_2] + [b_1, b_2] = [a_1 + b_1, a_2 + b_2]$

$$[a_1, a_2] - [b_1, b_2] = [a_1 - b_2, a_2 - b_1]$$

$$[a_1, a_2] * [b_1, b_2] = [\min(a_1, b_1, a_1 \cdot b_1, a_2, b_2, a_2 \cdot b_2), \max(a_1, b_1, a_1 \cdot b_2, a_2, b_1, a_2 \cdot b_2)]$$

$[a_1, a_2] \div [b_1, b_2] = [a_1, a_2] * [1/b_2, 1/b_1]$ provided that 0 does not belong to $[b_1, b_2]$

$\alpha [a_1, a_2] = [\alpha a_1, \alpha a_2]$ for $\alpha > 0$ and $[\alpha a_2, \alpha a_1]$ for $\alpha < 0$

III. SUSPICIONS AND INSCRIPTIONS [3]

3.1 Suspicions

- Queuing Model with multiple server and unlimited capacity.
- Inter arrival time taken Poisson dispersion and overhauled time taken Exponential circulation.
- Arrival(entry) rate, service(overhauled) rate are pentagon fuzzy numbers.

3.2 Inscriptions

λ = Average number of clients arriving per unit of time.

μ = Average number of clients being overhauled per unit of time.

L_s = The average number of clients in the system.

L_q = The Average number of clients holding up in the queue(line).

W_s = Average holding up time of a client in the system.

W_q = Average holding up time of a client in the line.

X = set of the inter arrival time. Y = set of the service time.

A = inter arrival time, S = service times

IV. PROPOSED QUEUING MODEL & IMPLEMENTATION [1]

we propose an unlimited limit population having two or more service facilities in parallel and the service is provided identically with, first started things out served (FCFS) regulation queuing model, indicated as (FM/FM/C) : (∞ /FCFS), where the inter entry time and the overhauled time ensue Poisson and Exponential dispersions with fuzzy parameters λ and μ .

The execution proportions of the multi-server model are given beneath

Expected number of clients in the system

$$L_s = \frac{\lambda \mu \left(\frac{\lambda}{\mu} \right)^c}{(C-1)!(C\mu - \lambda)^2} P_0 + \frac{\lambda}{\mu} (1)$$

Expected number of clients holding up in the queue

$$L_q = \frac{\lambda \mu \left(\frac{\lambda}{\mu} \right)^c}{(C-1)!(C\mu - \lambda)^2} P_0 (2)$$

Average time a client spends in the framework

$$W_s = \frac{\mu \left(\frac{\lambda}{\mu} \right)^c}{(C-1)!(C\mu - \lambda)^2} P_0 + \frac{1}{\mu} (3)$$

Average holding up time of a customer in the line

$$W_q = \frac{\mu \left(\frac{\lambda}{\mu} \right)^c}{(C-1)!(C\mu - \lambda)^2} P_0 (4)$$

$$\text{Where, } P_0 = \frac{1}{\sum_{n=0}^{C-1} \frac{\left(\frac{\lambda}{\mu} \right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu} \right)^c}{c!} \cdot \frac{c\mu}{c\mu - \lambda}} (5)$$

V.RESOLUTION PERSPECTIVE [5]

DSW (Dong, Shah and Wong) is a rough technique utilizes intervals at various α – cut dimensions in characterizing execution proportions. It avoids variation from the output membership function because of use of the segregation reaching on the fuzzy variable area. Any persistent participation capacity can be spoken to by ceaseless scope of α -cut in term from $\alpha=0$ to $\alpha=1$. Let $\mu_{\bar{A}}(a)$, and $\mu_{\bar{S}}(s)$ be membership functions of the inter entry time and the overhauled time, separately. The inter landing time and administration times are fuzzy sets, depicted as:

$$\bar{A} = \{(a, \mu_{\bar{A}}(a)) / a \in X\};$$

$$\bar{S} = \{(s, \mu_{\bar{S}}(s)) / s \in Y\}$$

The α cuts of inter entry time, overhauled time are represented as

$$\bar{A}(\alpha) = \{a \in X / \mu_{\bar{A}}(a) \geq \alpha\};$$

$$\bar{S}(\alpha) = \{s \in Y / \mu_{\bar{S}}(s) \geq \alpha\}$$

Utilizing these α cuts we characterize the membership function as follows:

$$\mu_{P(A, S)}(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{x-b}{c-b}, & b \leq x \leq c \\ 1, & x = c \\ \frac{d-x}{d-c}, & c \leq x \leq d \\ \frac{e-x}{e-d}, & d \leq x \leq e \\ 0, & x > e \end{cases}$$

The DSW calculation comprises the accompanying advances

Stage 1. Stipulate α cut esteem where $0 \leq \alpha \leq 1$.

Stage 2. Discover the intervals in the input membership functions that compare to this α .

Stage3. Utilizing standard binary interval operations, process the interval for the output membership function for the chosen α cut dimension.

Stage4. Iterate stages 1-3 for various estimations of α to finish α cut portrayal of the arrangement.

VI. ANALOGY OF INTERVAL VALUED MODEL [6]

Consider a FM/FM/C queue where both the entry rate and administration rate as pentagonal fuzzy numbers communicated by $\lambda = [11 \ 12 \ 13 \ 14 \ 15]$ and $\mu = [21 \ 22 \ 23 \ 24 \ 25]$. The interval of certainty at probability level α as $[11+2\alpha, 15-2\alpha]$ and $[21+2\alpha, 25-2\alpha]$. where $x = [11+2\alpha, 15-2\alpha]$ & $y = [21+2\alpha, 25-2\alpha]$.

Pentagon fuzzy number with graphical portrayal

By taking α values from 0, 0.1, . . . 1 the accompanying outcomes are acquired as appeared in Table1 and Table 2.

Table 1: L_s and L_q for different values of α cuts

A	L_s	L_q
0	[0.4411, 0.7334]	[0.0011, 0.0191]
0.1	[0.4529, 0.7148]	[0.0013, 0.1067]
0.2	[0.4649, 0.6968]	[0.0015, 0.0145]
0.3	[0.4772, 0.6794]	[0.0018, 0.0127]
0.4	[0.4896, 0.6625]	[0.0020, 0.0111]
0.5	[0.5024, 0.6460]	[0.0024, 0.0097]
0.6	[0.5153, 0.6301]	[0.0027, 0.0084]
0.7	[0.5286, 0.6145]	[0.0032, 0.0073]
0.8	[0.5421, 0.5993]	[0.0036, 0.0064]
0.9	[0.5559, 0.5845]	[0.0042, 0.0056]
1	[0.5701, 0.5701]	[0.0048, 0.0048]

Table 2: W_s and W_q for different values of α cuts

A	W_s	W_q
0	[0.0401, 0.0488]	[0.0001, 0.0012]
0.1	[0.0404, 0.0482]	[0.0001, 0.0011]
0.2	[0.0407, 0.0477]	[0.0001, 0.0009]
0.3	[0.0411, 0.0471]	[0.0001, 0.0008]
0.4	[0.0415, 0.0466]	[0.0001, 0.0007]
0.5	[0.0418, 0.0461]	[0.0002, 0.0006]
0.6	[0.0422, 0.0456]	[0.0002, 0.0006]
0.7	[0.0426, 0.0451]	[0.0002, 0.0005]
0.8	[0.0430, 0.0447]	[0.0002, 0.0004]
0.9	[0.0434, 0.0442]	[0.0003, 0.0004]
1	[0.0438, 0.0438]	[0.0003, 0.0003]

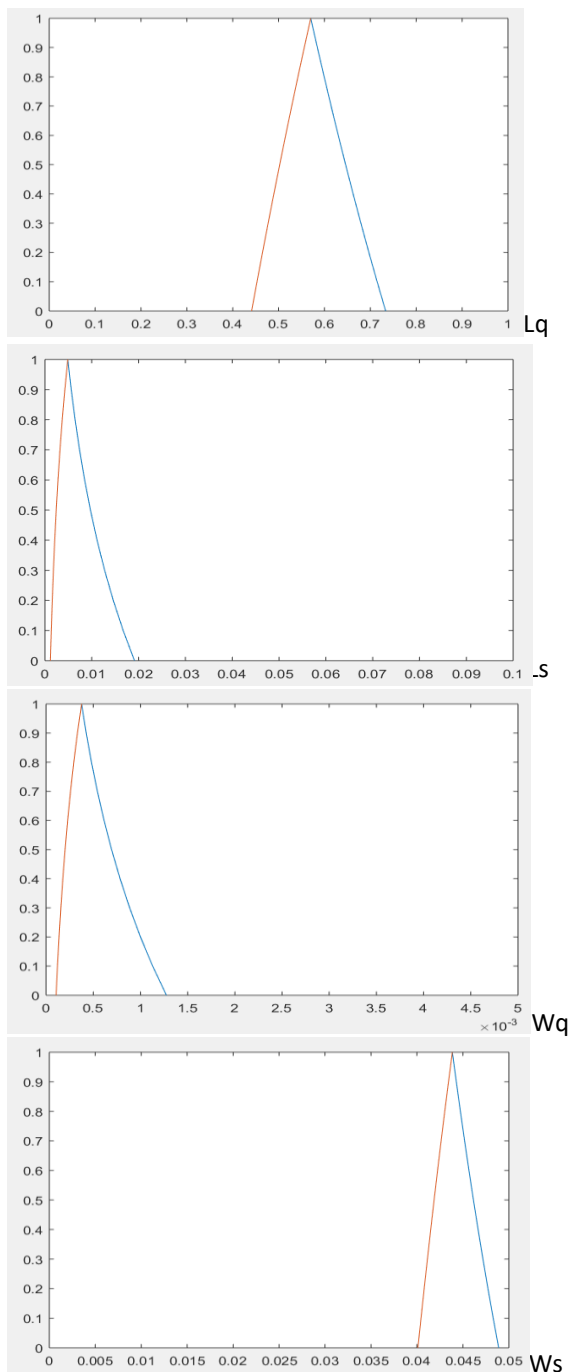


Figure 3: Plotting of Various Performance Measures versus α -cut Values

VII. RESULTS AND DISCUSSIONS

Utilizing MATLAB, we achieve α cuts of entry rate, administration rate and fuzzy anticipated number of occupations in queue just as system at eleven patent dimensions: 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1. Crisp intervals for fuzzy anticipated number of employments in line and framework at various probability α levels are exhibited in Table-1, 2 and 3. The execution proportions for example anticipated number of clients in the system (L_s), anticipated length of the line (L_q), the average holding up time of a client in the system (W_s) and the average holding up time of a client in the queue (W_q) likewise inferred in Tables 1-2.

For instance, the most likely estimation of the anticipated queue length L_q , falls at 0.0048 and its value is

unsustainable to fall outside the scope of 0.0011 and 0.0191 and for the system L_s , falls at 0.5701 and its value is difficult to fall outside the scope of 0.4411 and 0.7334. so also, for average holding up time in system W_s , the most likely esteem is 0.0438 and it is unsustainable to fall outside the scope 0.0401 and 0.0488 and for the line W_q , the most likely esteem is 0.0003 and it is unsustainable to fall outside the range 0.0001 and 0.0012

These above outcomes are extremely valuable for characterizing a lining framework and for decision process.

VIII. CONCLUSION

In this paper, the idea of pentagon fuzzy number has been examined with interval math operations to FM/FM/C for example multi server lining model. The execution proportions of FM/FM/C miniature with pentagon fuzzy number are given with the help of numerical precedent by utilising α cut tasks. The inter entry time and administration times are fuzzy nature. The execution of this framework is likewise fuzzy nature.

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